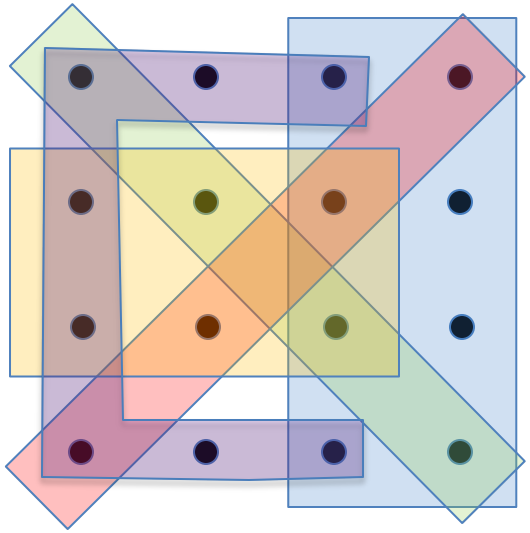
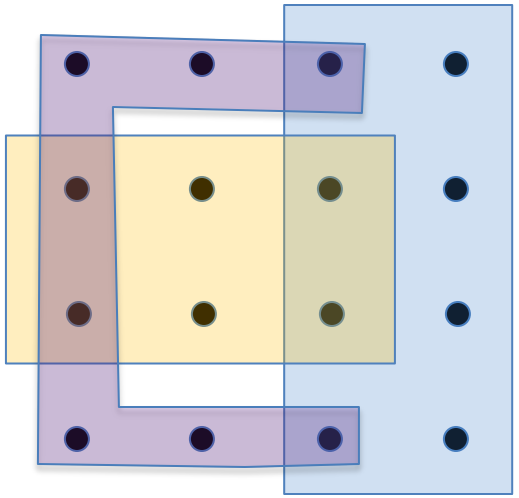
Set cover

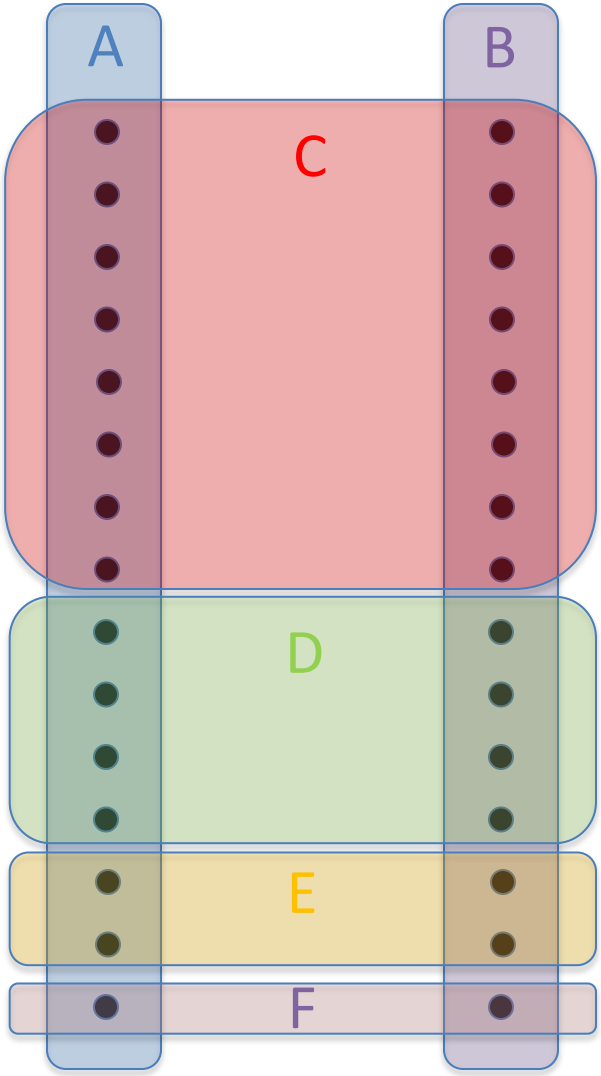


A universe of 16 elements

Five sets over this universe:
purple, orange, red, green, blue



A minimum cover



Optimal cover = {A,B}

Greedy cover = {C,D,E,F}

<u>Generalize</u>:

Universe of n = 2^m-2 elements

Optimal cover: 2 sets

Greedy cover: m-1 sets

Greedy/Optimal = $\Theta(\log n)$

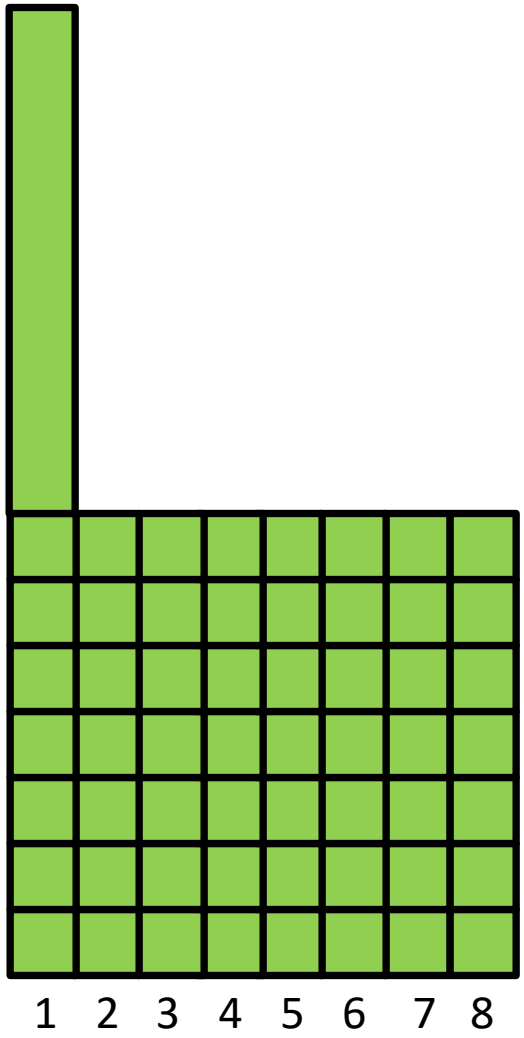
Not a constant approximation factor

Universe of 30 = 2^5-2 elements

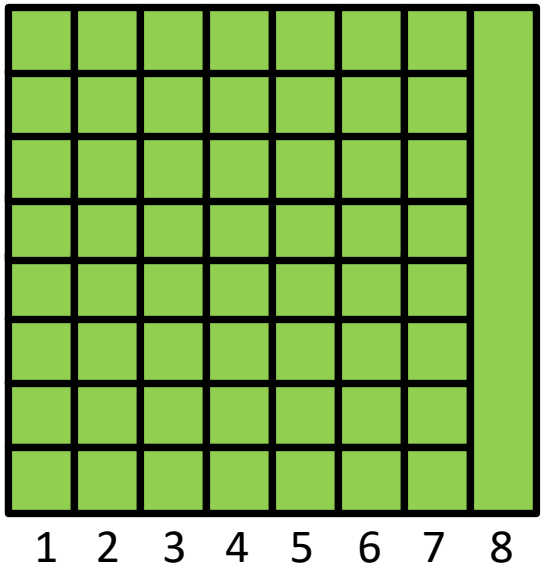
Example for tight analysis of greedy makespan algorithm

- $m(m-1)$ jobs, each of length 1
- 1 job of length m

$$\frac{\text{Greedy makespan}}{\text{Optimal makespan}} = \frac{2m-1}{m} = 2 - \frac{1}{m}$$

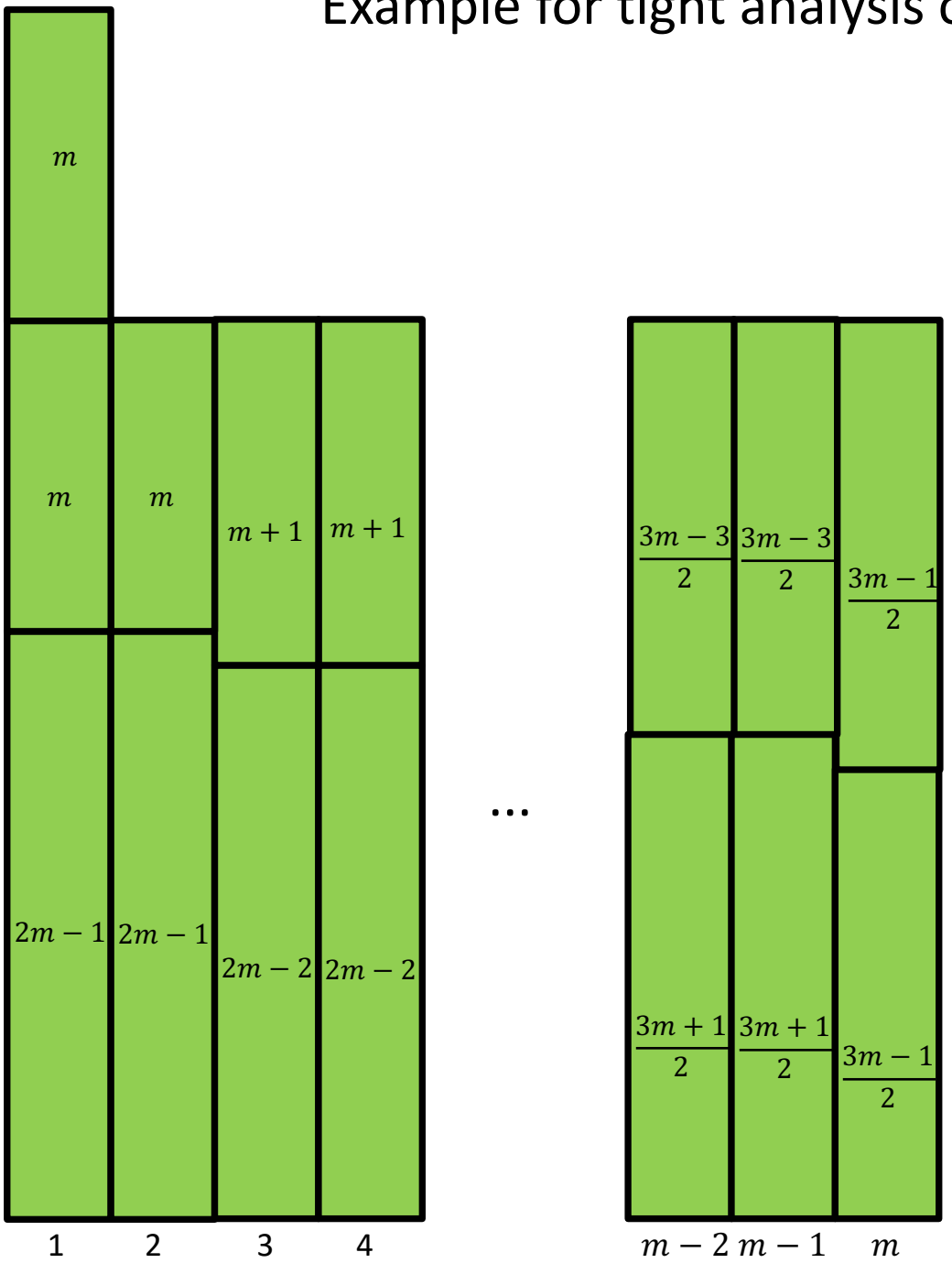


Greedy assignment



Optimal assignment

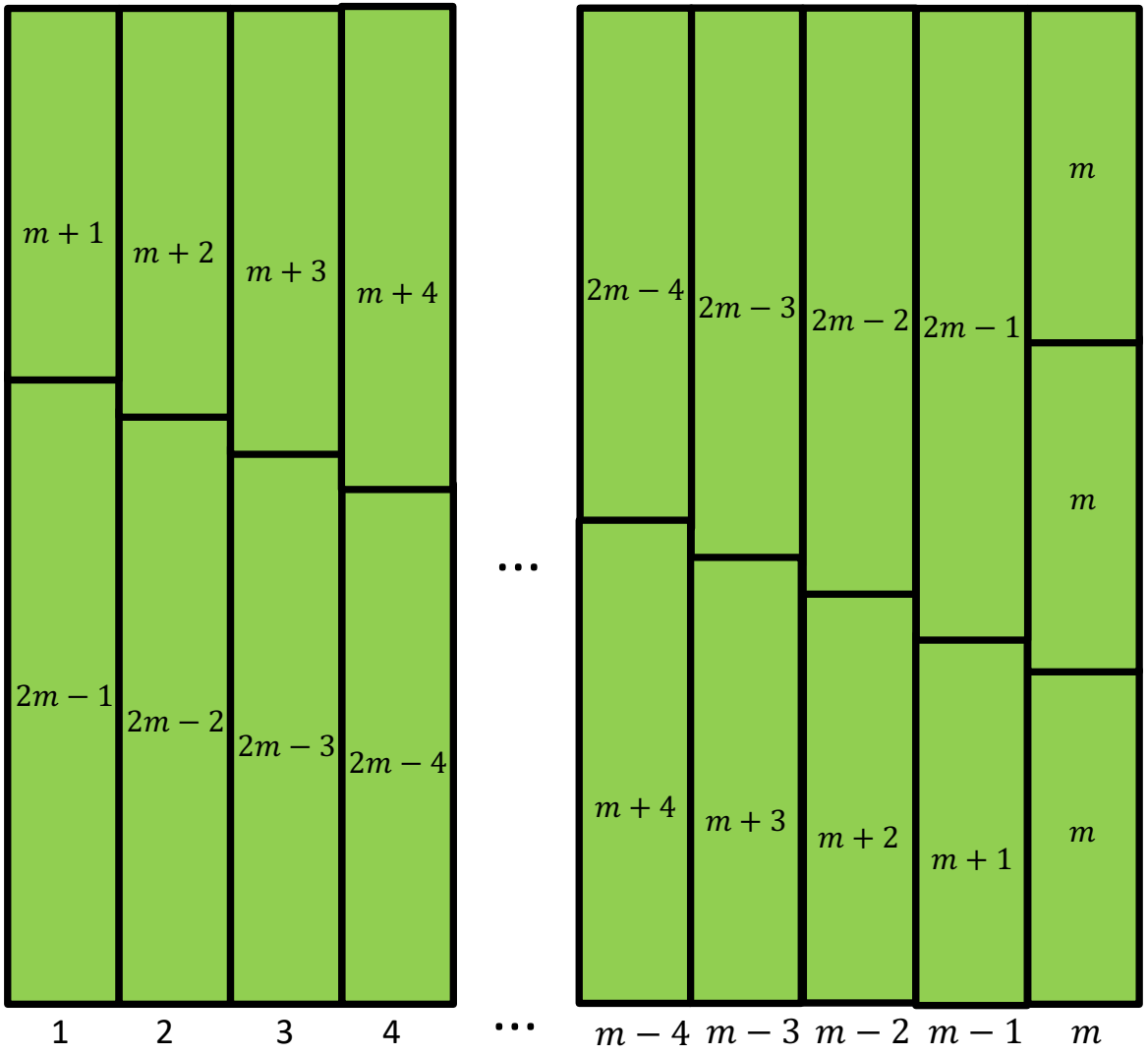
Example for tight analysis of longest-first greedy algorithm



- odd m
- 2 jobs of length $2m-1$
- 2 jobs of length $2m-2$
- 2 jobs of length $2m-3$
- ...
- 2 jobs of length $m+1$
- 3 jobs of length m

Makespan $= 4m-1$

Example for tight analysis of longest-first greedy algorithm



- odd m
- 2 jobs of length $2m-1$
- 2 jobs of length $2m-2$
- 2 jobs of length $2m-3$
- ...
- 2 jobs of length $m+1$
- 3 jobs of length m

Makespan $= 3m$

$$\frac{\text{Longest first}}{\text{Optimal}} = \frac{4m-1}{3m} = \frac{4}{3} - \frac{1}{3m}$$