A Tutorial on Energy-Based Learning

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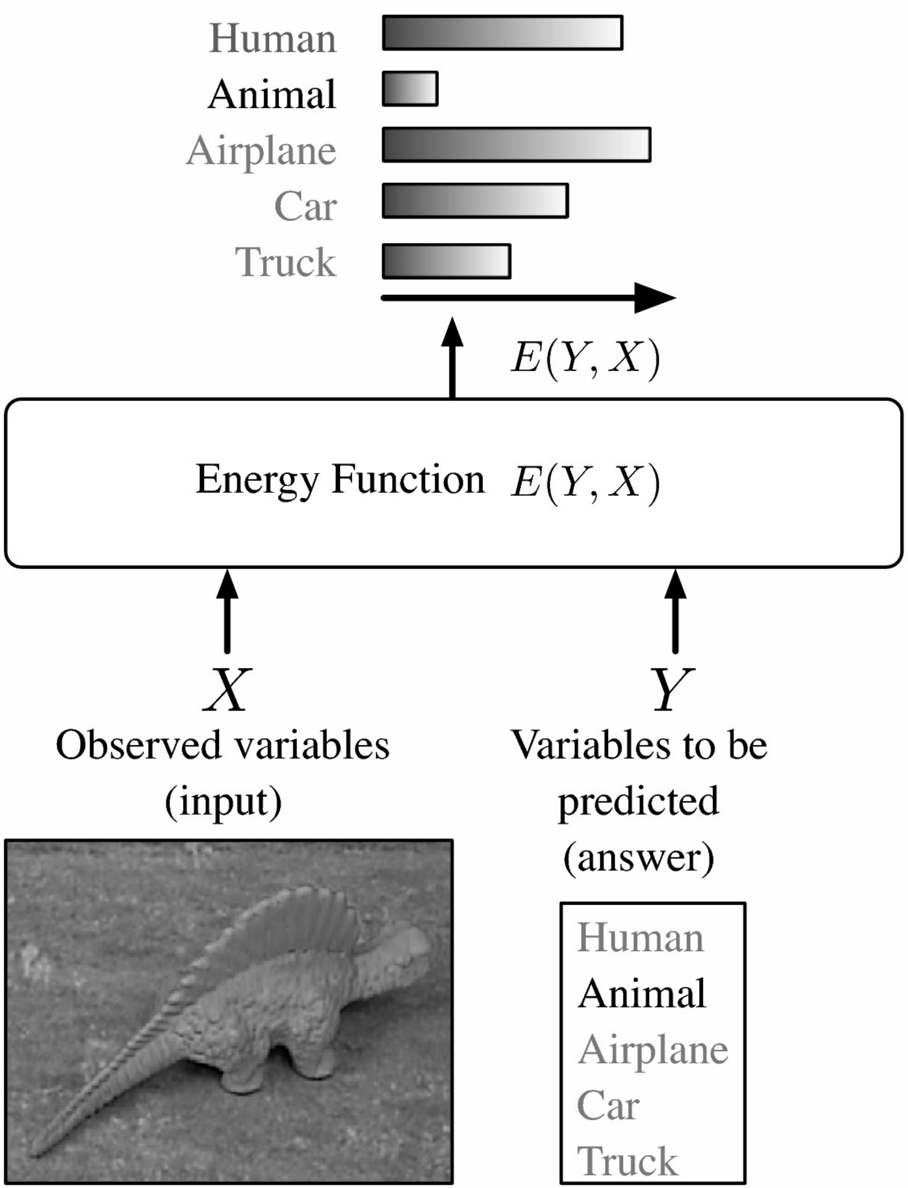
Two Problems in Machine Learning

- **1. The "Deep Learning Problem"**
  - "Deep" architectures are necessary to solve the invariance problem in vision (and perception in general)
  - How do we train deep architectures with lots of non-linear stages
- **2. The "Partition Function Problem"**
  - Give high probability (or low energy) to good answers
  - Give low probability (or high energy) to bad answers
  - There are too many bad answers!
- **This tutorial discusses problem #2**
  - The partition function problem arises with probabilistic approaches
  - Non-probabilistic approaches may allow us to get around it.
- **Energy-Based Learning provides a framework in which to describe probabilistic and non-probabilistic approaches to learning**

Energy-Based Model for Decision-Making

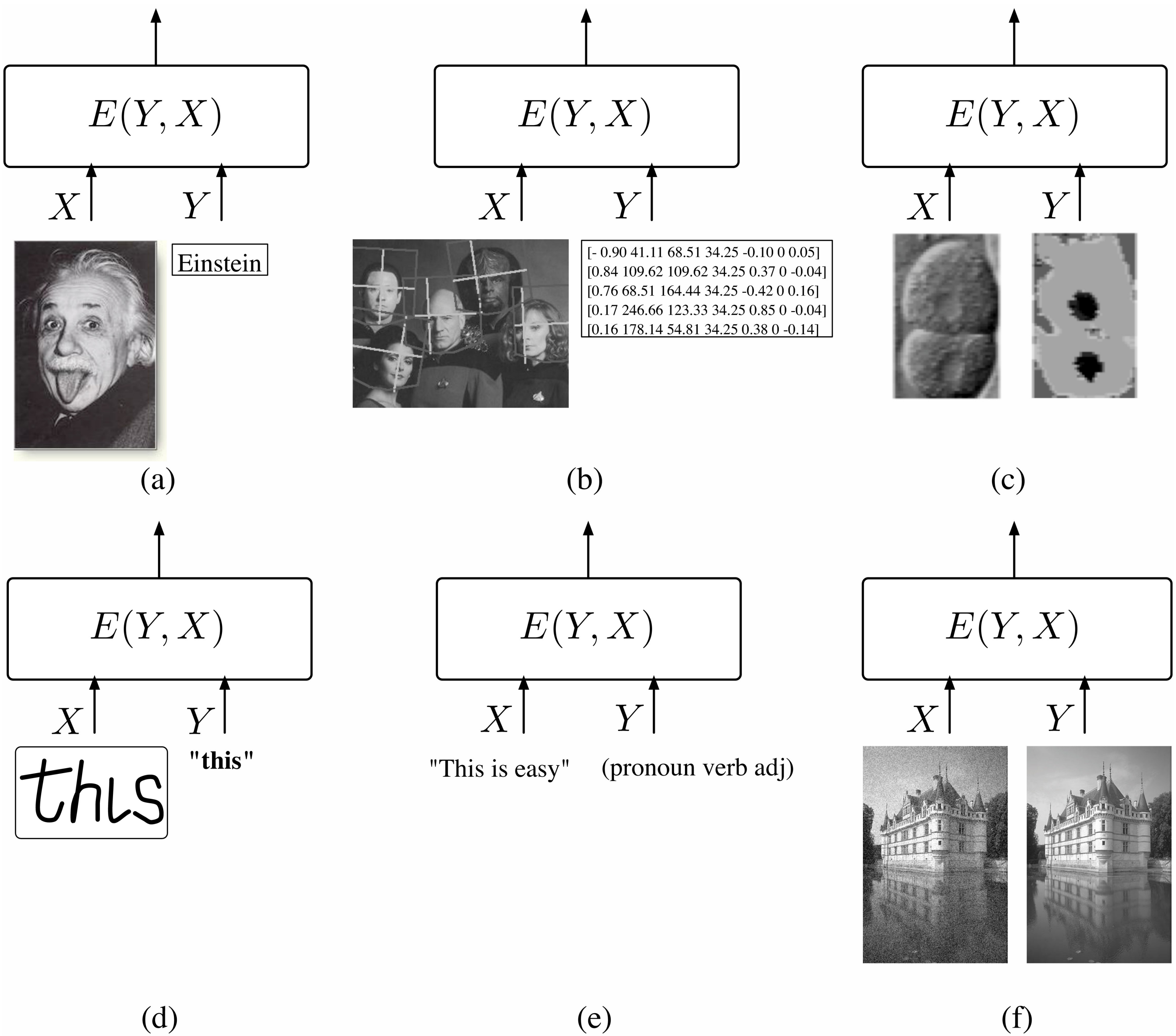


- **Model:** Measures the compatibility between an observed variable X and a variable to be predicted Y through an energy function E(Y,X).

$$Y^* = \text{argmin}_{Y \in \mathcal{Y}} E(Y, X).$$

- **Inference:** Search for the Y that minimizes the energy within a set $\mathcal{Y}$
- If the set has low cardinality, we can use exhaustive search.

Complex Tasks: Inference is non-trivial



- **When the cardinality or dimension of Y is large, exhaustive search is impractical.**
- **We need to use a "smart" inference procedure: min-sum, Viterbi, ...**

What Questions Can a Model Answer?

1. Classification & Decision Making:

- “which value of Y is most compatible with X?”
- Applications: Robot navigation,.....
- Training: give the lowest energy to the correct answer

2. Ranking:

- “Is Y1 or Y2 more compatible with X?”
- Applications: Data-mining....
- Training: produce energies that rank the answers correctly

3. Detection:

- “Is this value of Y compatible with X”?
- Application: face detection....
- Training: energies that increase as the image looks less like a face.

4. Conditional Density Estimation:

- “What is the conditional distribution P(Y|X)?”
- Application: feeding a decision-making system
- Training: differences of energies must be just so.

Decision-Making versus Probabilistic Modeling

Energies are uncalibrated

- The energies of two separately-trained systems cannot be combined
- The energies are uncalibrated (measured in arbitrary untis)

How do we calibrate energies?

- We turn them into probabilities (positive numbers that sum to 1).
- Simplest way: Gibbs distribution
- Other ways can be reduced to Gibbs by a suitable redefinition of the energy.

$$P(Y|X) = \frac{e^{-\beta E(Y,X)}}{\int_{y \in \mathcal{Y}} e^{-\beta E(y,X)}},$$

Partition function

Inverse temperature

Architecture and Loss Function

- **Family of energy functions** $\mathcal{E} = \{E(W,Y,X) : \ W \in \mathcal{W}\}.$
- **Training set** $\mathcal{S} = \{(X^i, Y^i) : i = 1 \ldots P\}$
- **Loss functional / Loss function** $\mathcal{L}(E, \mathcal{S})$ $\mathcal{L}(W, \mathcal{S})$
  - Measures the quality of an energy function
- **Training** $W^* = \min_{W \in \mathcal{W}} \mathcal{L}(W, \mathcal{S}).$
- **Form of the loss functional**
  - invariant under permutations and repetitions of the samples

$$\mathcal{L}(E, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} L(Y^i, E(W, \mathcal{Y}, X^i)) + R(W).$$

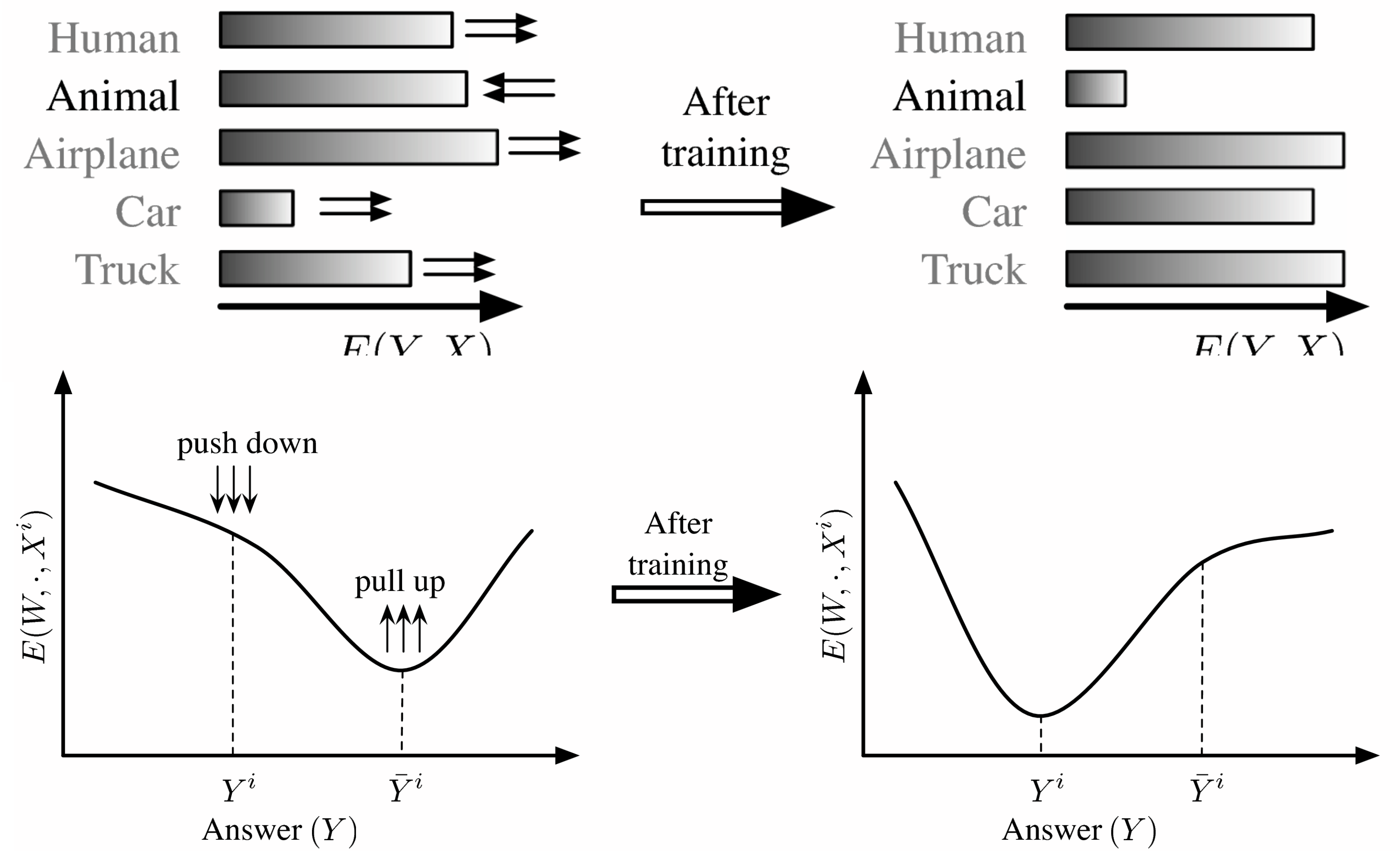
Per-sample loss

Desired answer

Energy surface for a given Xi as Y varies

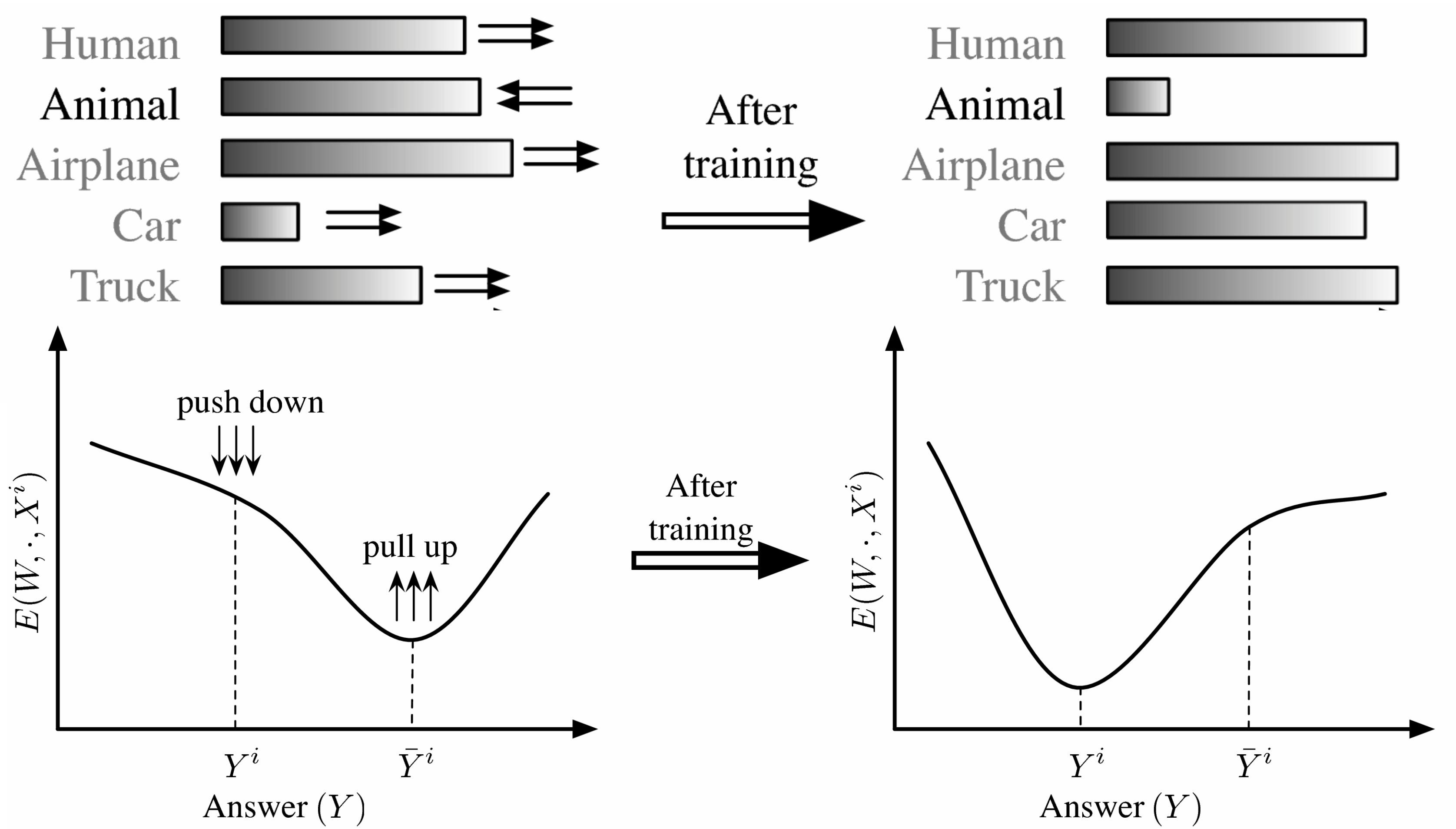
Regularizer

Designing a Loss Functional



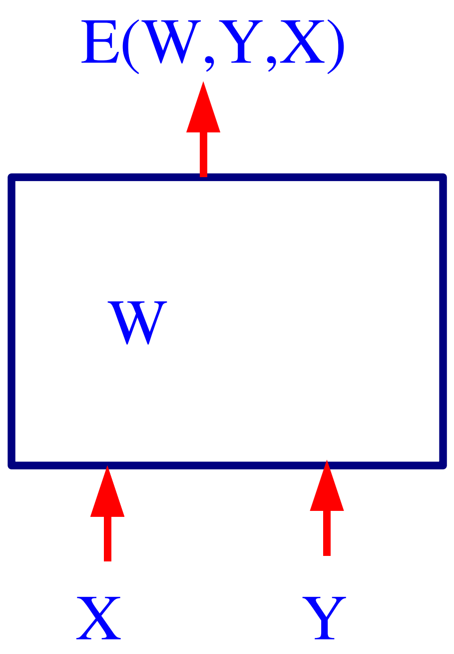
- **Correct answer has the lowest energy -> LOW LOSS**
- **Lowest energy is not for the correct answer -> HIGH LOSS**

Designing a Loss Functional



- **Push down on the energy of the correct answer**
- **Pull up on the energies of the incorrect answers, particularly if they are smaller than the correct one**

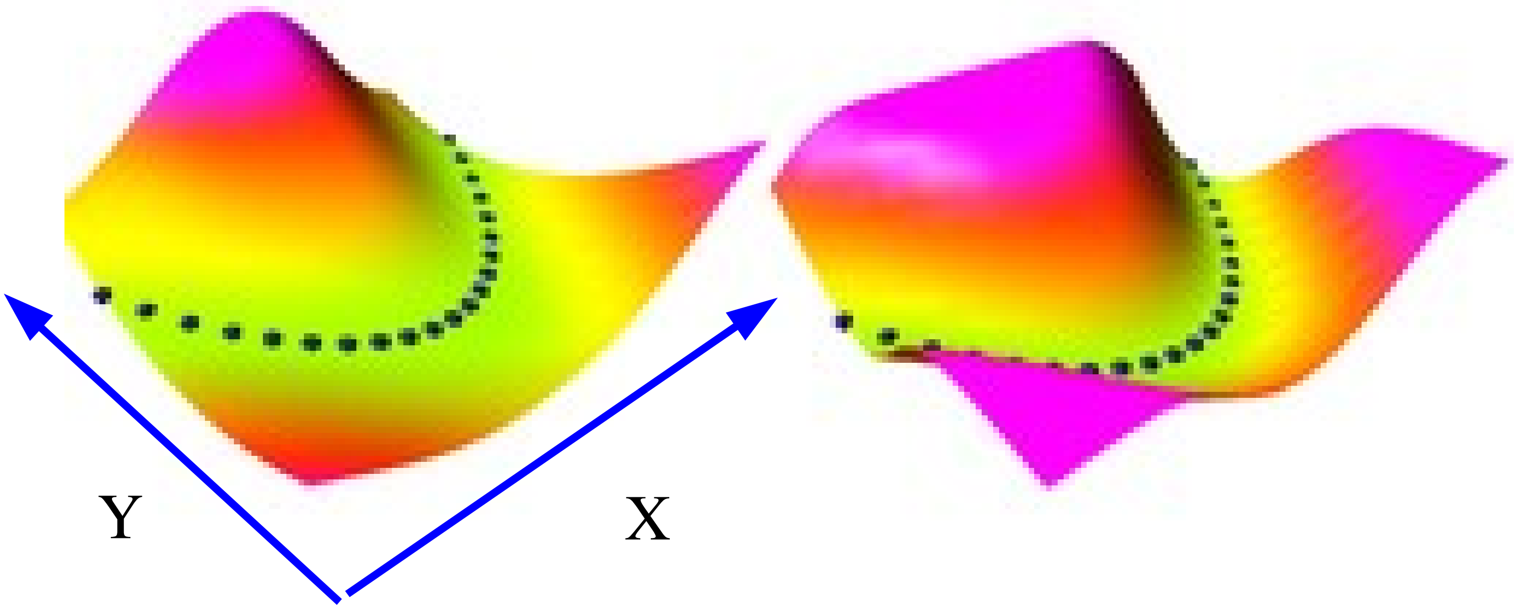
Architecture + Inference Algo + Loss Function = Model



- **1. Design an architecture:** a particular form for E(W,Y,X).
- **2. Pick an inference algorithm for Y:** MAP or conditional distribution, belief prop, min cut, variational methods, gradient descent, MCMC, HMC.....
- **3. Pick a loss function:** in such a way that minimizing it with respect to W over a training set will make the inference algorithm find the correct Y for a given X.
- **4. Pick an optimization method.**

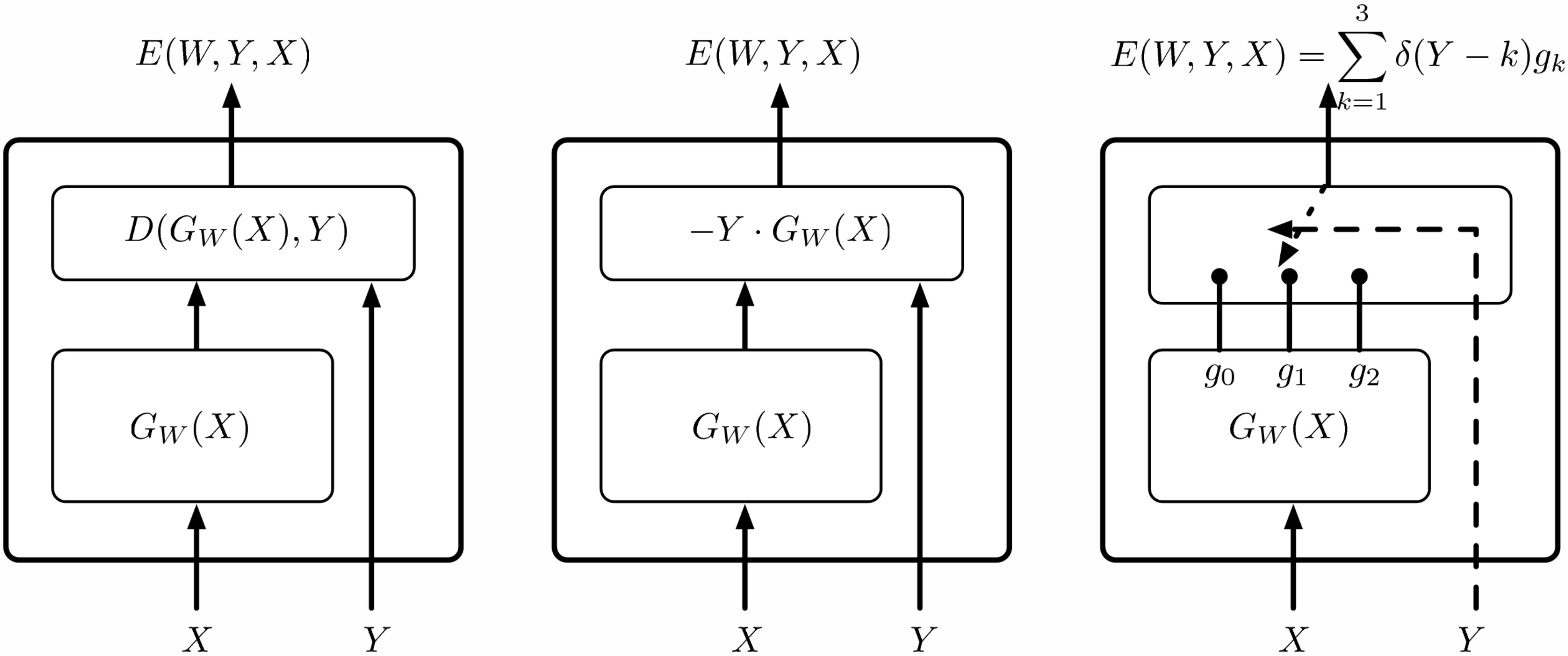
- **PROBLEM: What loss functions will make the machine approach the desired behavior?**

Several Energy Surfaces can give the same answers



- Both surfaces compute Y=X^2
- MINy E(Y,X) = X^2
- Minimum-energy inference gives us the same answer

Simple Architectures



- **Regression**

$$E(W,Y,X) = \frac{1}{2}||G_W(X) - Y||^2.$$

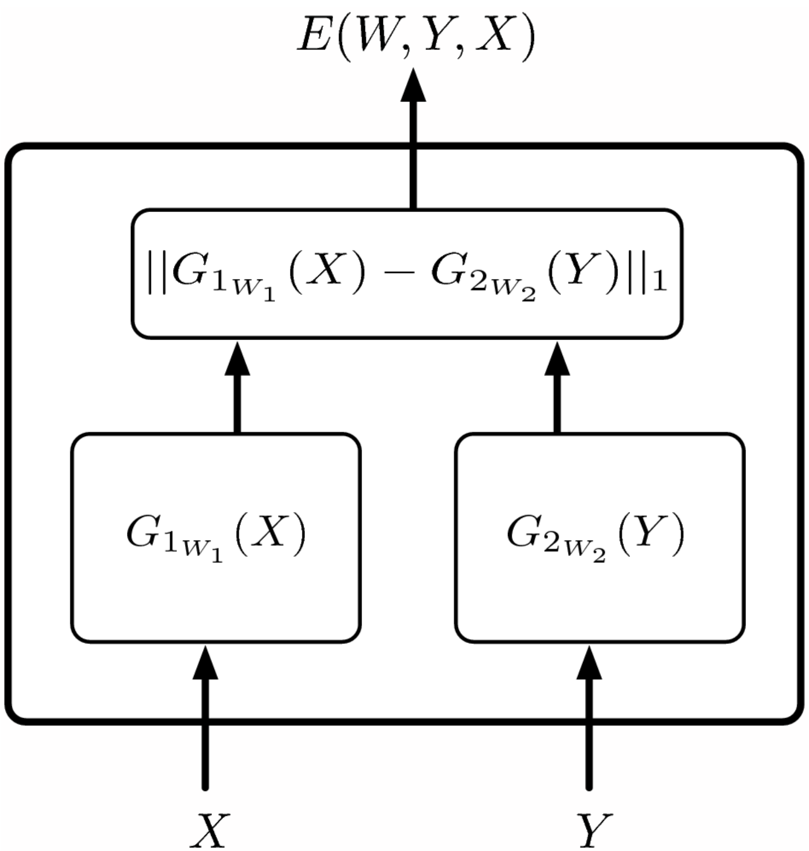
- **Binary Classification**

$$E(W,Y,X) = -YG_W(X),$$

- **Multi-class Classification**

Simple Architecture: Implicit Regression

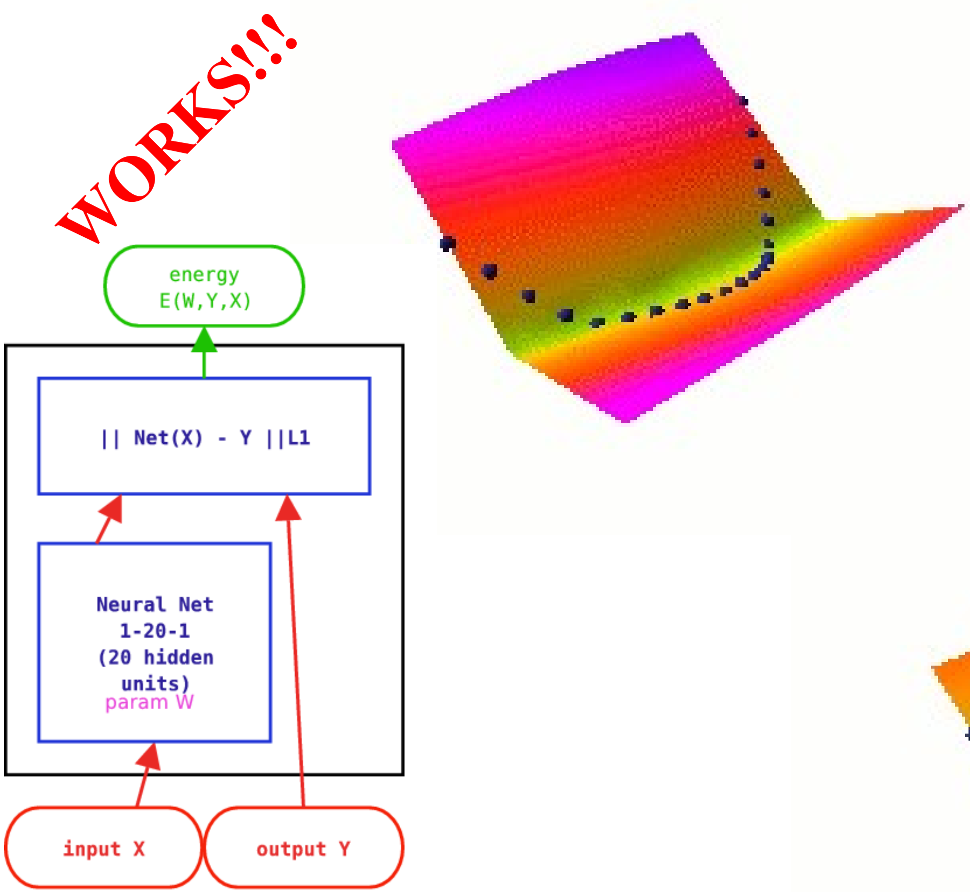
$$E(W,X,Y) = ||G_{1_{W_1}}(X) - G_{2_{W_2}}(Y)||_1,$$



- **The Implicit Regression architecture**
  - allows multiple answers to have low energy.
  - Encodes a constraint between X and Y rather than an explicit functional relationship
  - This is useful for many applications
  - Example: sentence completion: “The cat ate the {mouse,bird,homework,...}”
  - [Bengio et al. 2003]
  - But, inference may be difficult.

Examples of Loss Functions: Energy Loss

- **Energy Loss** $$L_{energy}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i).$$
  - Simply pushes down on the energy of the correct answer



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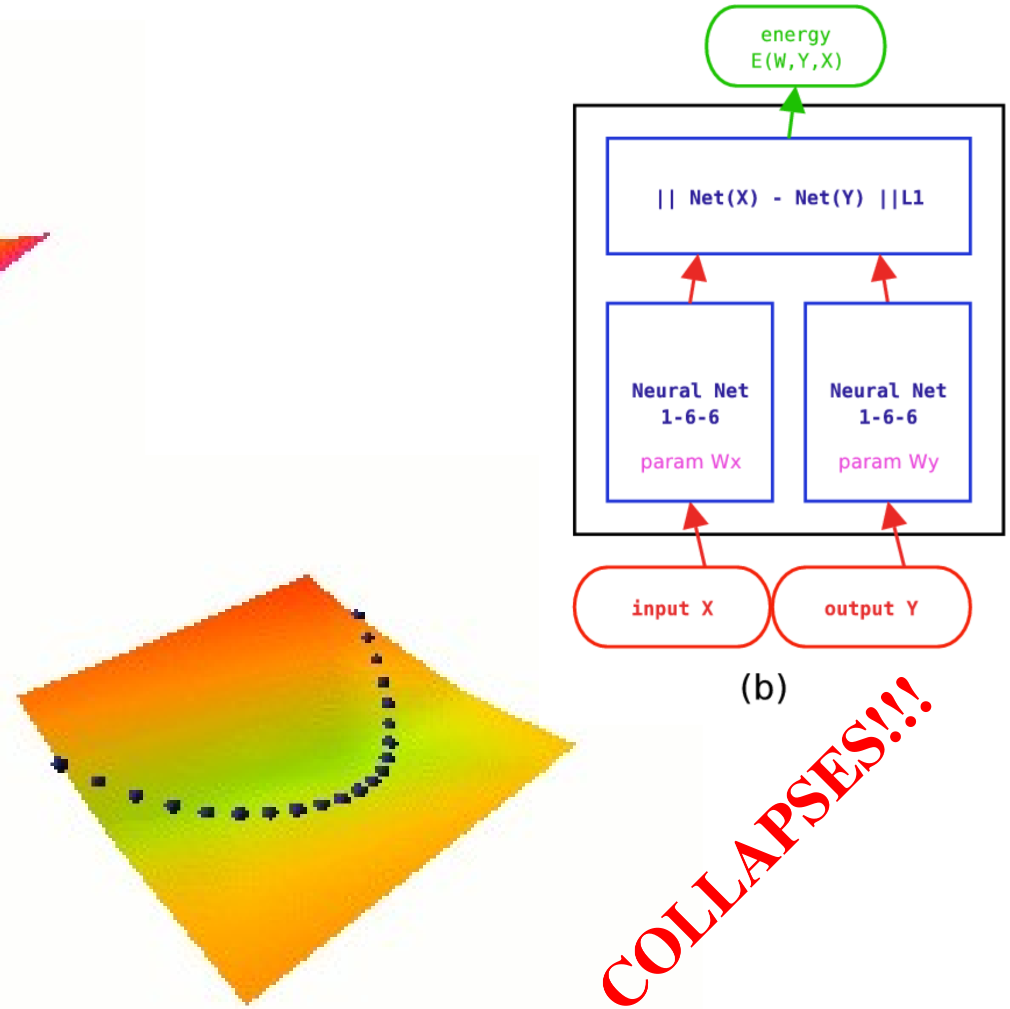


(b)

Examples of Loss Functions:Perceptron Loss

$$L_{perceptron}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i).$$

- **Perceptron Loss [LeCun et al. 1998], [Collins 2002]**
  - Pushes down on the energy of the correct answer
  - Pulls up on the energy of the machine's answer
  - Always positive. Zero when answer is correct
  - No “margin”: technically does not prevent the energy surface from being almost flat.
  - Works pretty well in practice, particularly if the energy parameterization does not allow flat surfaces.

Perceptron Loss for Binary Classification

$$L_{perceptron}(Y^i, E(W, \mathcal{Y}, X^i)) = E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i).$$

- **Energy:** $E(W, Y, X) = -Y G_W(X),$
- **Inference:** $Y^* = \text{argmin}_{Y \in \{-1,1\}} - Y G_W(X) = \text{sign}(G_W(X)).$
- **Loss:** $\mathcal{L}_{\text{perceptron}}(W, \mathcal{S}) = \frac{1}{P} \sum_{i=1}^{P} \left(\text{sign}(G_W(X^i)) - Y^i\right) G_W(X^i).$
- **Learning Rule:** $W \leftarrow W + \eta \left(Y^i - \text{sign}(G_W(X^i))\right) \frac{\partial G_W(X^i)}{\partial W},$
- **If Gw(X) is linear in W:** $E(W, Y, X) = -Y W^T \Phi(X)$

$$W \leftarrow W + \eta \left(Y^i - \text{sign}(W^T \Phi(X^i))\right) \Phi(X^i)$$

Examples of Loss Functions: Generalized Margin Losses

- **First, we need to define the Most Offending Incorrect Answer**

- **Most Offending Incorrect Answer: discrete case**

Definition 1 *Let Y be a discrete variable. Then for a training sample (X^i, Y^i), the **most offending incorrect answer** $\bar{Y}^i$ is the answer that has the lowest energy among all answers that are incorrect:*

$$\bar{Y}^i = \text{argmin}_{Y \in \mathcal{Y} and Y \neq Y^i} E(W, Y, X^i). \tag{8}$$

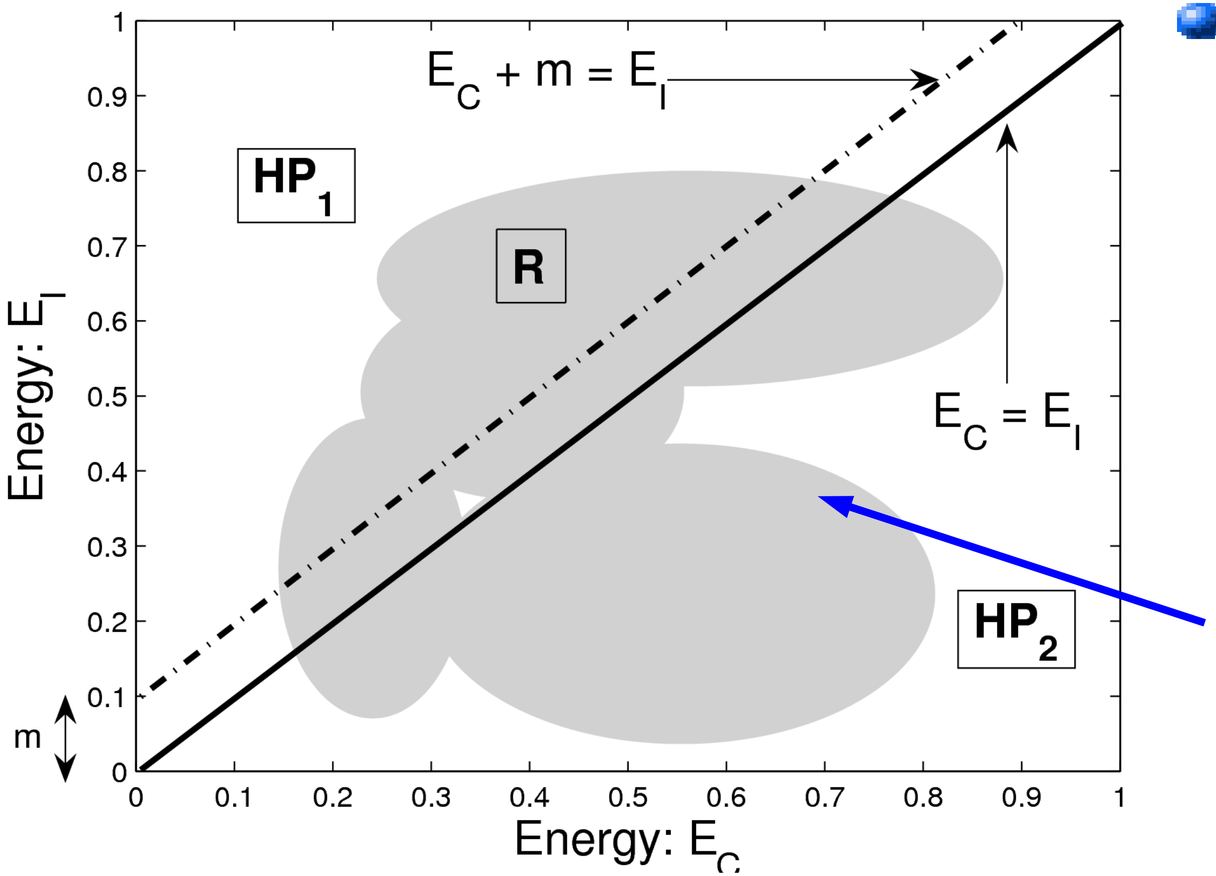
- **Most Offending Incorrect Answer: continuous case**

Definition 2 *Let Y be a continuous variable. Then for a training sample (X^i, Y^i), the **most offending incorrect answer** $\bar{Y}^i$ is the answer that has the lowest energy among all answers that are at least ϵ away from the correct answer:*

$$\bar{Y}^i = \text{argmin}_{Y \in \mathcal{Y}, \|Y - Y^i\| > \epsilon} E(W, Y, X^i). \tag{9}$$

Examples of Loss Functions: Generalized Margin Losses

$$L_{\text{margin}}(W, Y^i, X^i) = Q_m\left(E(W, Y^i, X^i), E(W, \bar{Y}^i, X^i)\right).$$

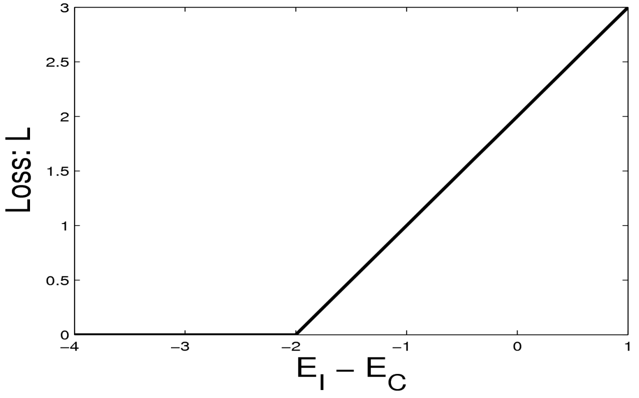


Generalized Margin Loss

- Qm increases with the energy of the correct answer
- Qm decreases with the energy of the most offending incorrect answer
- whenever it is less than the energy of the correct answer plus a margin m.

Examples of Generalized Margin Losses

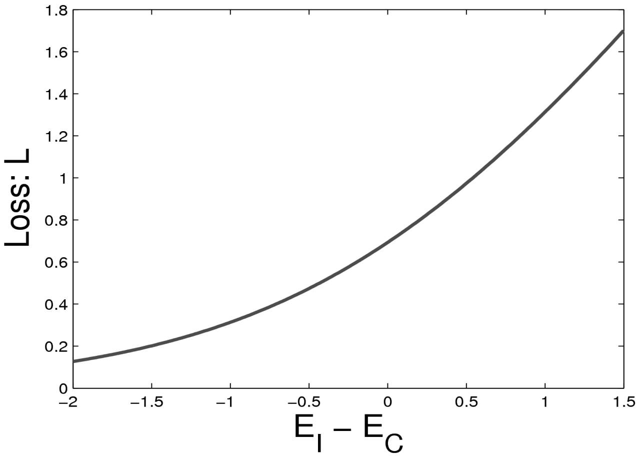
$$L_{\text{hinge}}(W, Y^i, X^i) = \max\left(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right),$$



Hinge Loss

- [Altun et al. 2003], [Taskar et al. 2003]
- With the linearly-parameterized binary classifier architecture, we get linear SVM

$$L_{\log}(W, Y^i, X^i) = \log\left(1 + e^{E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)}\right).$$



Log Loss

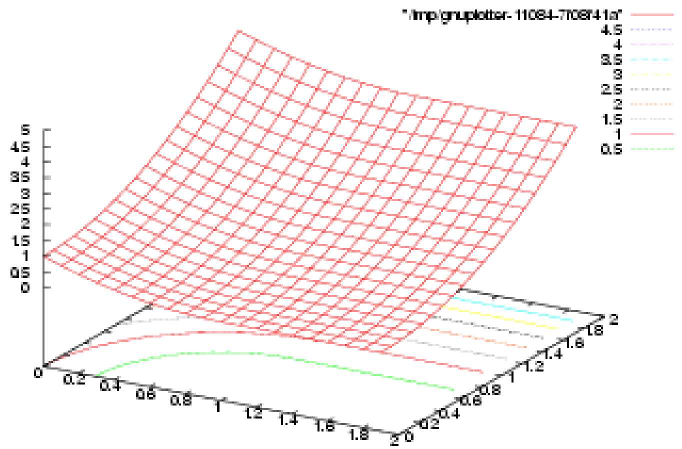
- "soft hinge" loss
- With the linearly-parameterized binary classifier architecture, we get linear Logistic Regression

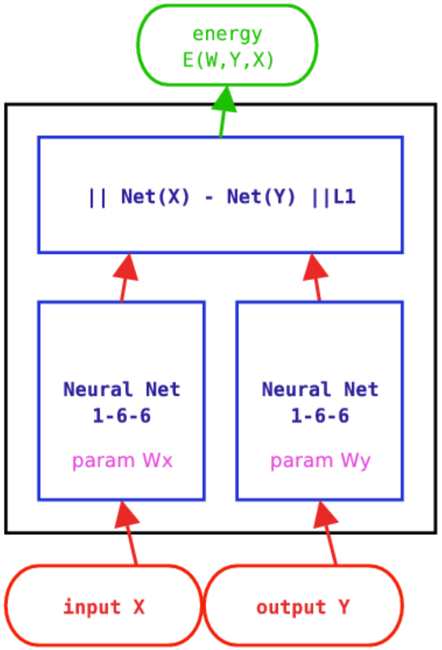
Examples of Margin Losses: Square-Square Loss

$$L_{\mathrm{sq-sq}}(W, Y^i, X^i) = E(W, Y^i, X^i)^2 + \left(\max(0, m - E(W, \bar{Y}^i, X^i))\right)^2.$$

Square-Square Loss

- [LeCun-Huang 2005]
- Appropriate for positive energy functions





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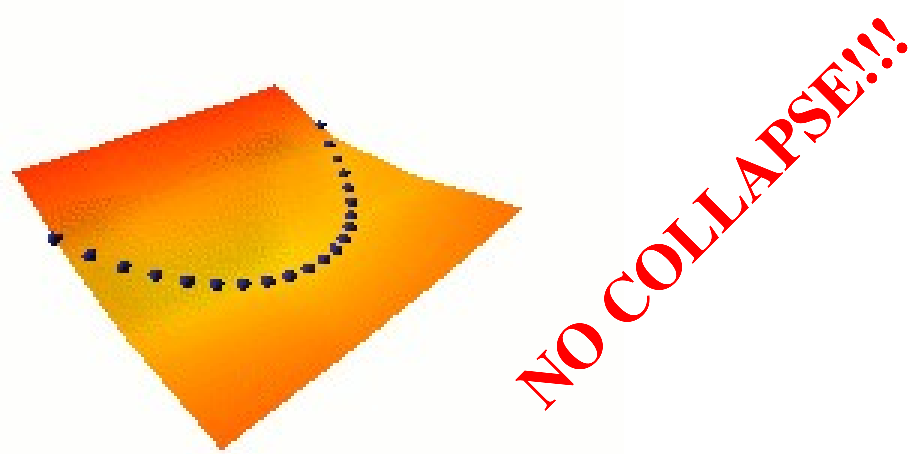
Learning Y = X^2



Other Margin-Like Losses

- **LVQ2 Loss** [Kohonen, Oja], Driancourt-Bottou 1991]

$$L_{\text{lvq2}}(W, Y^i, X^i) = \min\left(1, \max\left(0, \frac{E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)}{\delta E(W, \bar{Y}^i, X^i)}\right)\right),$$

- **Minimum Classification Error Loss** [Juang, Chou, Lee 1997]

$$L_{\text{mce}}(W, Y^i, X^i) = \sigma\left(E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right),$$

$$\sigma(x) = (1 + e^{-x})^{-1}$$

- **Square-Exponential Loss** [Osadchy, Miller, LeCun 2004]

$$L_{\text{sq-exp}}(W, Y^i, X^i) = E(W, Y^i, X^i)^2 + \gamma e^{-E(W, \bar{Y}^i, X^i)},$$

Negative Log-Likelihood Loss

- **Conditional probability of the samples (assuming independence)**

$$P(Y^1,\ldots,Y^P|X^1,\ldots,X^P,W) = \prod_{i=1}^{P} P(Y^i|X^i,W).$$

$$-\log \prod_{i=1}^{P} P(Y^i|X^i,W) = \sum_{i=1}^{P} -\log P(Y^i|X^i,W).$$

- **Gibbs distribution:** $$P(Y|X^i,W) = \frac{e^{-\beta E(W,Y,X^i)}}{\int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}}.$$

$$-\log \prod_{i=1}^{P} P(Y^i|X^i,W) = \sum_{i=1}^{P} \beta E(W,Y^i,X^i) + \log \int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}.$$

- **We get the NLL loss by dividing by P and Beta:**

$$\mathcal{L}_{\rm nll}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\left(E(W,Y^i,X^i) + \frac{1}{\beta}\log \int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}\right).$$

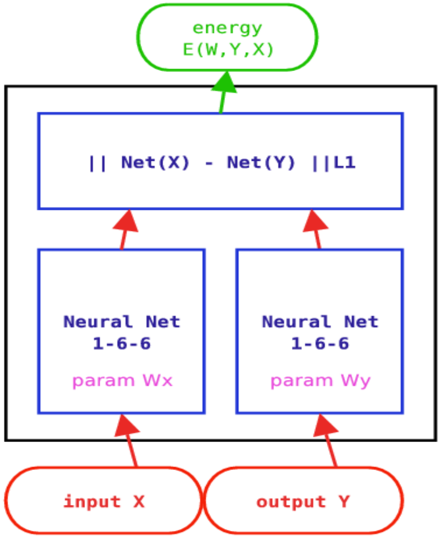
- **Reduces to the perceptron loss when Beta->infinity**

Negative Log-Likelihood Loss

- **Pushes down on the energy of the correct answer**
- **Pulls up on the energies of all answers in proportion to their probability**

$$\mathcal{L}_{\text{nll}}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\left(E(W,Y^i,X^i) + \frac{1}{\beta}\log\int_{y\in\mathcal{Y}} e^{-\beta E(W,y,X^i)}\right).$$

$$\frac{\partial L_{\text{nll}}(W,Y^i,X^i)}{\partial W} = \frac{\partial E(W,Y^i,X^i)}{\partial W} - \int_{Y\in\mathcal{Y}} \frac{\partial E(W,Y,X^i)}{\partial W} P(Y|X^i,W),$$



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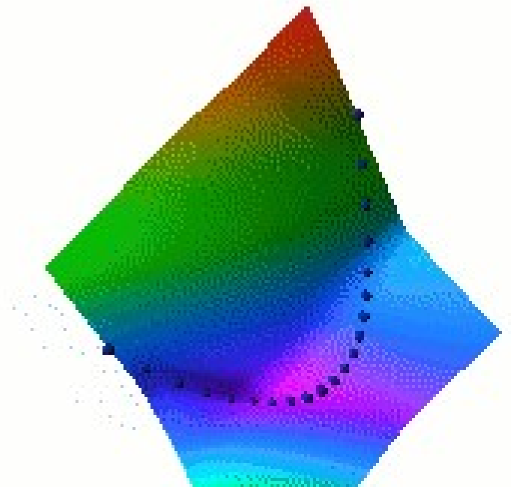


Negative Log-Likelihood Loss: Binary Classification

- **Binary Classifier Architecture:**

$$\mathcal{L}_{\text{nll}}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\left[-Y^i G_W(X^i) + \log\left(e^{Y^i G_W(X^i)} + e^{-Y^i G_W(X^i)}\right)\right].$$

$$\mathcal{L}_{\text{nll}}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\log\left(1 + e^{-2Y^i G_W(X^i)}\right),$$

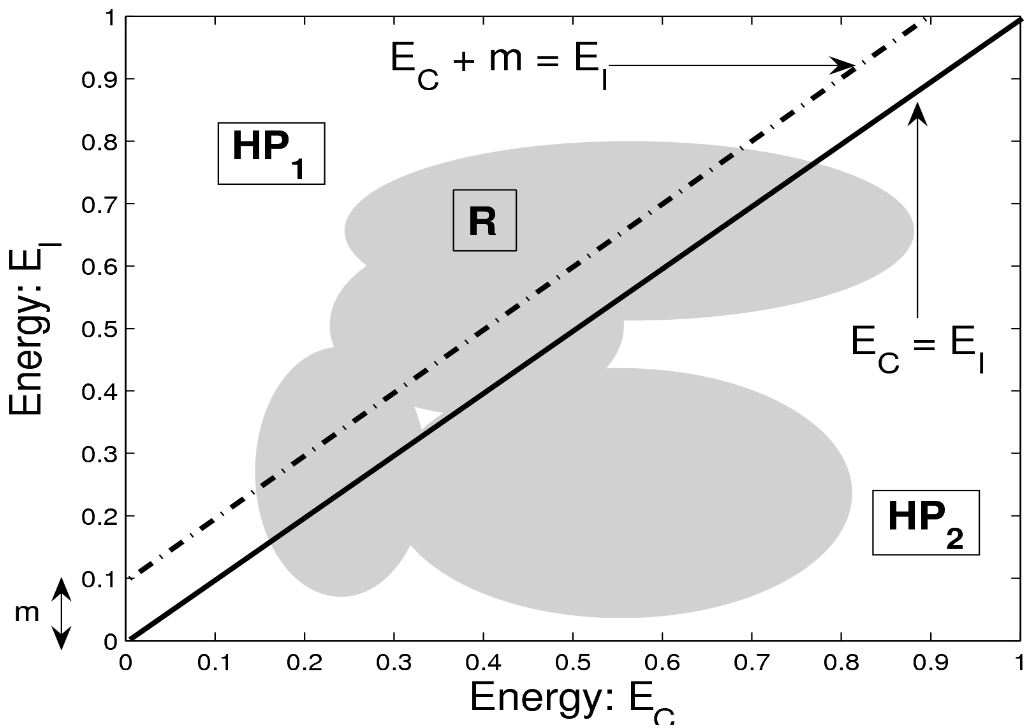
- **Linear Binary Classifier Architecture:**

$$\mathcal{L}_{\text{nll}}(W,\mathcal{S}) = \frac{1}{P}\sum_{i=1}^{P}\log\left(1 + e^{-2Y^i W^T \Phi(X^i)}\right).$$

- **Learning Rule: logistic regression**

What Makes a "Good" Loss Function

- **Good loss functions make the machine produce the correct answer**
  - Avoid collapses and flat energy surfaces



Sufficient Condition on the Loss

Let (X^i, Y^i) be the i^{th} training example and m be a positive margin. Minimizing the loss function L will cause the machine to satisfy $E(W, Y^i, X^i) < E(W, Y, X^i) - m$ for all $Y \neq Y^i$, if there exists at least one point (e_1, e_2) with $e_1 + m < e_2$ such that for all points (e'_1, e'_2) with $e'_1 + m \geq e'_2$, we have

$$Q_{[E_y]}(e_1, e_2) < Q_{[E_y]}(e'_1, e'_2),$$

where $Q_{[E_y]}$ is given by

$$L(W, Y^i, X^i) = Q_{[E_y]}(E(W, Y^i, X^i), E(W, \bar{Y}^i, X^i)).$$

What Make a “Good” Loss Function

Good and bad loss functions

Loss (equation #)	Formula	Margin
energy loss	$E(W, Y^i, X^i)$	none
perceptron	$E(W, Y^i, X^i) - \min_{Y \in \mathcal{Y}} E(W, Y, X^i)$	0
hinge	$\max\left(0, m + E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)$	m
log	$\log\left(1 + e^{E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)}\right)$	> 0
LVQ2	$\min\left(M, \max(0, E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)$	0
MCE	$\left(1 + e^{-\left(E(W, Y^i, X^i) - E(W, \bar{Y}^i, X^i)\right)}\right)^{-1}$	> 0
square-square	$E(W, Y^i, X^i)^2 - \left(\max(0, m - E(W, \bar{Y}^i, X^i))\right)^2$	m
square-exp	$E(W, Y^i, X^i)^2 + \beta e^{-E(W, \bar{Y}^i, X^i)}$	> 0
NLL/MMI	$E(W, Y^i, X^i) + \frac{1}{\beta} \log \int_{y \in \mathcal{Y}} e^{-\beta E(W, y, X^i)}$	> 0
MEE	$1 - e^{-\beta E(W, Y^i, X^i)} / \int_{y \in \mathcal{Y}} e^{-\beta E(W, y, X^i)}$	> 0

Advantages/Disadvantages of various losses

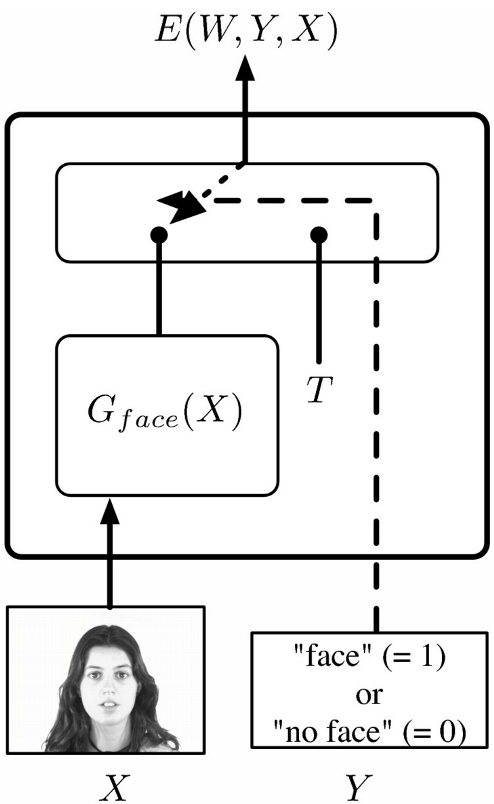
- **Loss functions differ in how they pick the point(s) whose energy is pulled up, and how much they pull them up**
- **Losses with a log partition function in the contrastive term pull up all the bad answers simultaneously.**
  - This may be good if the gradient of the contrastive term can be computed efficiently
  - This may be bad if it cannot, in which case we might as well use a loss with a single point in the contrastive term
- **Variational methods pull up many points, but not as many as with the full log partition function.**
- **Efficiency of a loss/architecture: how many energies are pulled up for a given amount of computation?**
  - The theory for this is to be developed

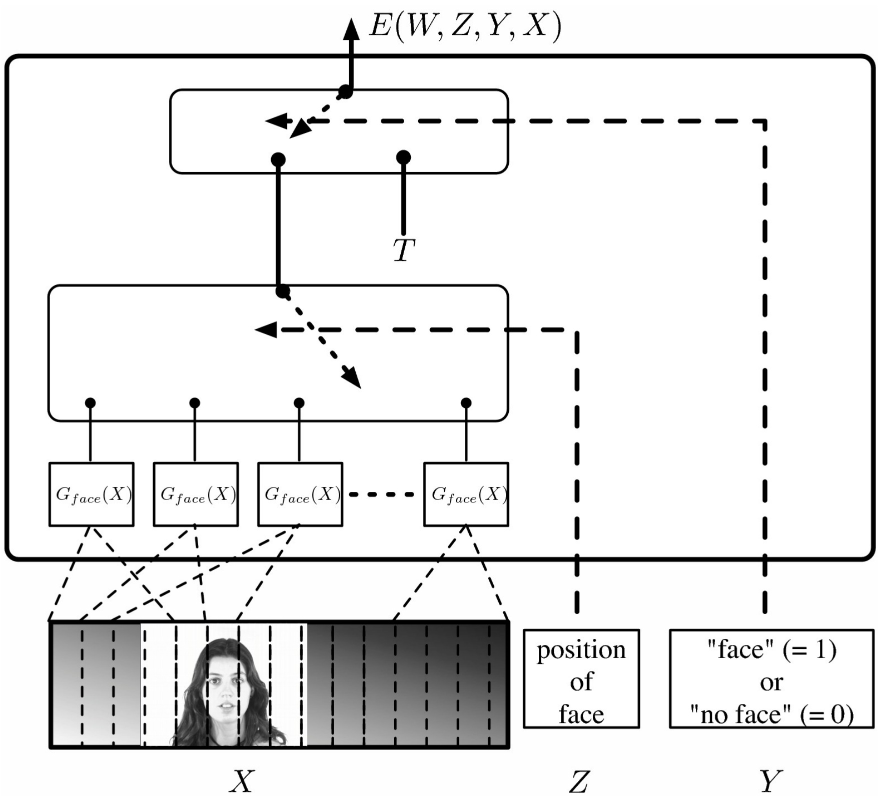
Latent Variable Models

- **The energy includes "hidden" variables Z whose value is never given to us**

$$E(Y, X) = \min_{Z \in \mathcal{Z}} E(Z, Y, X).$$

$$Y^* = \operatorname{argmin}_{Y \in \mathcal{Y}, Z \in \mathcal{Z}} E(Z, Y, X).$$





What can the latent variables represent?

- **Variables that would make the task easier if they were known:**
  - **Face recognition**: the gender of the person, the orientation of the face.
  - **Object recognition**: the pose parameters of the object (location, orientation, scale), the lighting conditions.
  - **Parts of Speech Tagging**: the segmentation of the sentence into syntactic units, the parse tree.
  - **Speech Recognition**: the segmentation of the sentence into phonemes or phones.
  - **Handwriting Recognition**: the segmentation of the line into characters.
- **In general, we will search for the value of the latent variable that allows us to get an answer (Y) of smallest energy.**

Probabilistic Latent Variable Models

- **Marginalizing over latent variables instead of minimizing.**

$$P(Z,Y|X) = \frac{e^{-\beta E(Z,Y,X)}}{\int_{y\in\mathcal{Y},\ z\in\mathcal{Z}} e^{-\beta E(y,z,X)}}.$$

$$P(Y|X) = \frac{\int_{z\in\mathcal{Z}} e^{-\beta E(Z,Y,X)}}{\int_{y\in\mathcal{Y},\ z\in\mathcal{Z}} e^{-\beta E(y,z,X)}}.$$

- **Equivalent to traditional energy-based inference with a redefined energy function:**

$$Y^* = \text{argmin}_{Y\in\mathcal{Y}} - \frac{1}{\beta}\log\int_{z\in\mathcal{Z}} e^{-\beta E(z,Y,X)}.$$

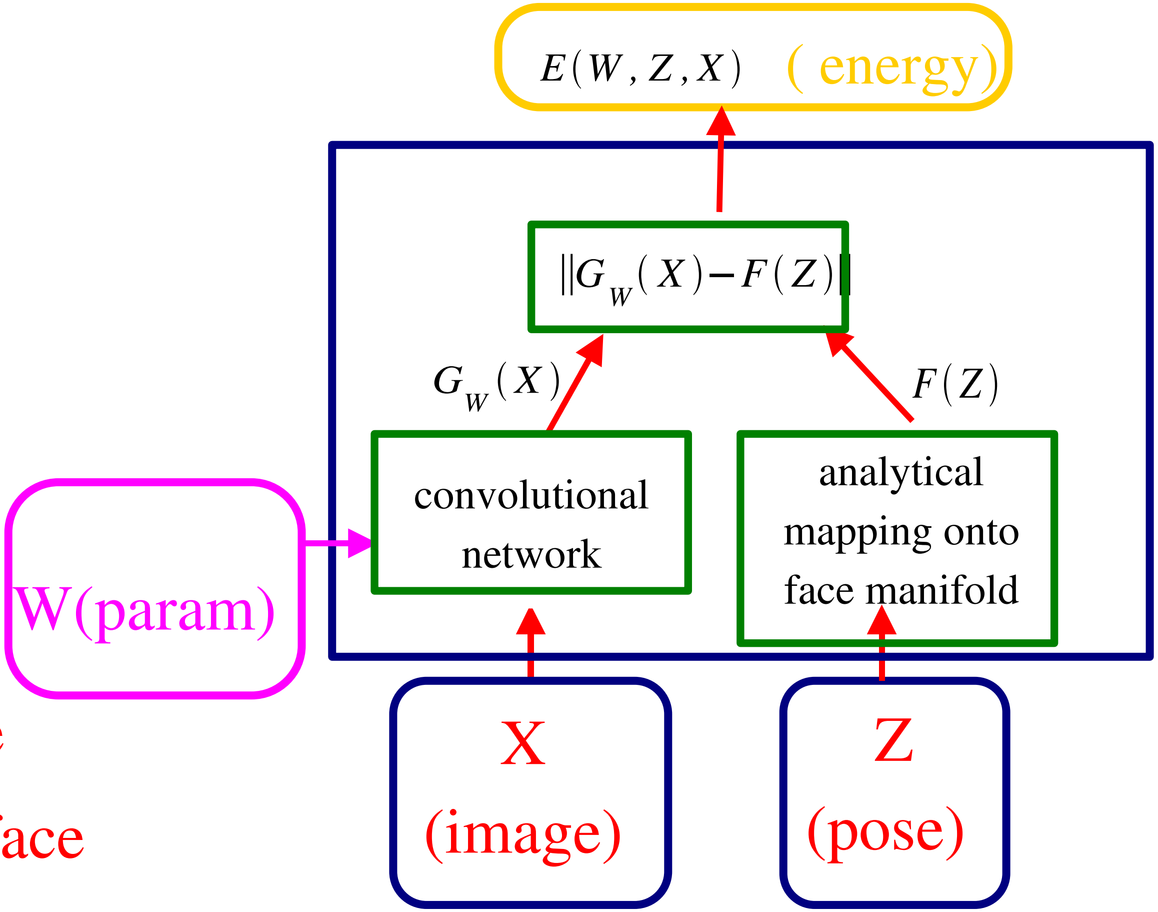
- **Reduces to traditional minimization when Beta->infinity**

Face Detection and Pose Estimation with a Convolutional EBM

- **Training:** 52,850, 32x32 grey-level images of faces, 52,850 selected non-faces.
- Each training image was used 5 times with random variation in scale, in-plane rotation, brightness and contrast.
- **2nd phase:** half of the initial negative set was replaced by false positives of the initial version of the detector .

$$E^*(W, X) = \min_Z ||G_W(X) - F(Z)||$$

$$Z^* = \text{argmin}_Z ||G_W(X) - F(Z)||$$

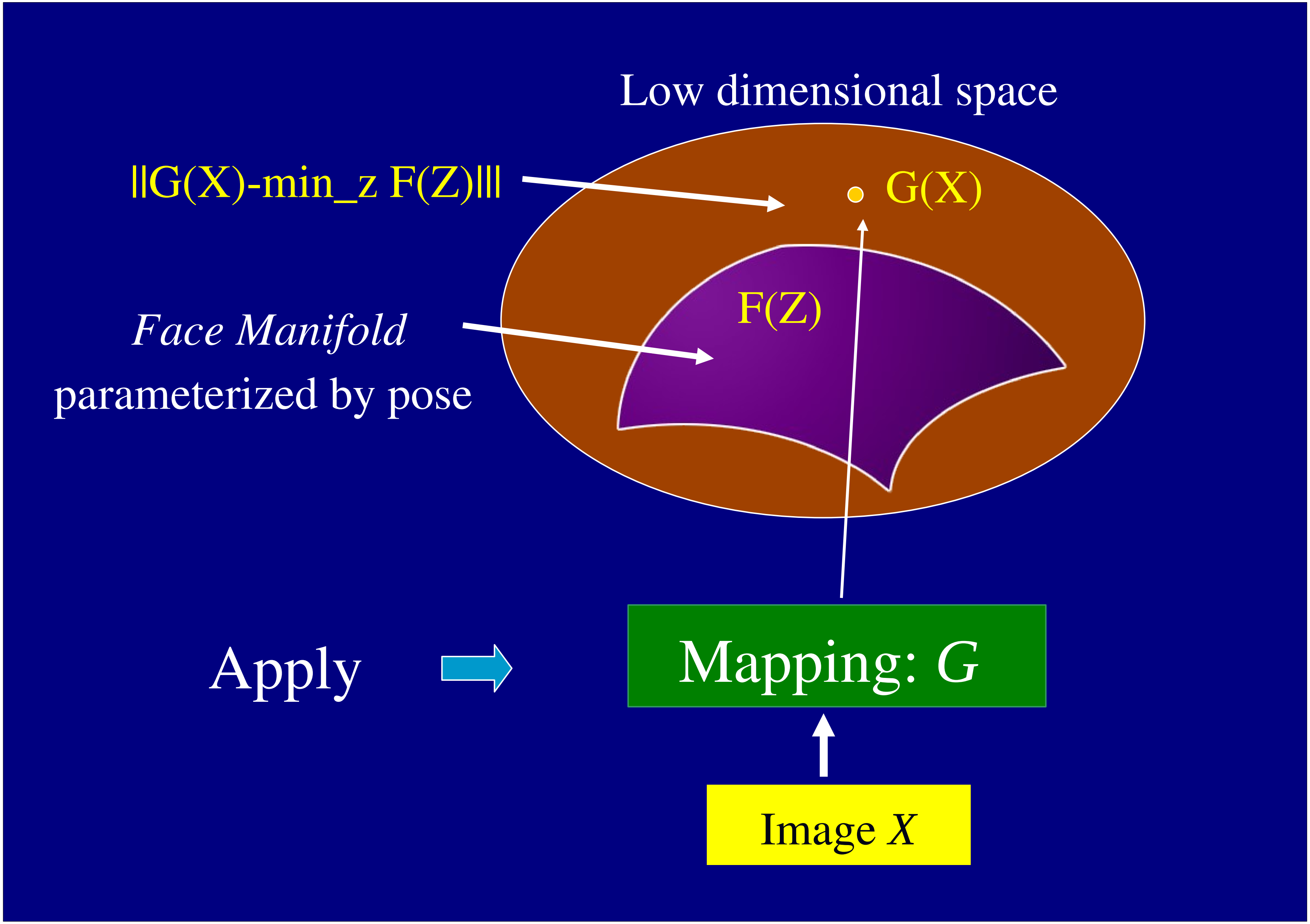


Small E*(W,X): face

Large E*(W,X): no face

[Osadchy, Miller, LeCun, NIPS 2004]

Face Manifold



Probabilistic Approach: Density model of joint P(face,pose)

Probability that image

X is a face with pose Z

$$P(X,Z) = \frac{\exp(-E(W,Z,X))}{\int_{X,Z\in \text{images,poses}} \exp(-E(W,Z,X))}$$

Given a training set of faces annotated with pose, find the W that maximizes the likelihood of the data under the model:

$$P(\text{faces}+\text{pose}) = \prod_{X,Z\in \text{faces+pose}} \frac{\exp(-E(W,Z,X))}{\int_{X,Z\in \text{images,poses}} \exp(-E(W,Z,X))}$$

Equivalently, minimize the negative log likelihood:

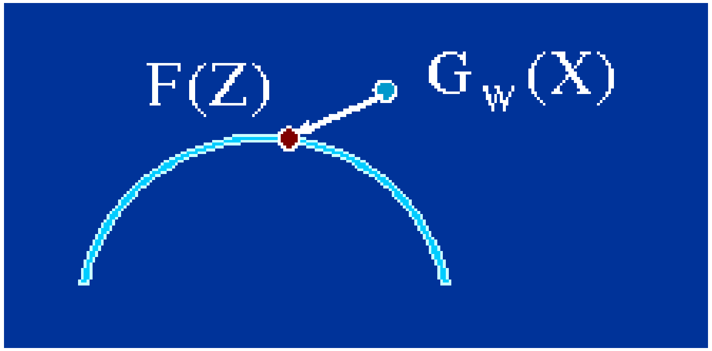
$$\mathcal{L}(W, \text{faces}+\text{pose}) = \sum_{X,Z\in \text{faces+pose}} E(W,Z,X) + \log\left[\int_{X,Z\in \text{images,poses}} \exp(-E(W,Z,X))\right]$$

COMPLICATED

Energy-Based Contrastive Loss Function

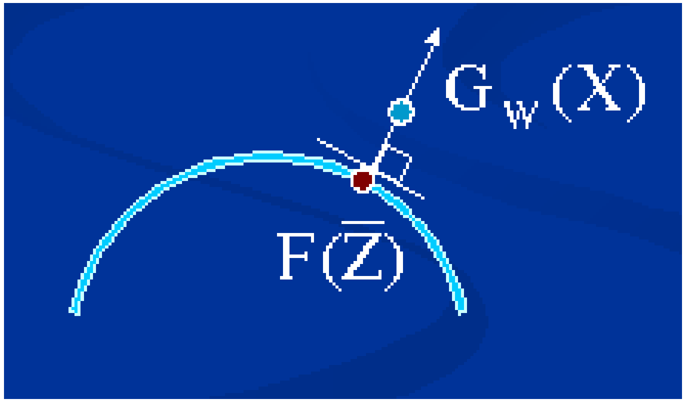
$$\mathcal{L}(W) = \frac{1}{|\mathrm{f}+\mathrm{p}|} \sum_{X,Z \in \mathrm{faces+pose}} \left[L^{+}\left(E(W,Z,X)\right)\right] + L^{-}\left(\min_{X,Z \in \mathrm{bckgnd,poses}} E(W,Z,X)\right)$$

$$L^{+}\left(E(W,Z,X)\right) = E(W,Z,X)^{2} = ||G_W(X) - F(Z)||^{2}$$



Attract the network output Gw(X) to the location of the desired pose F(Z) on the manifold

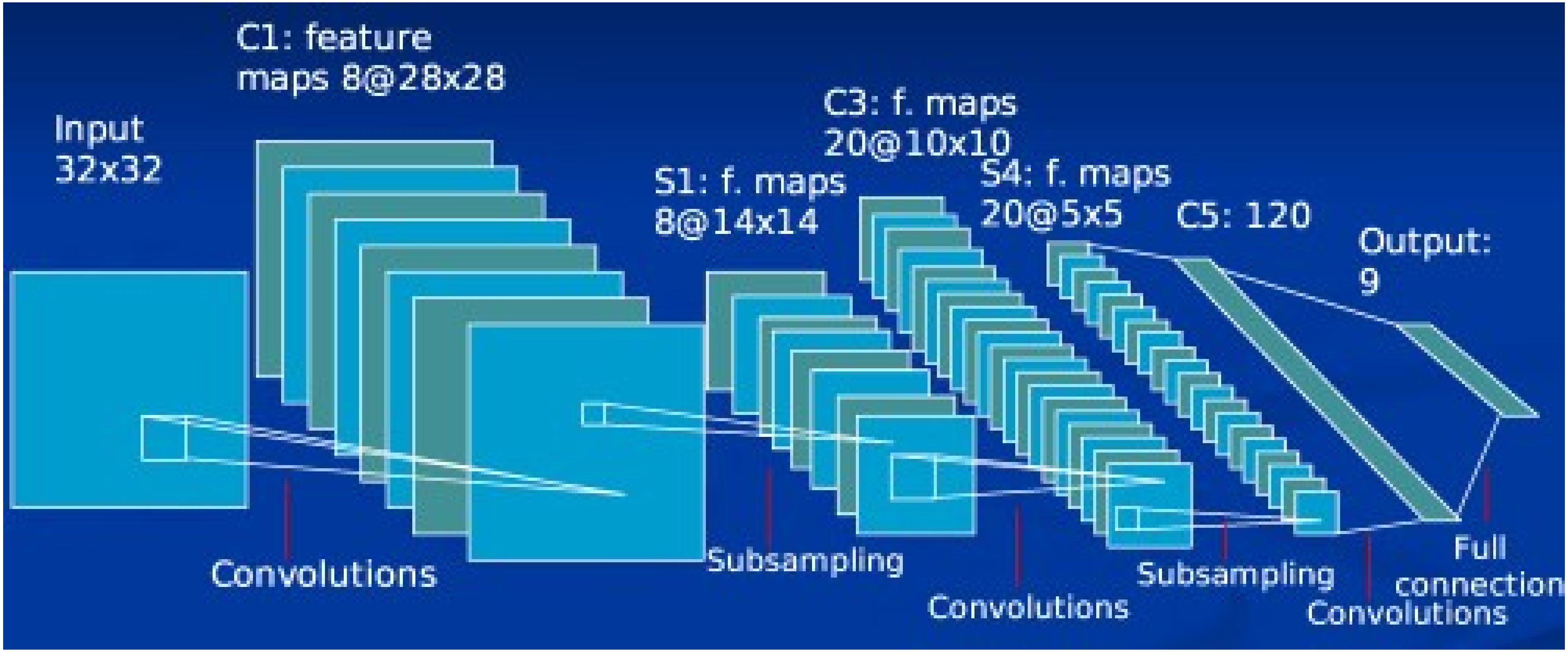
$$L^{-}\left(\min_{X,Z \in \mathrm{bckgnd,poses}} E(W,Z,X)\right) = K \exp\left(-\min_{X,Z \in \mathrm{bckgnd,poses}} ||G_W(X) - F(Z)||\right)$$



Repel the network output Gw(X) away from the face/pose manifold

Convolutional Network Architecture

[LeCun et al. 1988, 1989, 1998, 2005]



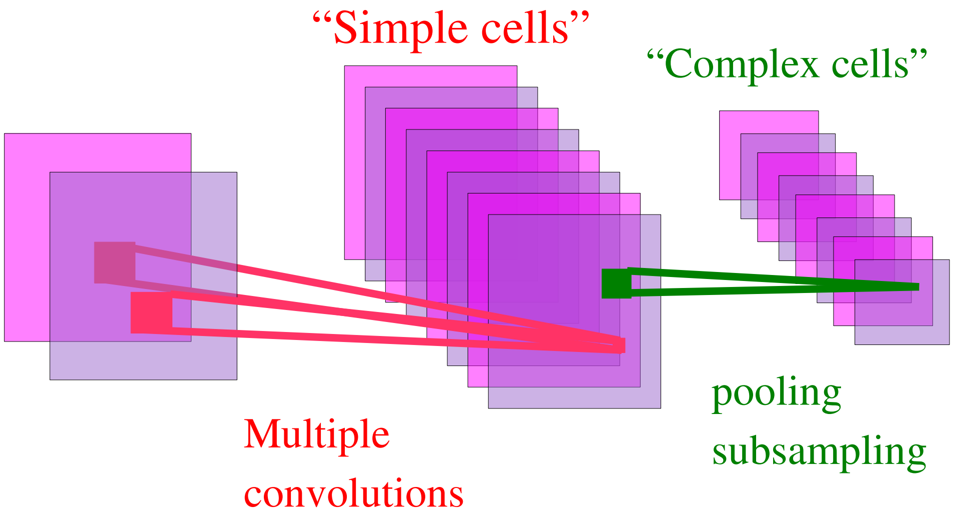
Hierarchy of local filters (convolution kernels),

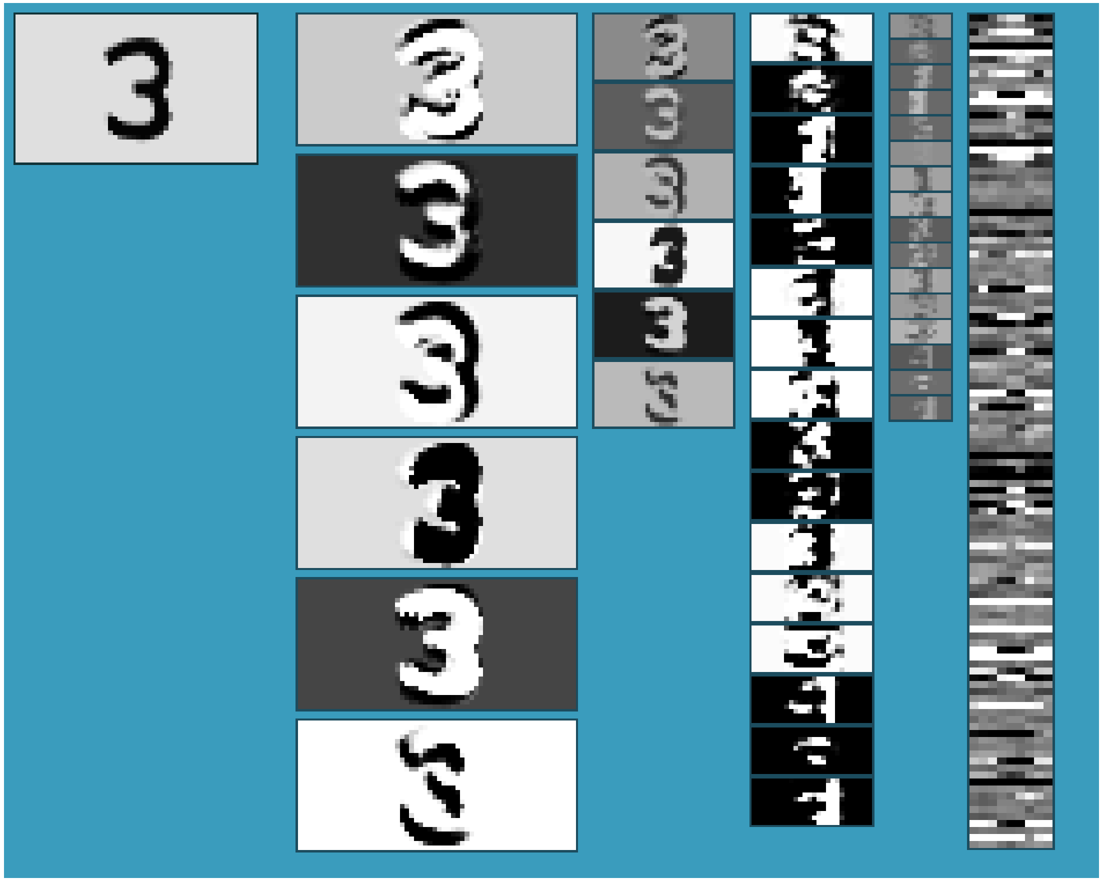
sigmoid pointwise non-linearities, and spatial subsampling

All the filter coefficients are learned with gradient descent (back-prop)

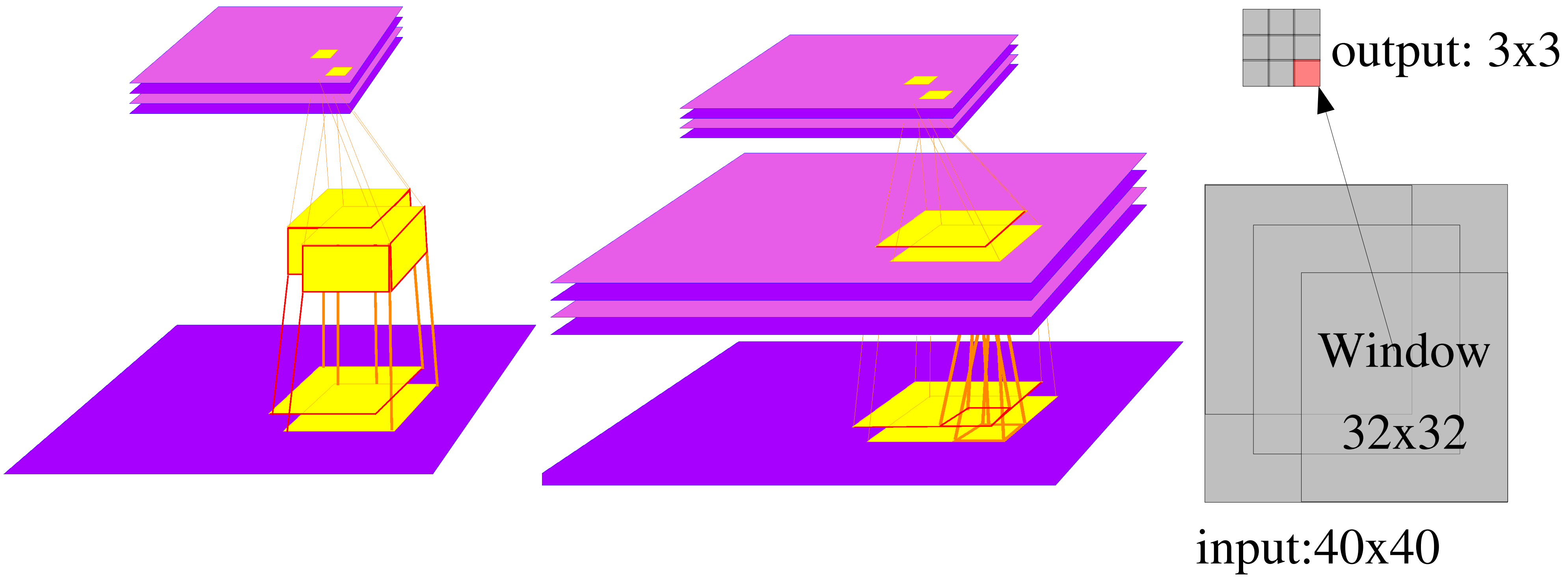
Alternated Convolutions and Pooling/Subsampling

- Local features are extracted everywhere.
- pooling/subsampling layer builds robustness to variations in feature locations.
- Long history in neuroscience and computer vision:
  - Hubel/Wiesel 1962,
  - Fukushima 1971-82,
  - LeCun 1988-06
  - Poggio, Riesenhuber, Serre 02-06
  - Ullman 2002-06
  - Triggs, Lowe,....





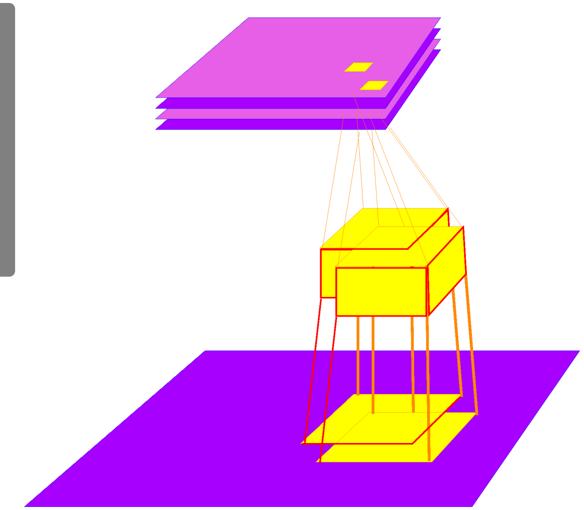
Building a Detector/Recognizer: Replicated Conv. Nets

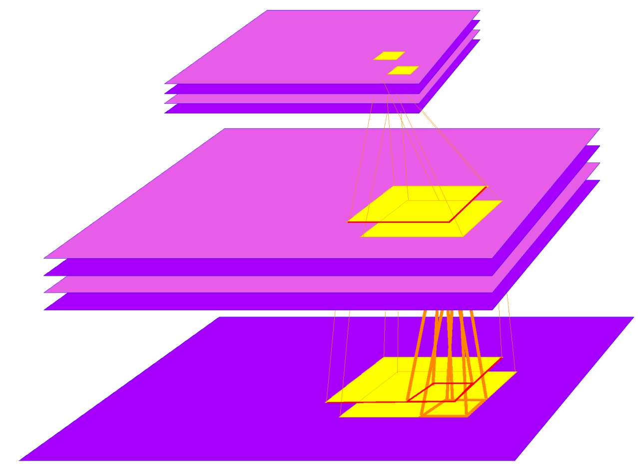


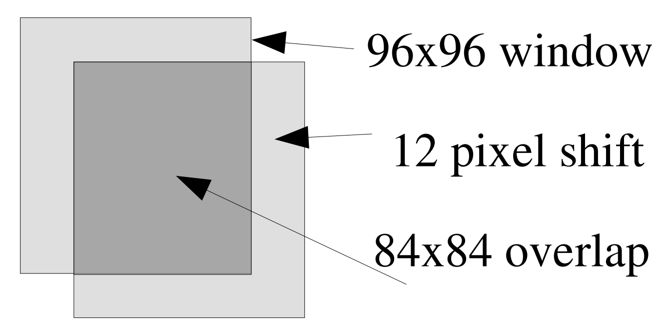
- Traditional Detectors/Classifiers must be applied to every location on a large input image, at multiple scales.
- Convolutional nets can replicated over large images very cheaply.
- The network is applied to multiple scales spaced by sqrt(2)
- Non-maximum suppression with exclusion window

Building a Detector/Recognizer: Replicated Convolutional Nets

- Computational cost for replicated convolutional net:
  - 96x96 -> 4.6 million multiply-accumulate operations
  - 120x120 -> 8.3 million multiply-accumulate operations
  - 240x240 -> 47.5 million multiply-accumulate operations
  - 480x480 -> 232 million multiply-accumulate operations
- Computational cost for a non-convolutional detector of the same size, applied every 12 pixels:
  - 96x96 -> 4.6 million multiply-accumulate operations
  - 120x120 -> 42.0 million multiply-accumulate operations
  - 240x240 -> 788.0 million multiply-accumulate operations
  - 480x480 -> 5,083 million multiply-accumulate operations

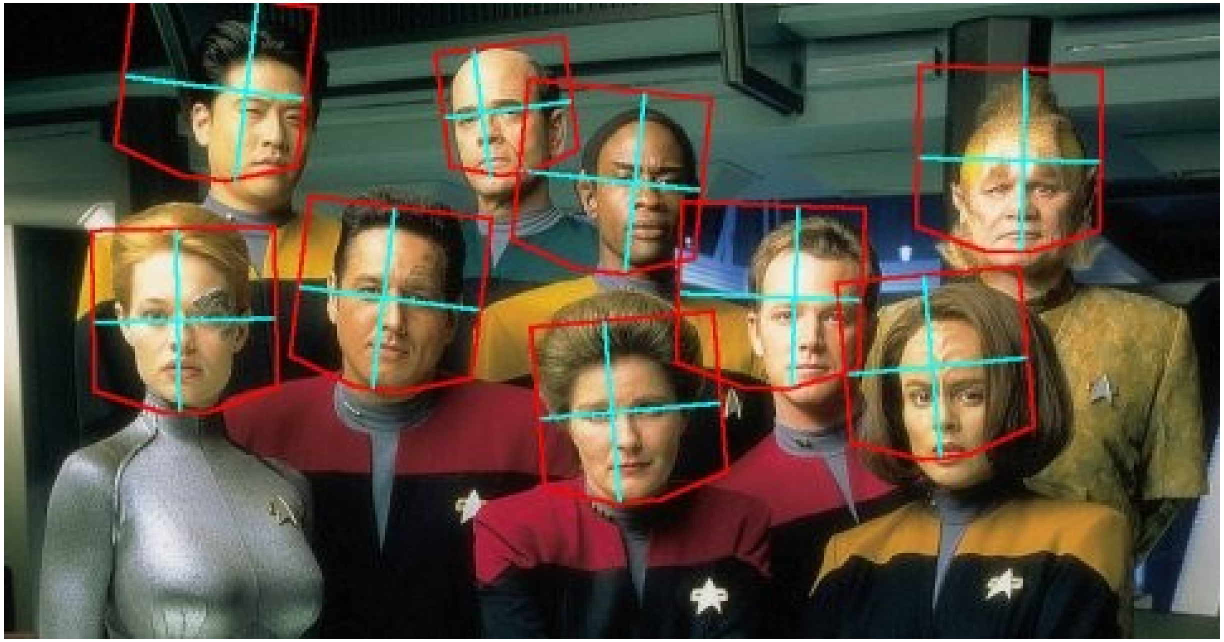


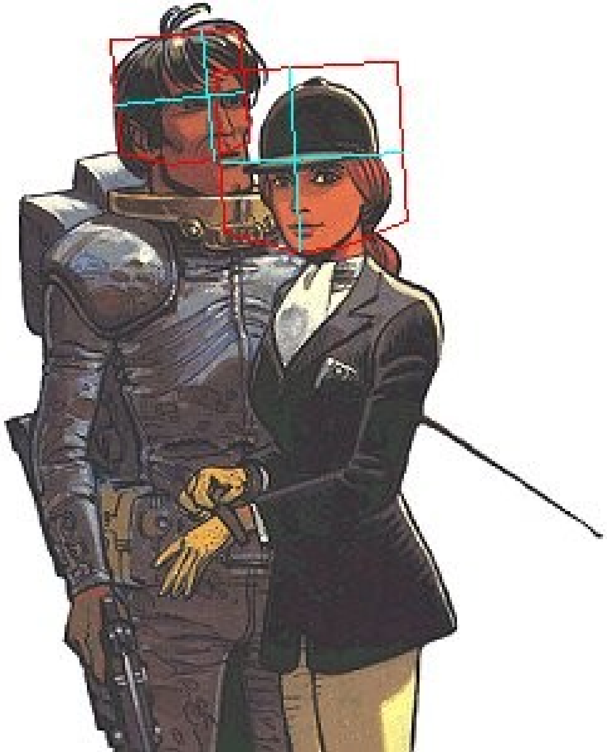




Face Detection: Results

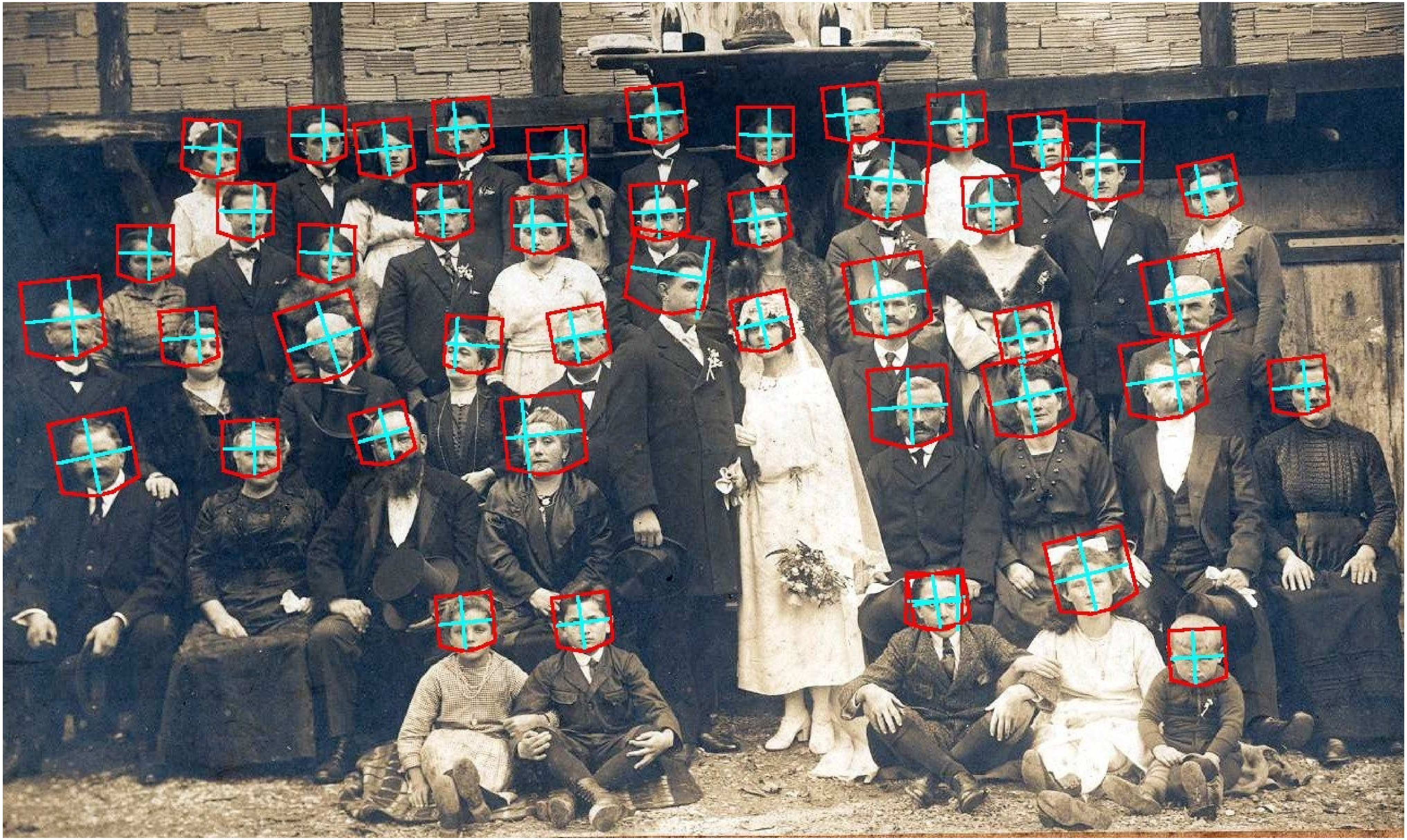
Data Set-> *False positives per image->*	TILTED		PROFILE		MIT+CMU	
	4.42	26.9	0.47	3.36	0.5	1.28
Our Detector	90%	97%	67%	83%	83%	88%
Jones & Viola (tilted)	90%	95%	x		x	
Jones & Viola (profile)	x		70%	83%	x	





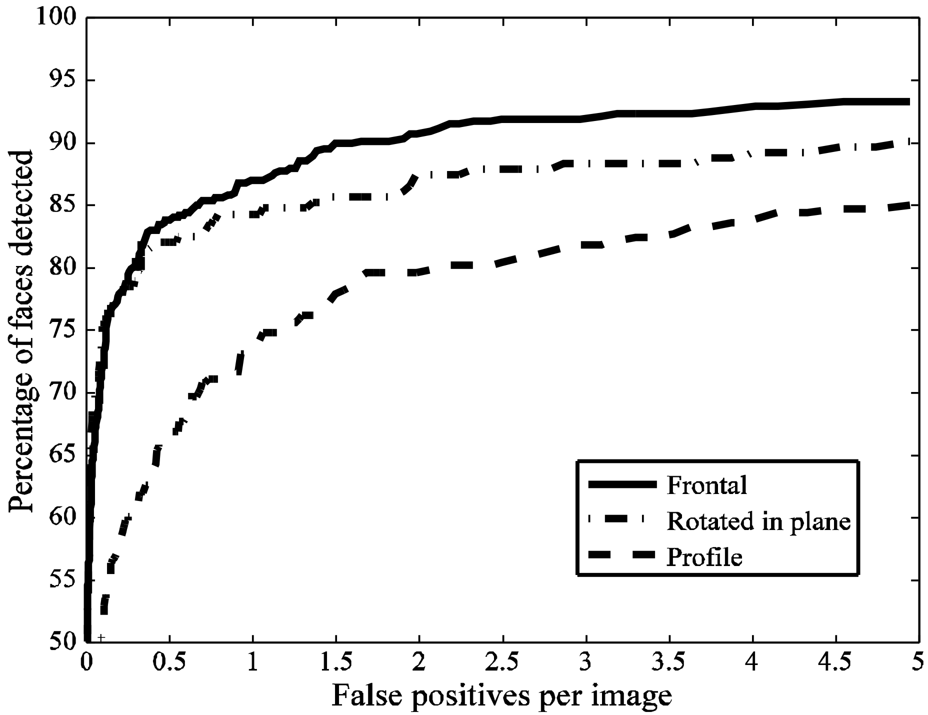
Face Detection and Pose Estimation: Results

Face Detection with a Convolutional Net

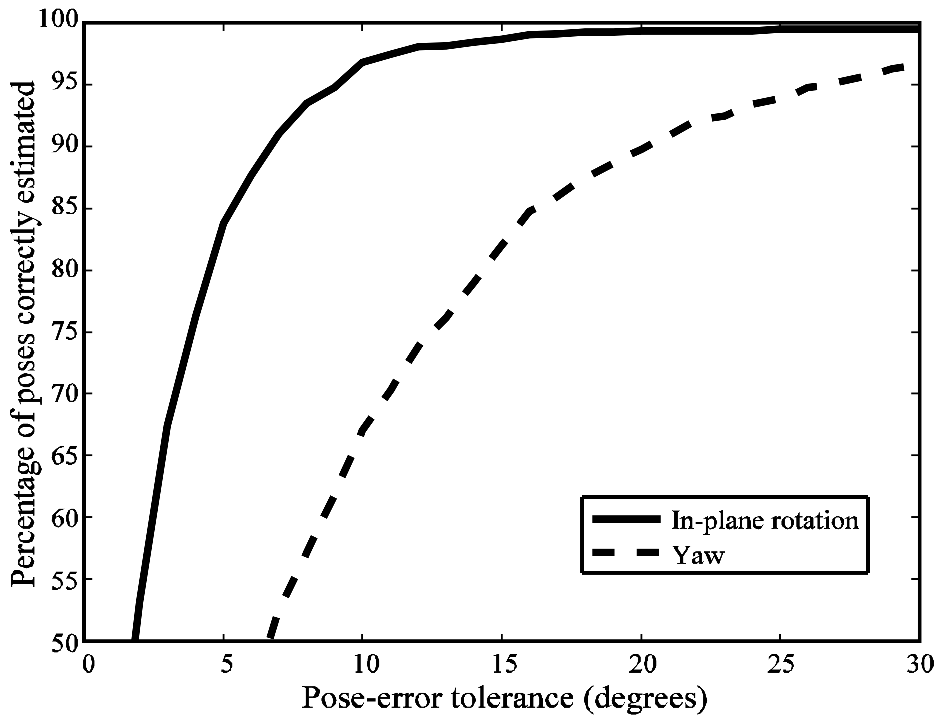


Performance on standard dataset

Detection



Pose estimation

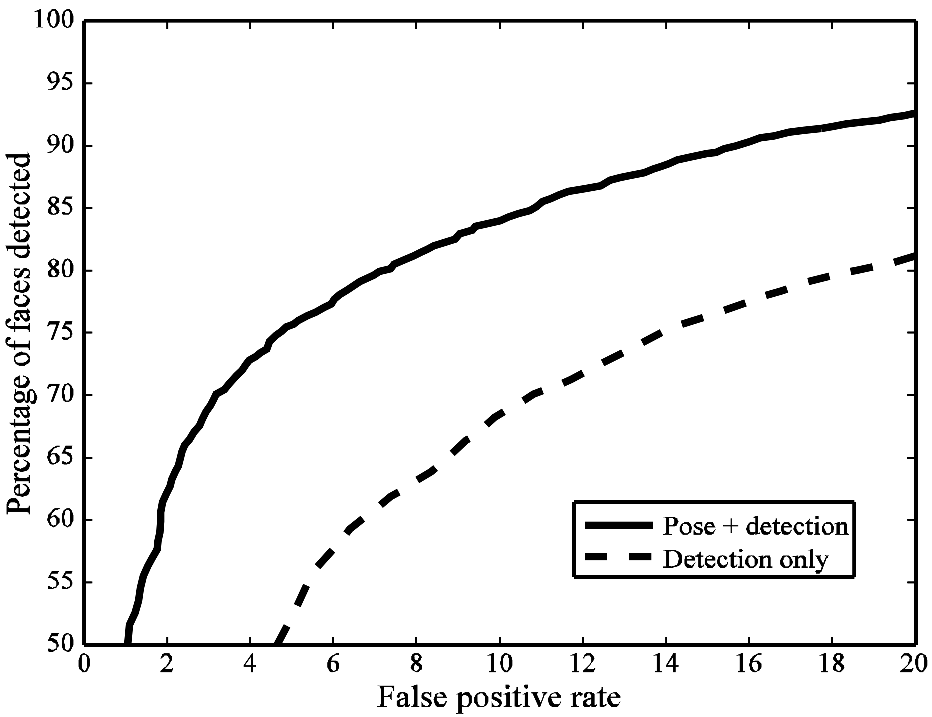


Pose estimation is performed on faces located automatically by the system

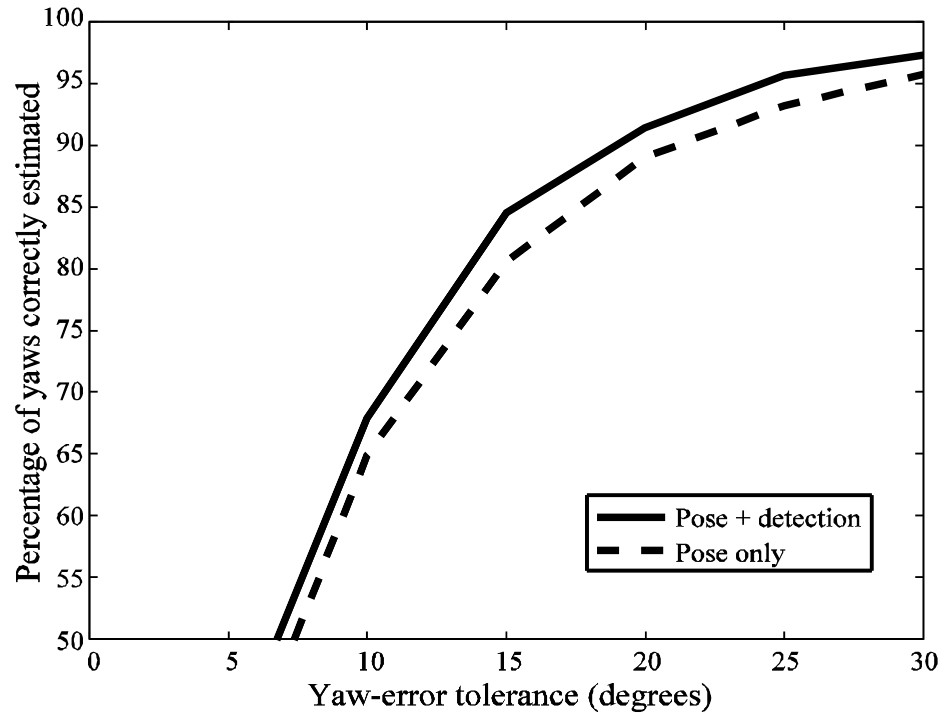
when the faces are localized by hand we get: 89% of yaw and 100% of in-plane rotations within 15 degrees.

Synergy Between Detection and Pose Estimation

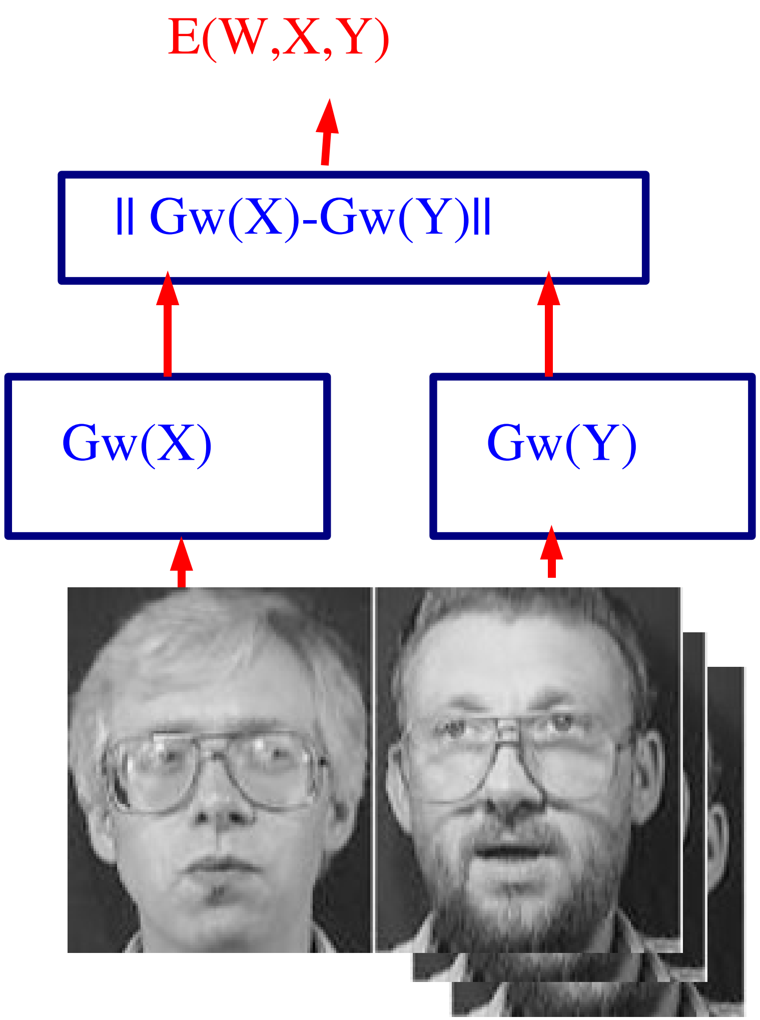
Pose Estimation Improves Detection



Detection improves pose estimation



EBM for Face Recognition

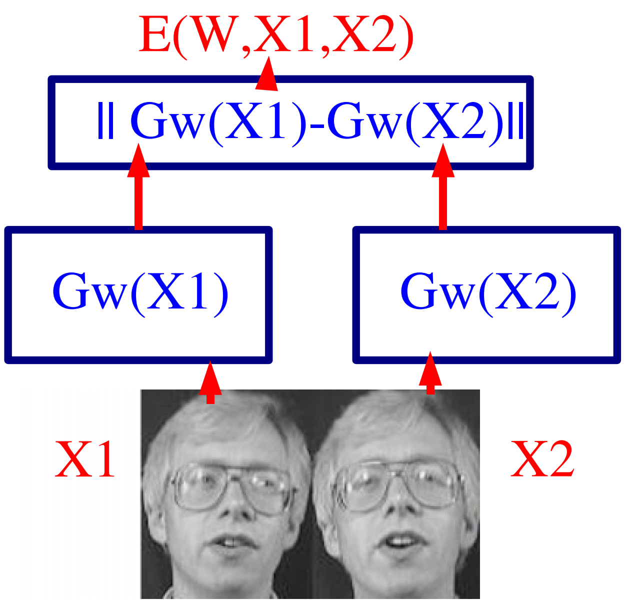


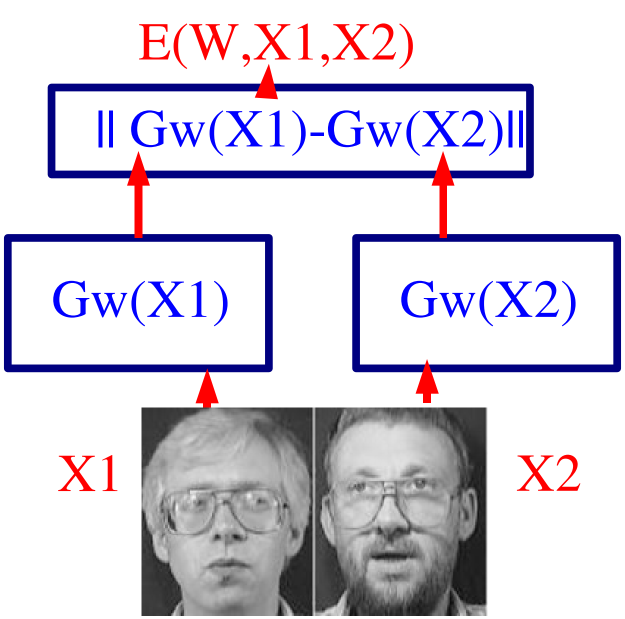
- **X and Y are images**
- **Y is a discrete variable with many possible values**
  - All the people in our gallery
- **Example of architecture:**
  - A function G(X) maps input images into a low-dimensional space in which the Euclidean distance measures dissemblance.
- **Inference:**
  - Find the Y in the gallery that minimizes the energy (find the Y that is most similar to X)
  - Minimization through exhaustive search.

Learning an Invariant Dissimilarity Metric with EBM

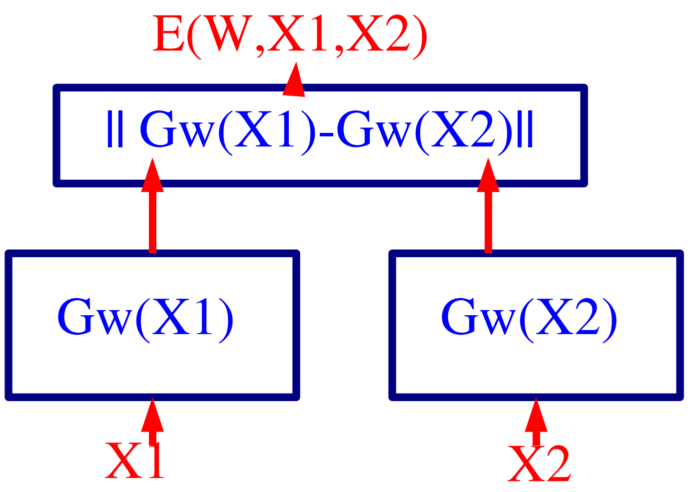
[Chopra, Hadsell, LeCun CVPR 2005]

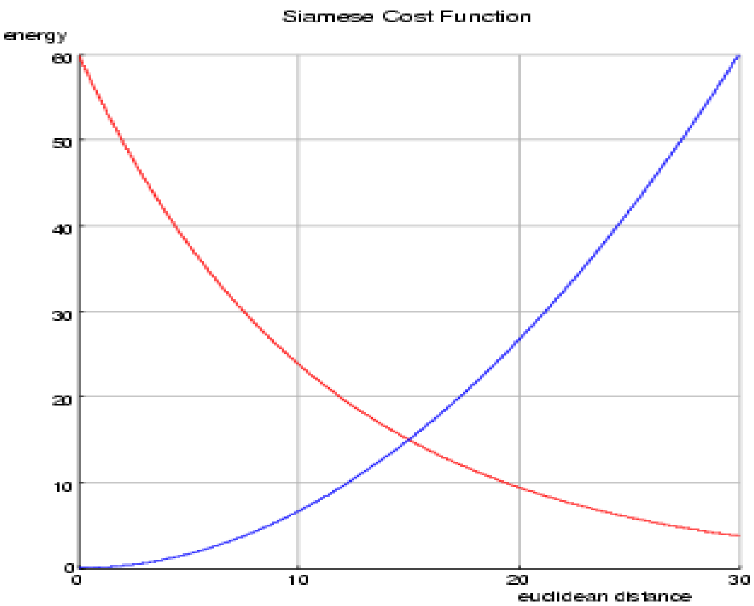
- Training a **parameterized, invariant dissimilarity metric** may be a solution to the **many-category problem**.
- Find a mapping **Gw(X)** such that the Euclidean distance **||Gw(X1)- Gw(X2)||** reflects the "semantic" distance between X1 and X2.
- Once trained, a trainable dissimilarity metric can be used to classify **new categories using a very small number of training samples** (used as prototypes).
- This is an example where probabilistic models are too constraining, because we would have to limit ourselves to models that can be normalized over the space of input pairs.
- With EBMs, we can put what we want in the box (e.g. A convolutional net).
- **Siamese Architecture**
- **Application:** face verification/recognition





Loss Function





- **Siamese models:** distance between the outputs of two identical copies of a model.
- **Energy function**: E(W,X1,X2) = ||Gw(X1)-Gw(X2)||
- If X1 and X2 are from the **same category (genuine pair)**, train the two copies of the model to produce **similar outputs (low energy)**
- If X1 and X2 are from **different categories (impostor pair)**, train the two copies of the model to produce **different outputs (high energy)**
- **Loss function: increasing function of genuine pair energy, decreasing function of impostor pair energy.**

Loss Function

Our Loss function for a single training pair (X1,X2):

$$L(W, X_1, X_2) = (1-Y) L_G(E_W(X_1, X_2)) + Y L_I(E_W(X_1, X_2))$$

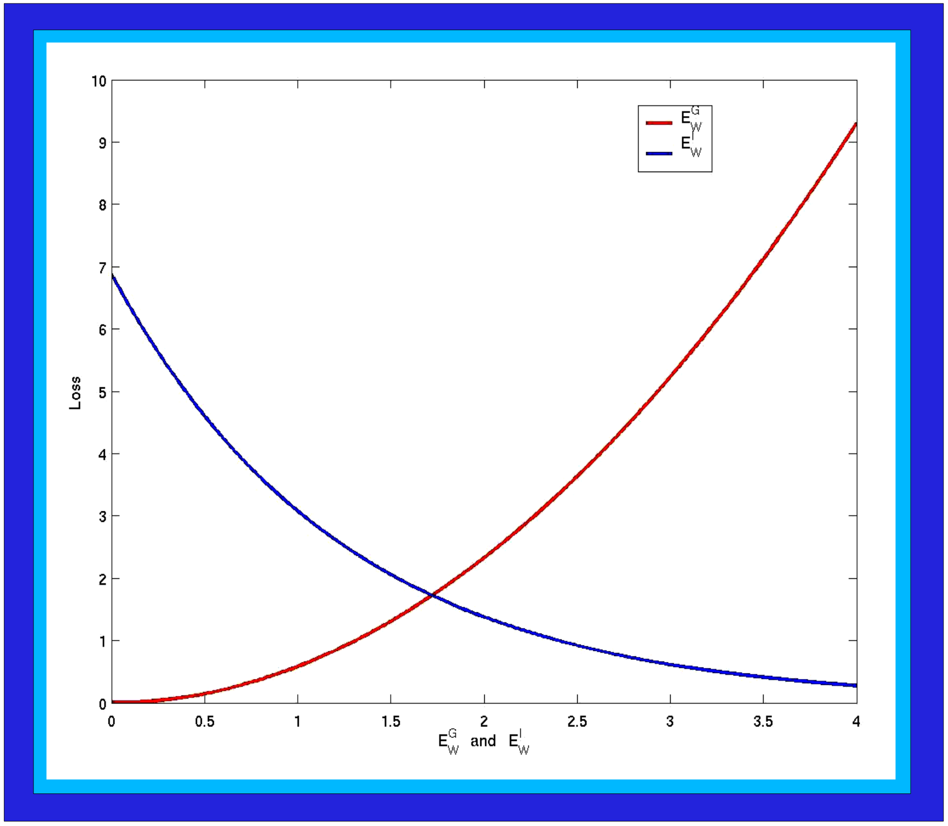
$$= (1-Y)\frac{2}{R}(E_W(X_1, X_2)^2) + (Y)\, 2R\, e^{-2.77\frac{E_W(X_1, X_2)}{R}}$$

$$E_W(X_1, X_2) = \| G_W(X_1) - G_W(X_2) \|_{L1}$$

And R is the largest possible value of

$$E_W(X_1, X_2)$$

Y=0 for a genuine pair, and Y=1 for an impostor pair.



Face Verification datasets: AT&T, FERET, and AR/Purdue

- The AT&T/ORL dataset
- Total subjects: **40**. Images per subject: **10**. Total images: **400**.
- Images had a moderate degree of variation in pose, lighting, expression and head position.
- Images from **35** subjects were used for training. Images from **5** remaining subjects for testing.
- Training set was taken from: **3500** genuine and **119000** impostor pairs.
- Test set was taken from: **500** genuine and **2000** impostor pairs.
- http://www.uk.research.att.com/facedatabase.html

AT&T/ORL Dataset

Face Verification datasets: AT&T, FERET, and AR/Purdue

- The FERET dataset. part of the dataset was used only for training.
- Total subjects: **96**. Images per subject: **6**. Total images: **1122**.
- Images had high degree of variation in pose, lighting, expression and head position.
- The images were used for training only.
- http://www.itl.nist.gov/iad/humanid/feret/

FERET Dataset

Face Verification datasets: AT&T, FERET, and AR/Purdue

- The AR/Purdue dataset
- Total subjects: **136**. Images per subject: **26**. Total images: **3536**.
- Each subject has 2 sets of 13 images taken 14 days apart.
- Images had very high degree of variation in pose, lighting, expression and position. Within each set of 13, there are 4 images with expression variation, 3 with lighting variation, 3 with dark sun glasses and lighting variation, and 3 with face obscuring scarfs and lighting variation.
- Images from **96** subjects were used for training. The remaining **40** subjects were used for testing.
- Training set drawn from: **64896** genuine and **6165120** impostor pairs.
- Test set drawn from: **27040** genuine and **1054560** impostor pairs.
- http://rv11.ecn.purdue.edu/aleix/aleix_face_DB.html

Face Verification dataset: AR/Purdue

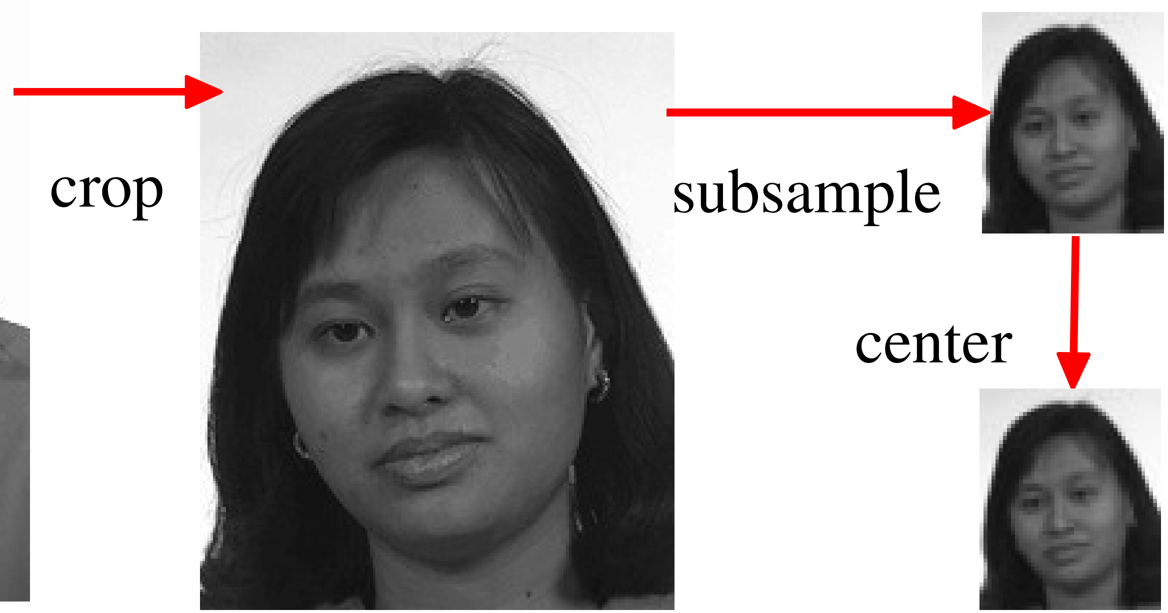
Preprocessing

The 3 datasets each required a small amount of preprocessing.

FERET: Cropping, subsampling, and centering (see below)

AR/PURDUE: Cropping and subsampling

AT&T: Subsampling only

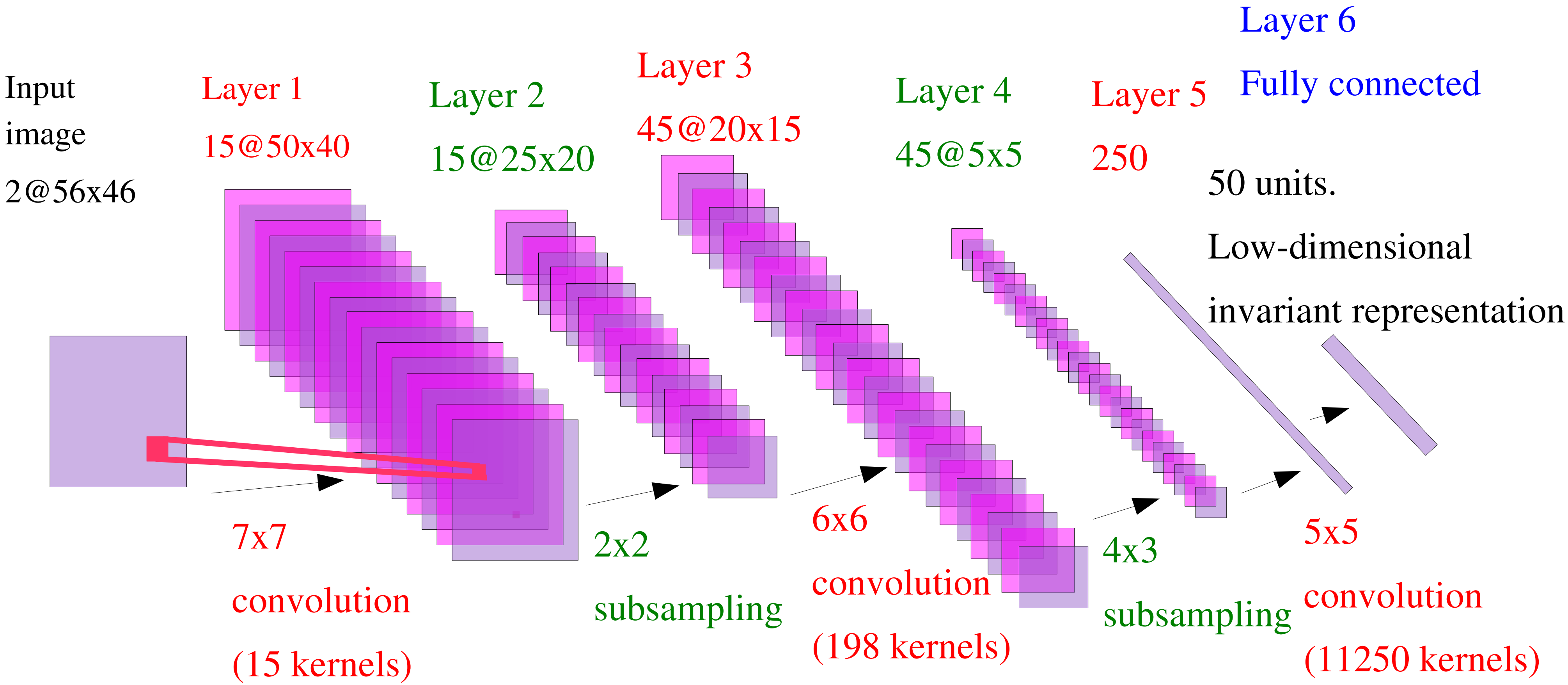


Centering with a Gaussian-blurred face template

- Coarse centering was done on the FERET database images
  1. Construct a template by blurring a well-centered face.
  2. Convolve the template with an uncentered image.
  3. Choose the 'peak' of the convolution as the center of the image.

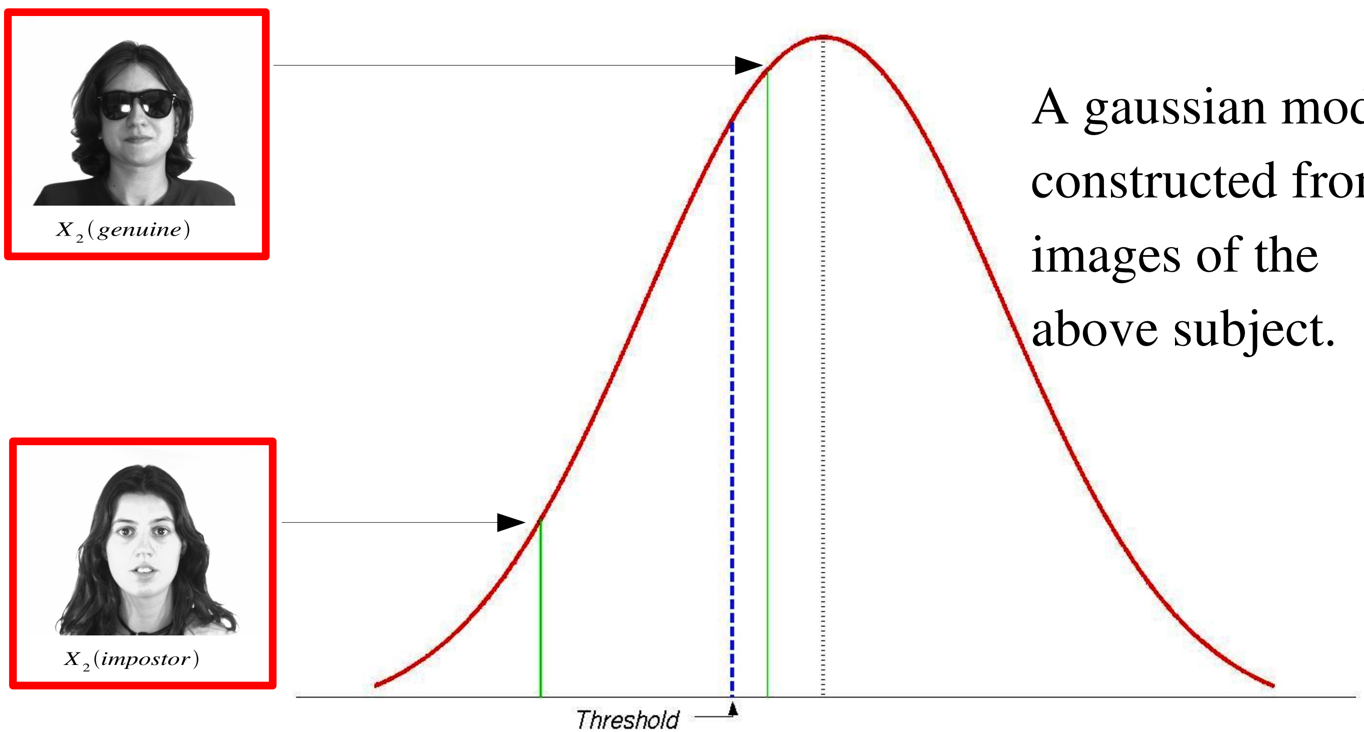
Architecture for the Mapping Function Gw(X)

Convolutional net



Internal state for genuine and impostor pairs

Gaussian Face Model in the output space



A gaussian model constructed from 5 images of the above subject.

Dataset for Verification

- tested on AT&T and AR/Purdue
- AT&T dataset

  Number of subjects: 5

  Images/subject: 10

  Images/Model: 5

  Total test size: 5000

  Number of Genuine: 500

  Number of Impostors: 4500
- Purdue/AR dataset

  Number of subjects: 40

  Images/subject: 26

  Images/Model: 13

  Total test size: 5000

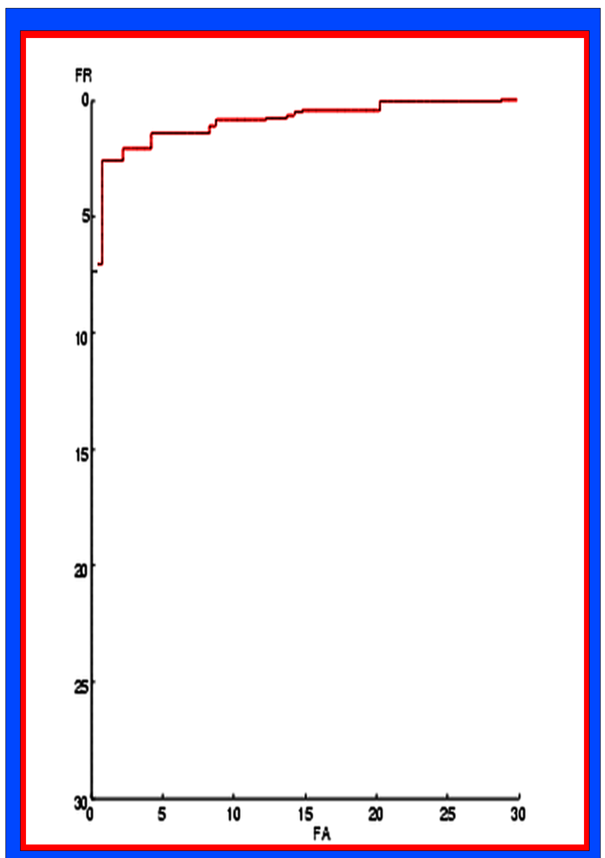
  Number of Genuine: 500

  Number of Impostors: 4500

Verification Results

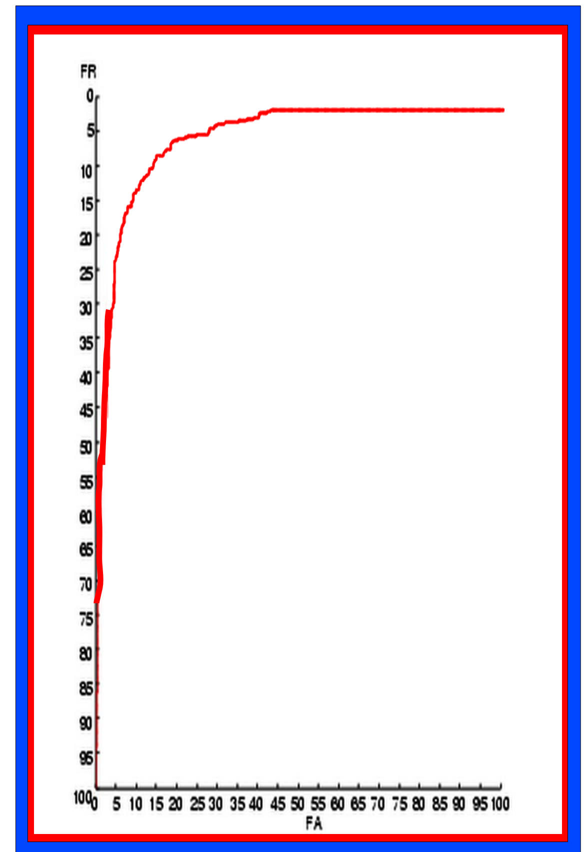
- The AT&T dataset

False Accept	False Reject
10.00%	0.00%
7.50%	1.00%
5.00%	1.00%



- The AR/Purdue dataset

False Accept	False Reject
10.00%	11.00%
7.50%	14.60%
5.00%	19.00%



Classification Examples

Example: Correctly classified genuine pairs

energy: 0.3159

energy: 0.0043

energy: 0.0046

Example: Correctly classified impostor pairs

energy: 20.1259

energy: 32.7897

energy: 5.7186

Example: Mis-classified pairs

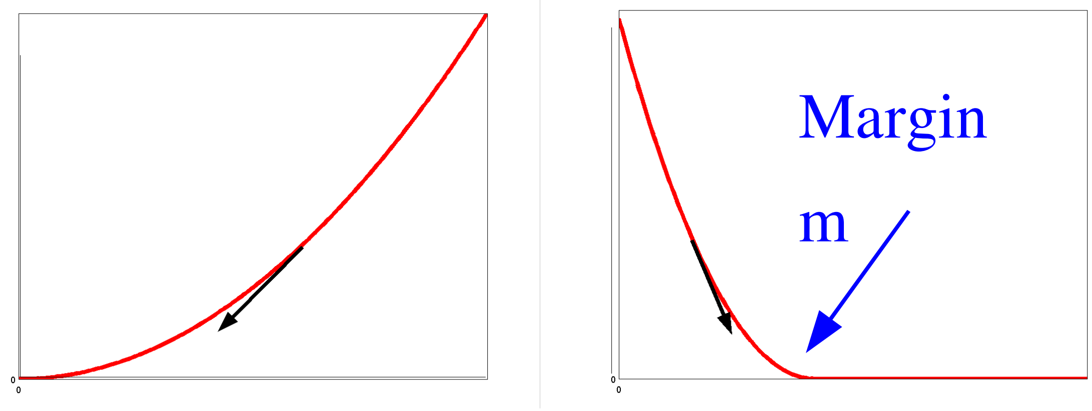
energy: 10.3209

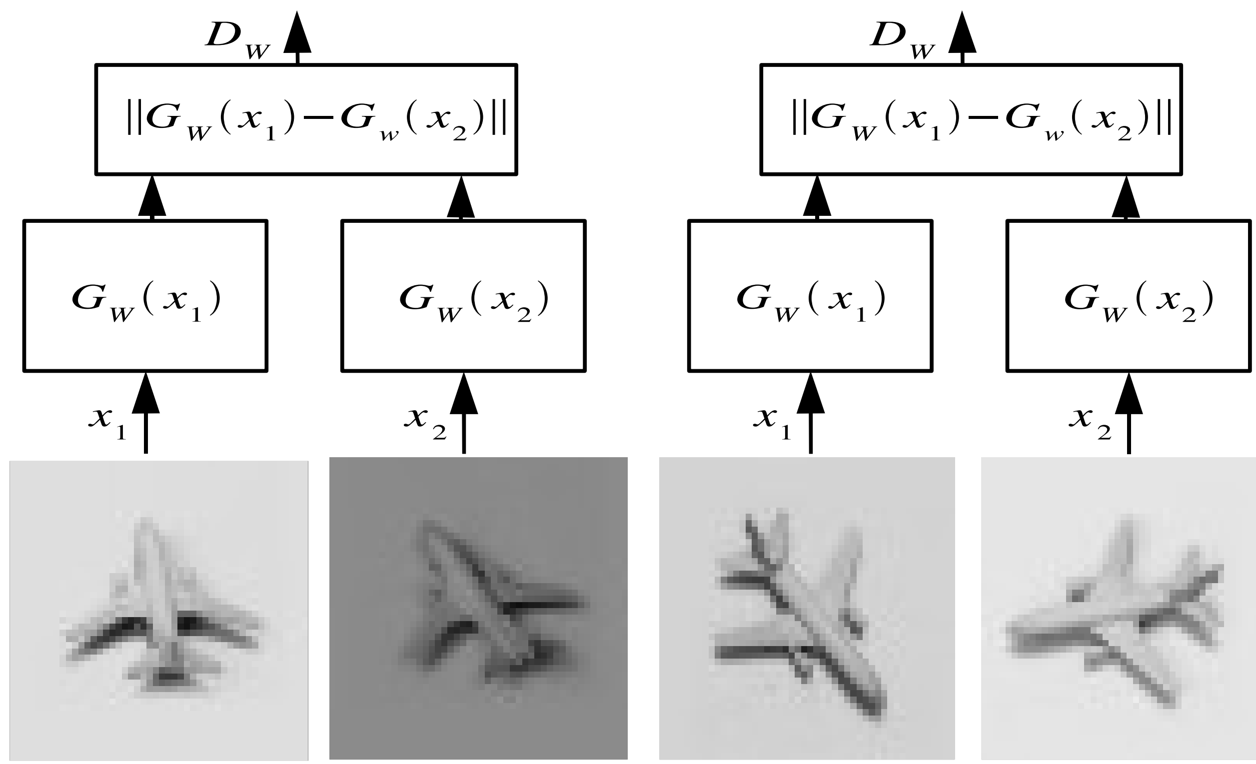
energy: 2.8243

Internal State

A similar idea for Learning a Manifold with Invariance Properties

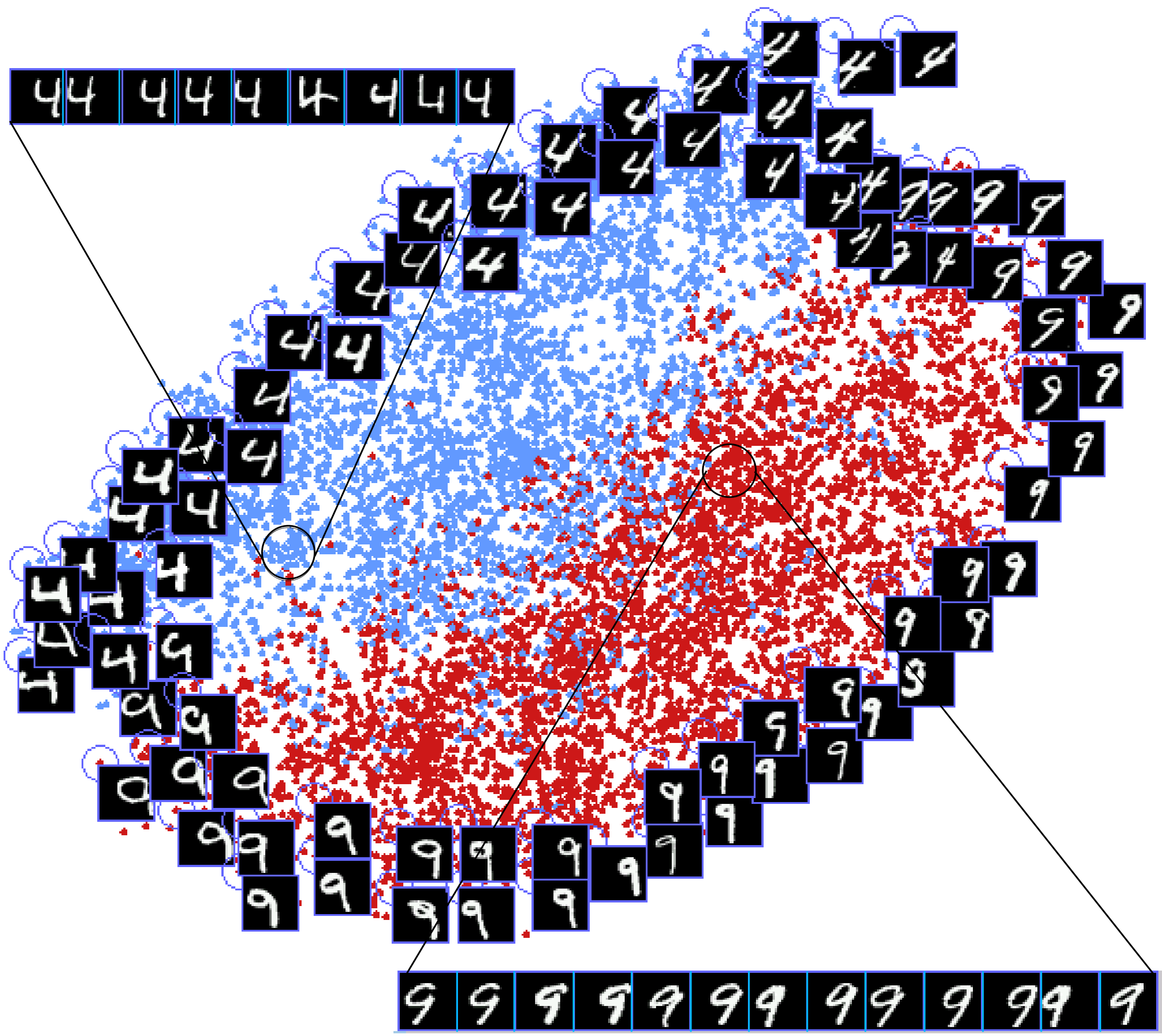
$$L_{similar} = \frac{1}{2} D_W^2 \qquad L_{dissimilar} = \frac{1}{2} \{ max(0, m - D_W) \}^2$$





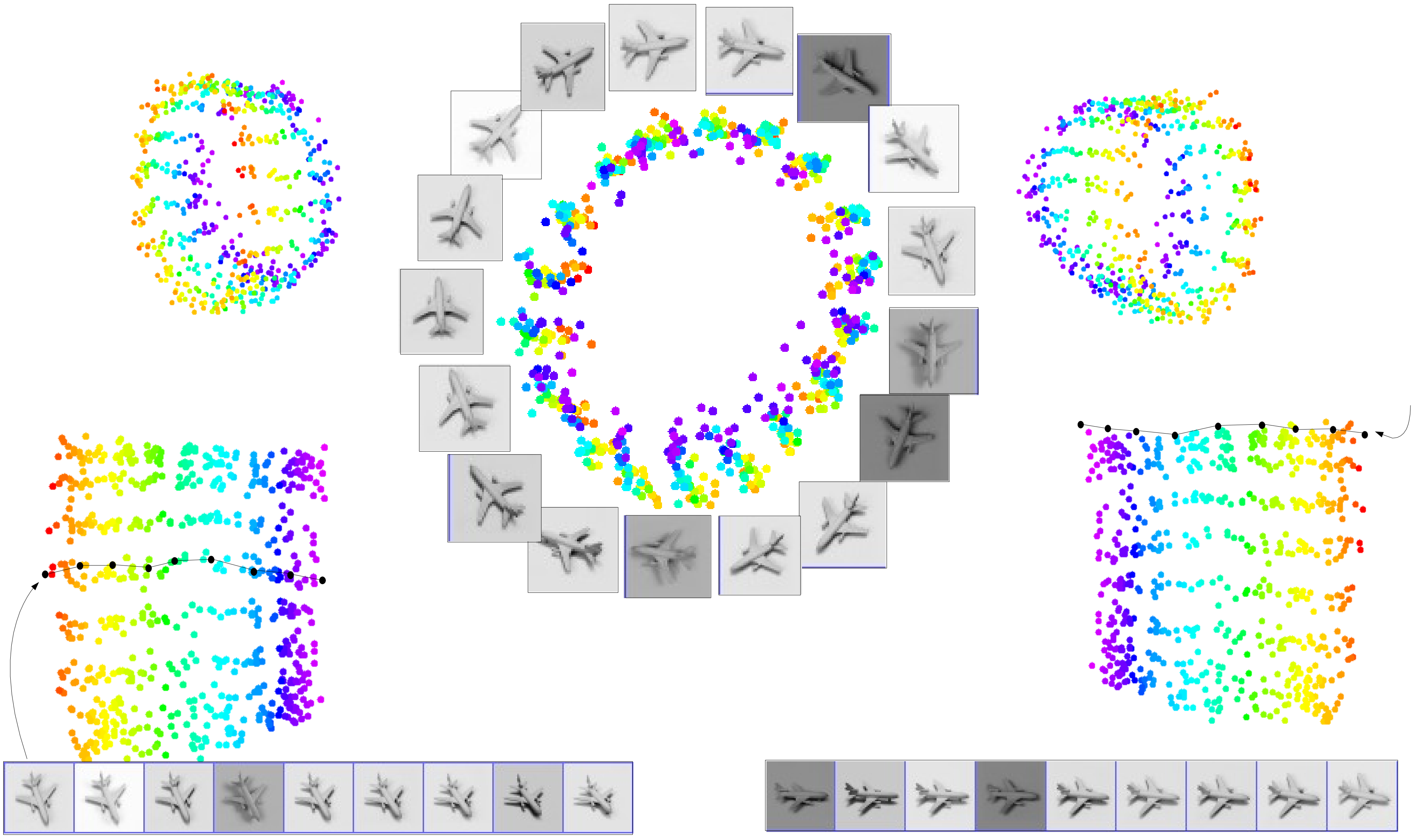
- **Loss function:**
  - Pay quadratically for making outputs of neighbors far apart
  - Pay quadratically for making outputs of non-neighbors smaller than a **margin** m

A Manifold with Invariance to Shifts



- Training set: 3000 “4” and 3000 “9” from MNIST. **Each digit is shifted horizontally by -6, -3, 3, and 6 pixels**
- Neighborhood graph: 5 nearest neighbors in Euclidean distance, **and shifted versions of self and nearest neighbors**
- Output Dimension: 2
- Test set (shown) 1000 “4” and 1000 “9”

Automatic Discovery of the Viewpoint Manifold with Invariant to Illumination

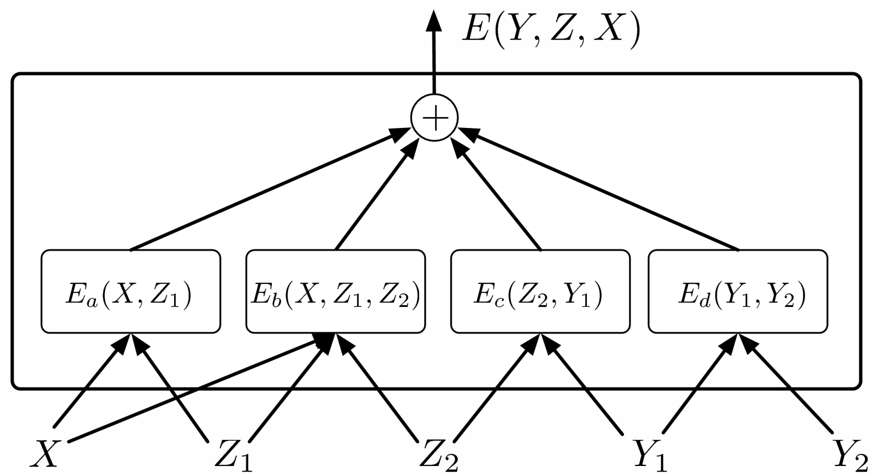


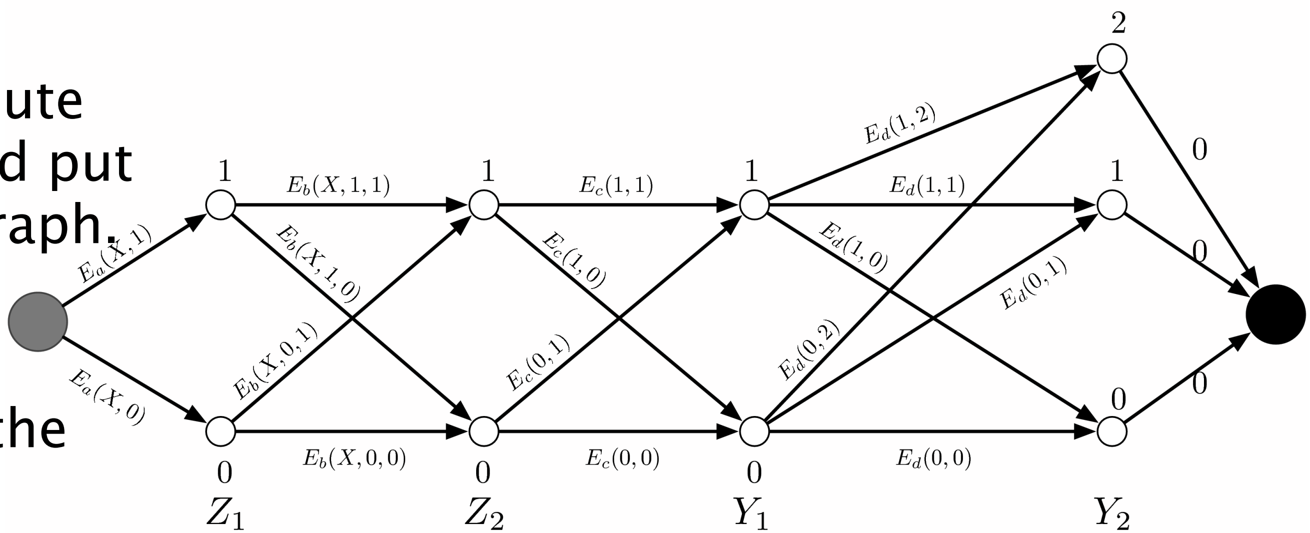
Efficient Inference: Energy-Based Factor Graphs

- Graphical models have brought us efficient inference algorithms, such as belief propagation and its numerous variations.
- Traditionally, graphical models are viewed as probabilistic models
- At first glance, is seems difficult to dissociate graphical models from the probabilistic view
- Energy-Based Factor Graphs are an extension of graphical models to non-probabilistic settings.
- An EBFG is an energy function that can be written as a sum of “factor” functions that take different subsets of variables as inputs.

Efficient Inference: Energy-Based Factor Graphs

- **The energy is a sum of "factor" functions**
- **Example:**
  - Z1, Z2, Y1 are binary
  - Z2 is ternary
  - A naïve exhaustive inference would require 2x2x2x3 energy evaluations (= 96 factor evaluations)
  - BUT: Ea only has 2 possible input configurations, Eb and Ec have 4, and Ed 6.
  - Hence, we can precompute the 16 factor values, and put them on the arcs in a graph.
  - A path in the graph is a config of variable
  - The cost of the path is the energy of the config



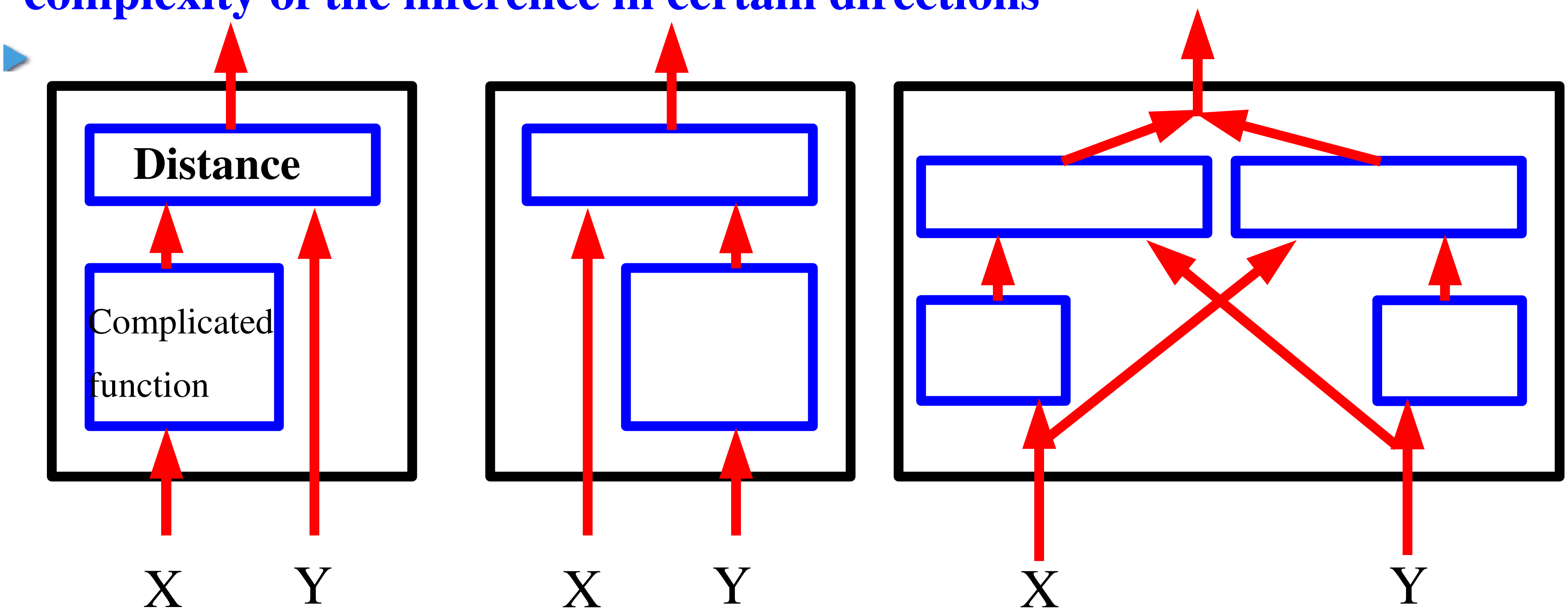


Energy-Based Belief Prop

- **The previous picture shows a chain graph of factors with 2 inputs.**
- **The extension of this procedure to trees, with factors that can have more than 2 inputs the "min-sum" algorithm (a non-probabilistic form of belief propagation)**
- **Basically, it is the sum-product algorithm with a different semi-ring algebra (min instead of sum, sum instead of product), and no normalization step.**
  - [Kschischang, Frey, Loeliger, 2001][McKay's book]

Feed-Forward, Causal, and Bi-directional Models

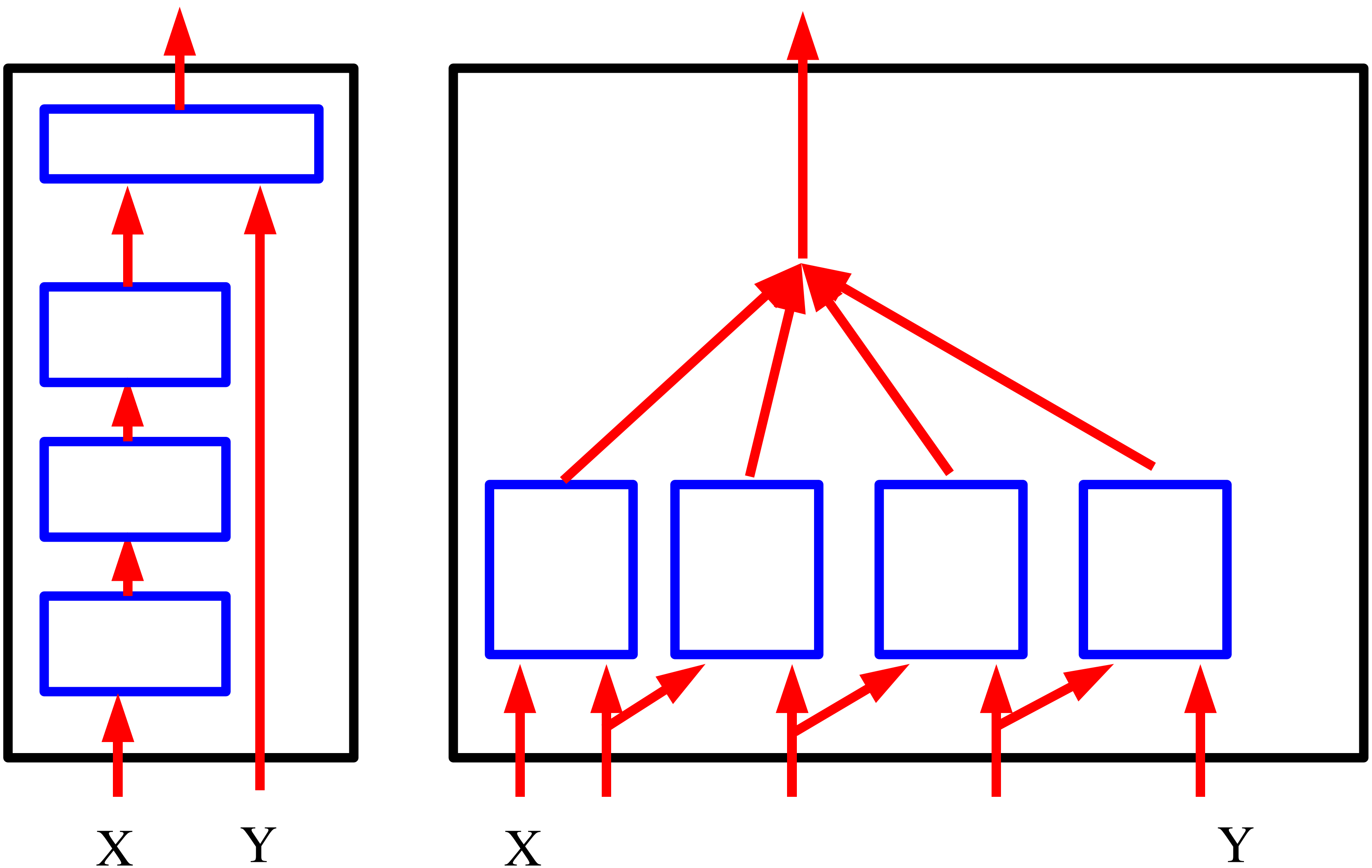
- **EBFG are all "undirected", but the architecture determines the complexity of the inference in certain directions**



- **Feed-Forward**
  - Predicting Y from X is easy
  - Predicting X from Y is hard
- **"Causal"**
  - Predicting Y from X is hard
  - Predicting X from Y is easy
- **Bi-directional**
  - X->Y and Y->X are both hard if the two factors don't agree.
  - They are both easy if the factors agree

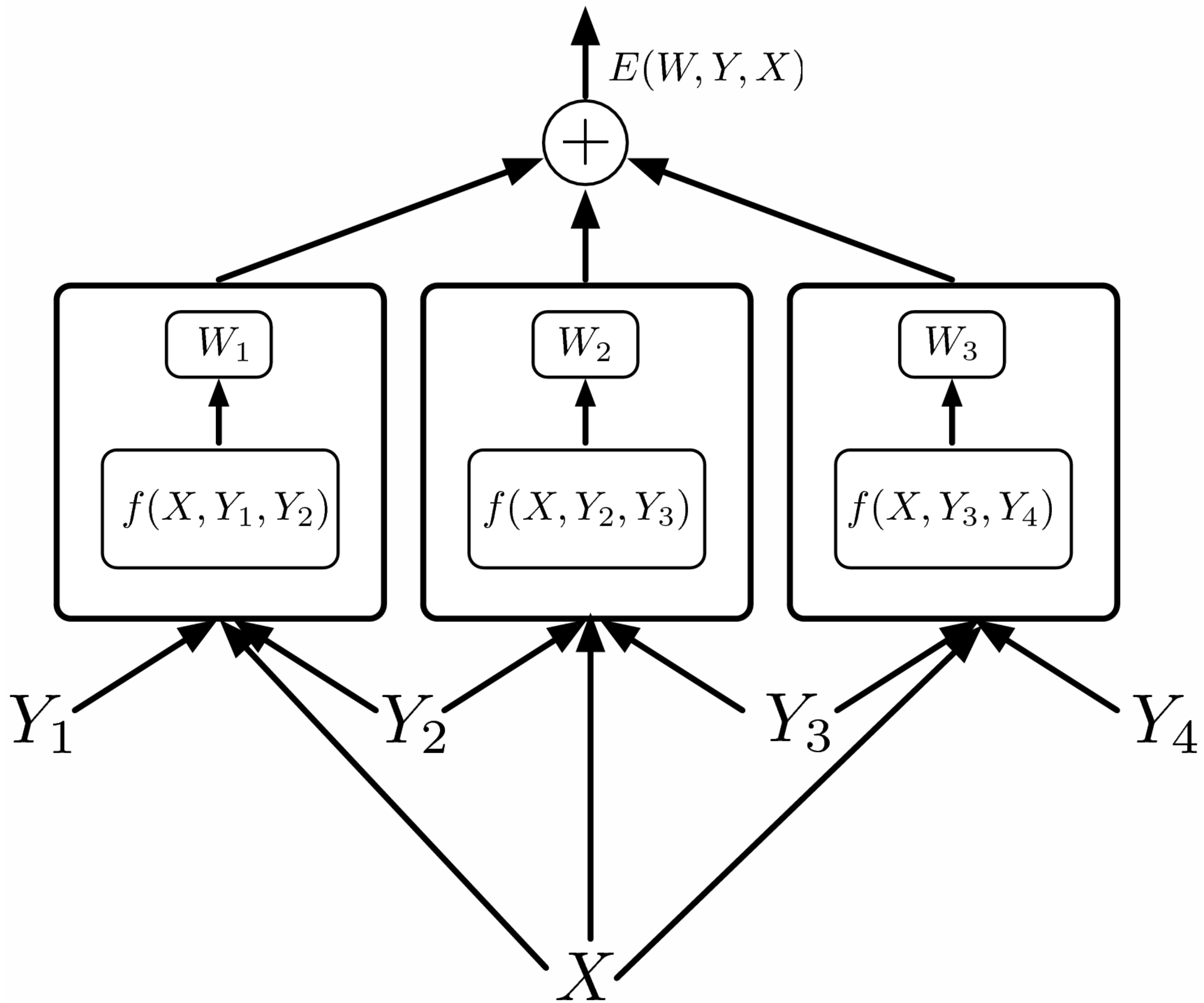
Two types of “deep” architectures

- **Factors are deep / graph is deep**



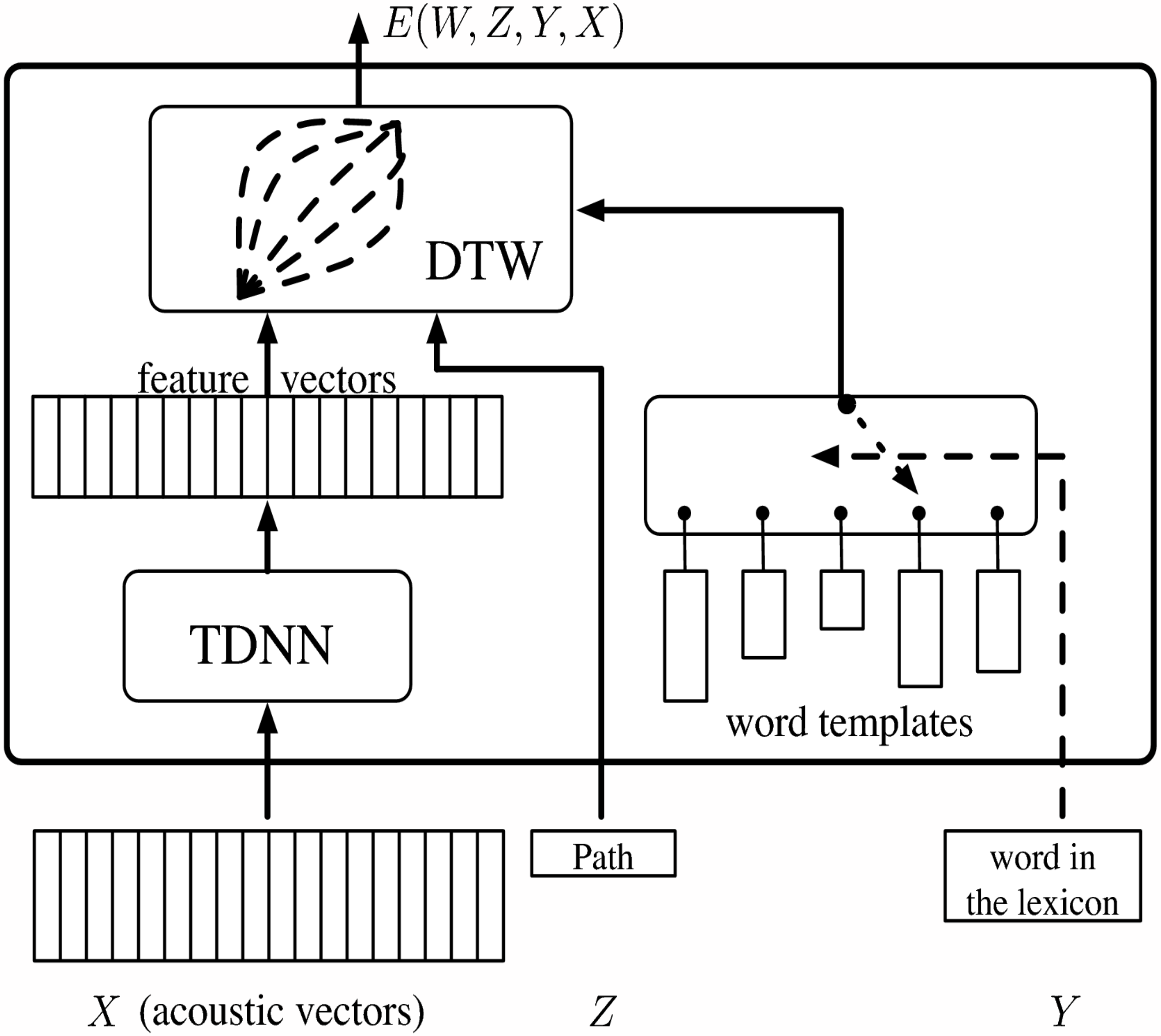
Shallow Factors / Deep Graph

- **Linearly Parameterized Factors**
- **with the NLL Loss :**
  - Lafferty's Conditional Random Field
- **with Hinge Loss:**
  - Taskar's Max Margin Markov Nets
- **with Perceptron Loss**
  - Collins's sequence labeling model
- **With Log Loss:**
  - Altun/Hofmann sequence labeling model



Deep Factors / Deep Graph: ASR with TDNN/DTW

- **Trainable Automatic Speech Recognition system with convolutional nets (TDNN) and dynamic time warping (DTW)**
- **Training the feature extractor as part of the whole process.**
- **with the LVQ2 Loss :**
  - Driancourt and Bottou's speech recognizer (1991)
- **with NLL:**
  - Bengio's speech recognizer (1992)
  - Haffner's speech recognizer (1993)



Deep Factors / Deep Graph: ASR with TDNN/HMM

- **Discriminative Automatic Speech Recognition system with HMM and various acoustic models**
  - Training the acoustic model (feature extractor) and a (normalized) HMM in an integrated fashion.
- **With Minimum Empirical Error loss**
  - Ljolje and Rabiner (1990)
- **with NLL:**
  - Bengio (1992)
  - Haffner (1993)
  - Bourlard (1994)
- **With MCE**
  - Juang et al. (1997)
- **Late normalization scheme (un-normalized HMM)**
  - Bottou pointed out the **label bias problem** (1991)
  - Denker and Burges proposed a solution (1995)

Really Deep Factors / Really Deep Graph

- **Handwriting Recognition with Graph Transformer Networks**
- **Un-normalized hierarchical HMMs**
  - Trained with Perceptron loss [LeCun, Bottou, Bengio, Haffner 1998]
  - Trained with NLL loss [Bengio, LeCun 1994], [LeCun, Bottou, Bengio, Haffner 1998]
- **Answer = sequence of symbols**
- **Latent variable = segmentation**