4 Tutorial

Thursday, October 3, 2019 3:12 PM

Augmented BSTs (Interval Trees)

Properties:

- Each node stores an interval $[a,b]$.

  Note: $[a,b] \leq [c,d]$ if $a < c$ or $(a = c$ and $b \leq d)$

- Stores maximum right endpoint in the subtree

Operations:

- Insertion / Deletion
- Intersection Input: $[a,b]$

  Output: Interval in a tree that intersects $[a,b]$

Intersection $(v, [a,b])$:

① If $v \cap [a,b] \neq \emptyset$: return v

② if v.left.max $\geq a$:

  return Intersection(v.left, $[a,b]$)

return Intersection(v.right, $[a,b]$)

Either  or 

Q2: Find the smallest $[c,d]$

Do ② first, then ①. (by BST property, left children smaller)

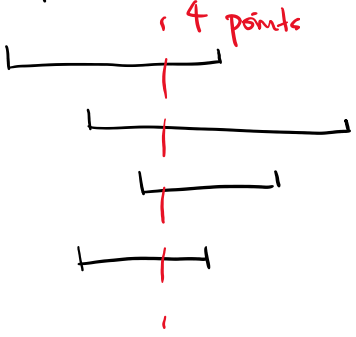
Q3: Find $[a,b]$ if it is in the tree

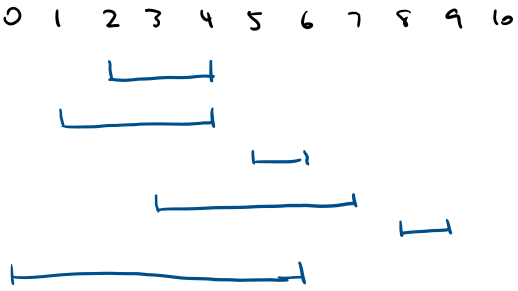
Find interval where left endpoint is a. After, search in the right subtree.

Q4: Given tree, find the max points of intersections

Do in $O(n \log n)$

Example:

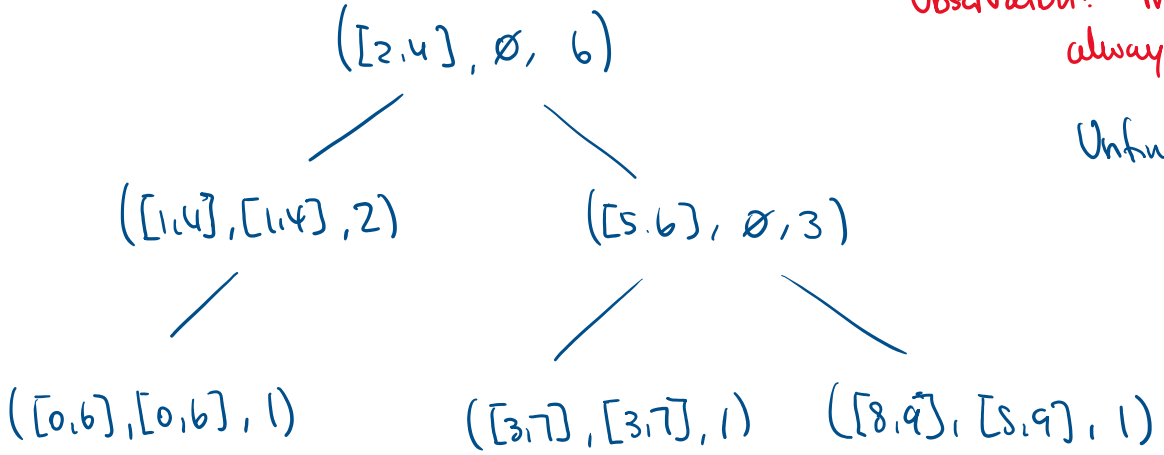




Size and smallest intersection

Observation: max intersection always at endpoints.

(Unfinished)



Q5: Rectangle Intersection

Input: Set of rectangles $\{(x_1, x_2, y_1, y_2)\}$

Output: Are there 2 that intersect