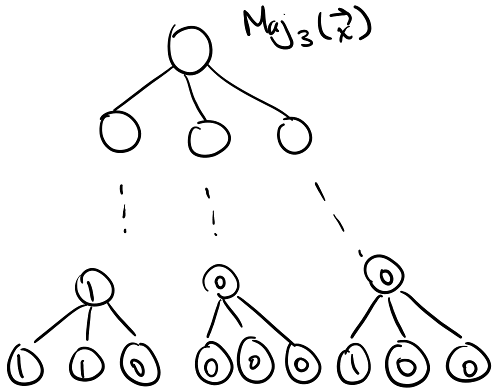
6 Tutorial

Thursday, October 17, 2019 3:13 PM

① Randomized Maj_3



Bottom layer $\vec{x}$

Problem: Construct randomized algorithm such that in expectation the algorithm reads c^h bits for $c < 3$.

```
def Maj(u):
    x1, x2 = Maj(pick 2 from {l, m, r})
    if x1 = x2:
        return x1
    x3 <- Maj(not picked)
    return mode(x1, x2, x3)
```

$$T(n) = \underbrace{P(x_1 = x_2)}_{\frac{1}{2}}\big(2T(n-1)\big) + \big(1 - \underbrace{P(x_1 = x_2)}_{\frac{1}{2}}\big)\big(3T(n-1)\big)$$

$c = 2.5$

② Reservoir Sampling

Let $p_2, \dots, p_n \in (0,1)$

```
def SAMPLE(p2, ..., pn):
    x = 1
    for i = 2...n
        x = i   if heads when
                flipping coin with
                prob. pi
    return x
```

a) If $p_2, \dots, p_n = \frac{1}{2}$, calculate $\forall i \in [n]$ $P[x = i \text{ as output}]$

b) Want $P[x = i] = \frac{1}{n}$ $\forall i \in [n]$. Find values $P_2, \dots, P_n$

a) $P[x = i] = \dfrac{1}{2^{n-i}}$

b) $P_n = \frac{1}{n}$. $P_{n-1} = \frac{1}{n}(1 - P_n)$

$$P_i = 1/i$$

③ Biased → Unbiased Coin

Given coin such that $P[\text{heads}] = p$ and $P[\text{tails}] = 1-p$

Problem: Come up with UNBIASED() which returns H/T with $p = \frac{1}{2}$.
Show your algorithm runs in time $\text{poly}\left(\frac{1}{p(1-p)}\right)$