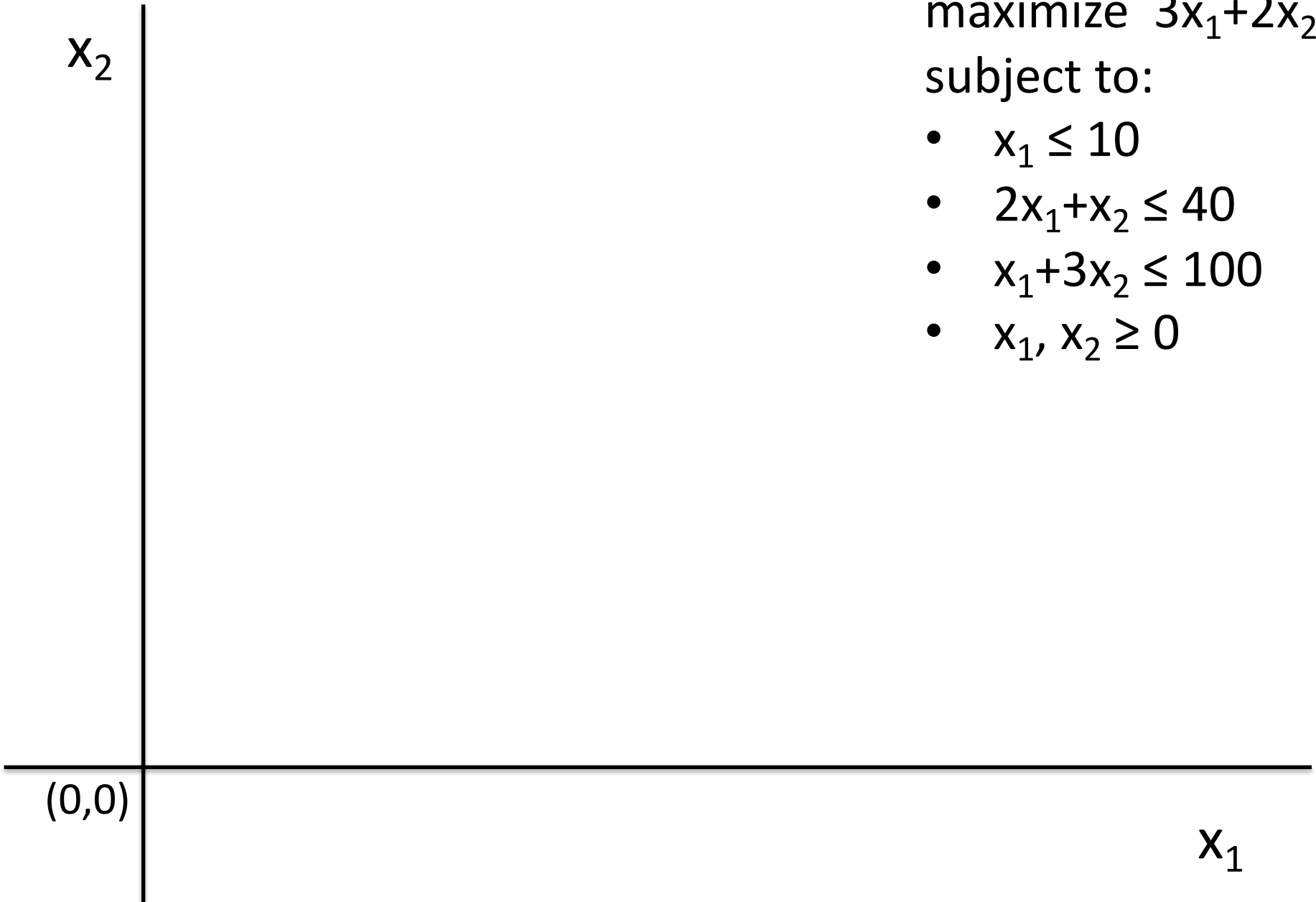


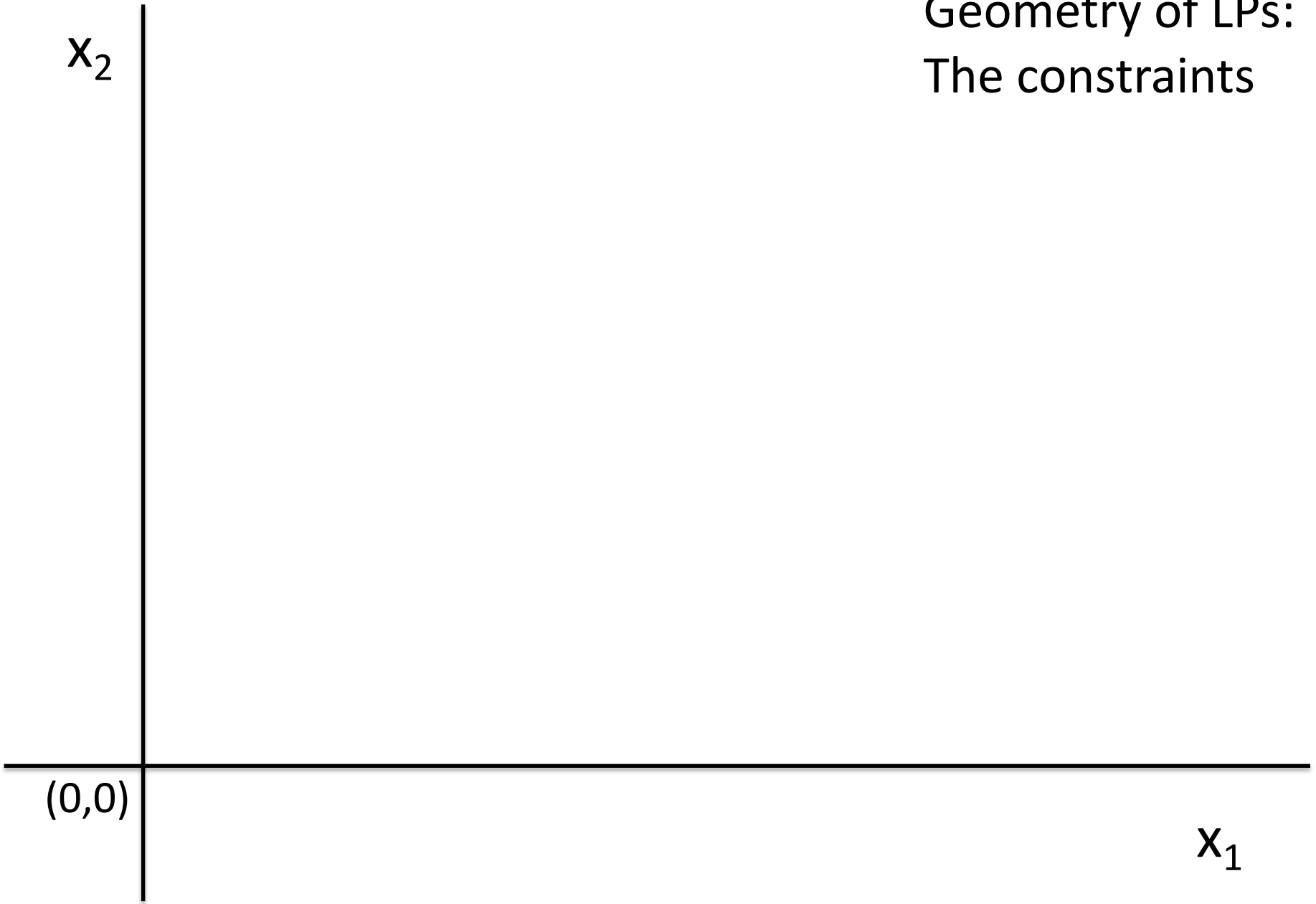
maximize  $3x_1+2x_2$

subject to:

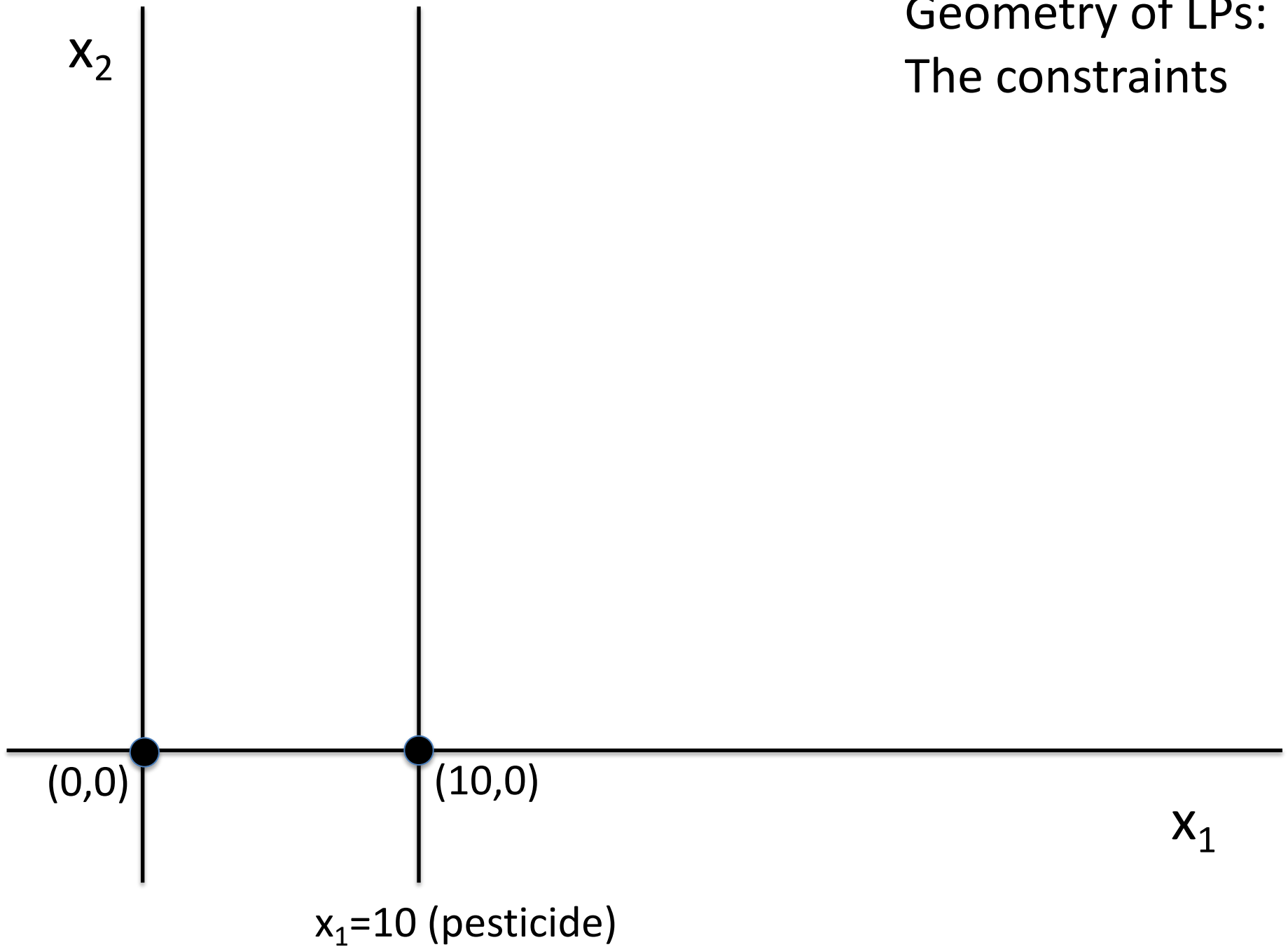
- $x_1 \leq 10$
- $2x_1+x_2 \leq 40$
- $x_1+3x_2 \leq 100$
- $x_1, x_2 \geq 0$



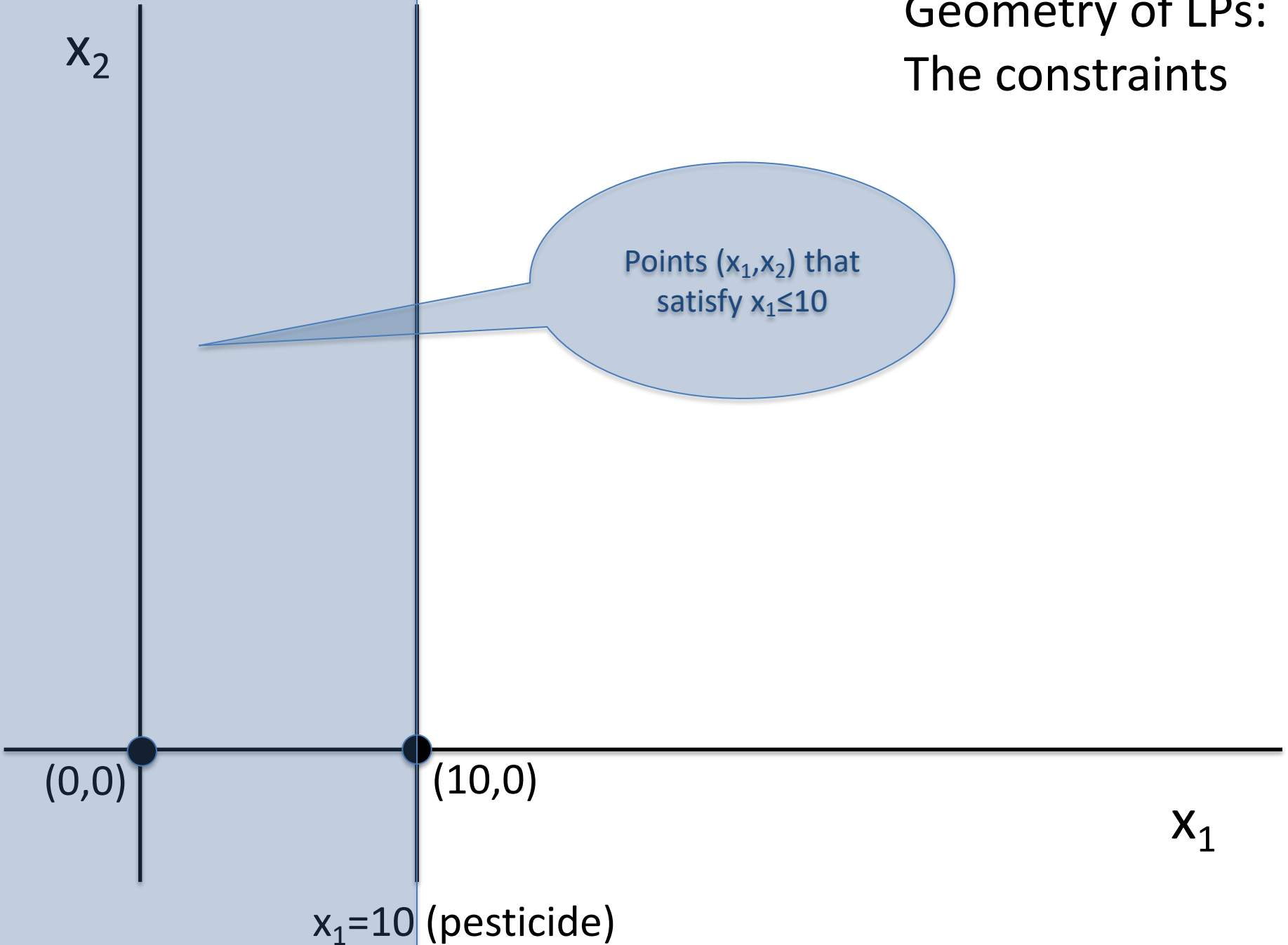
# Geometry of LPs: The constraints



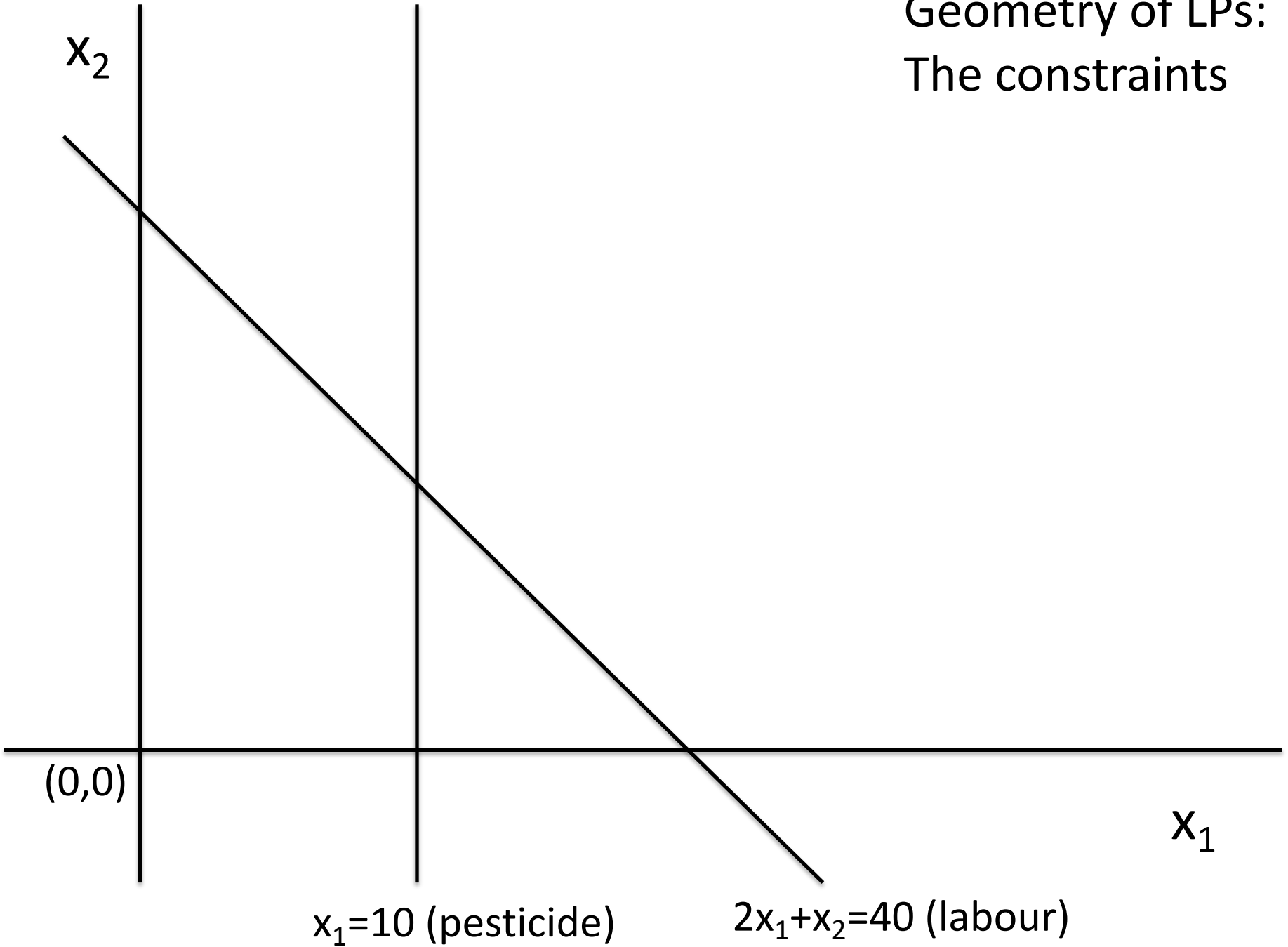
# Geometry of LPs: The constraints



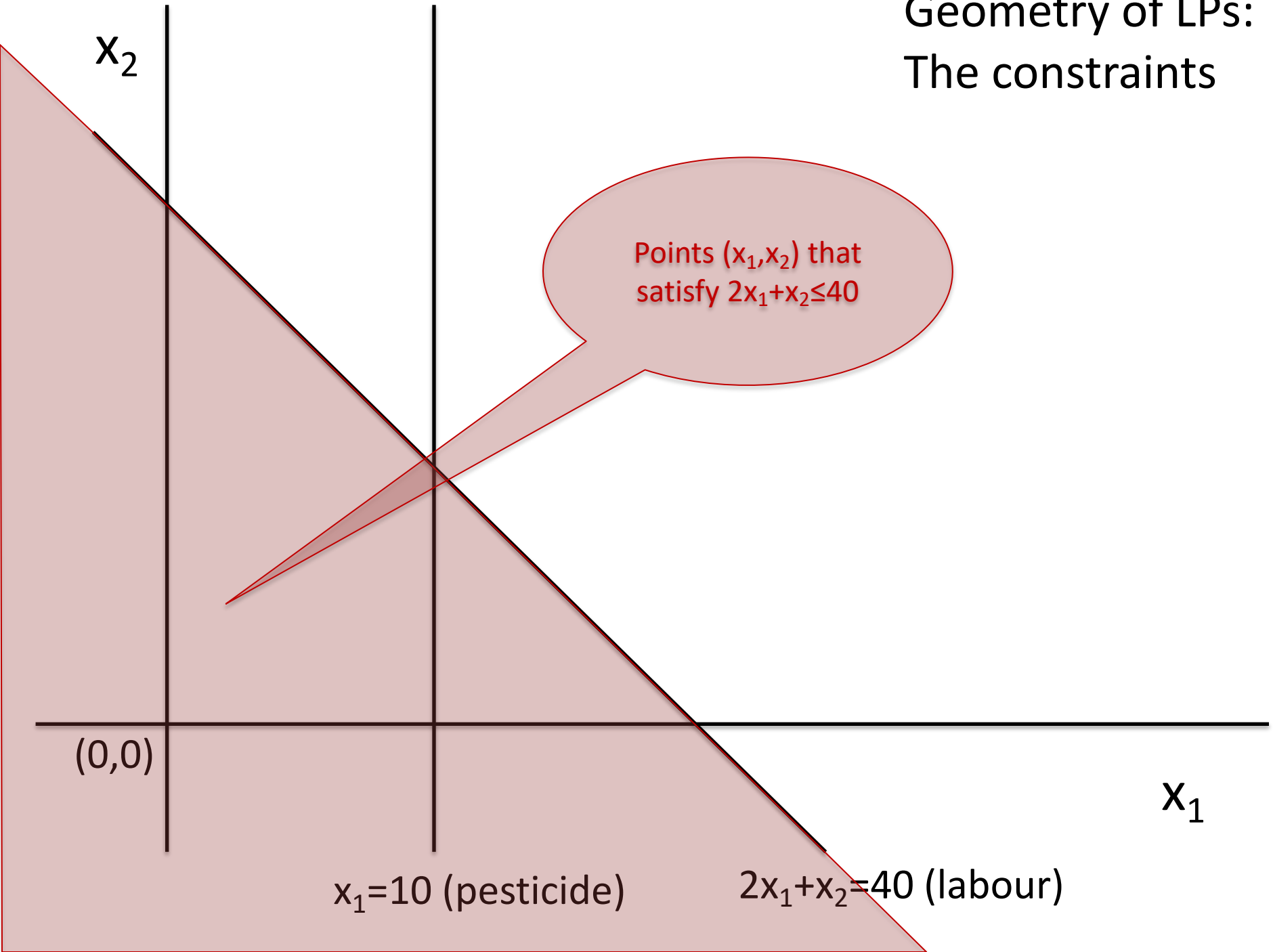
# Geometry of LPs: The constraints



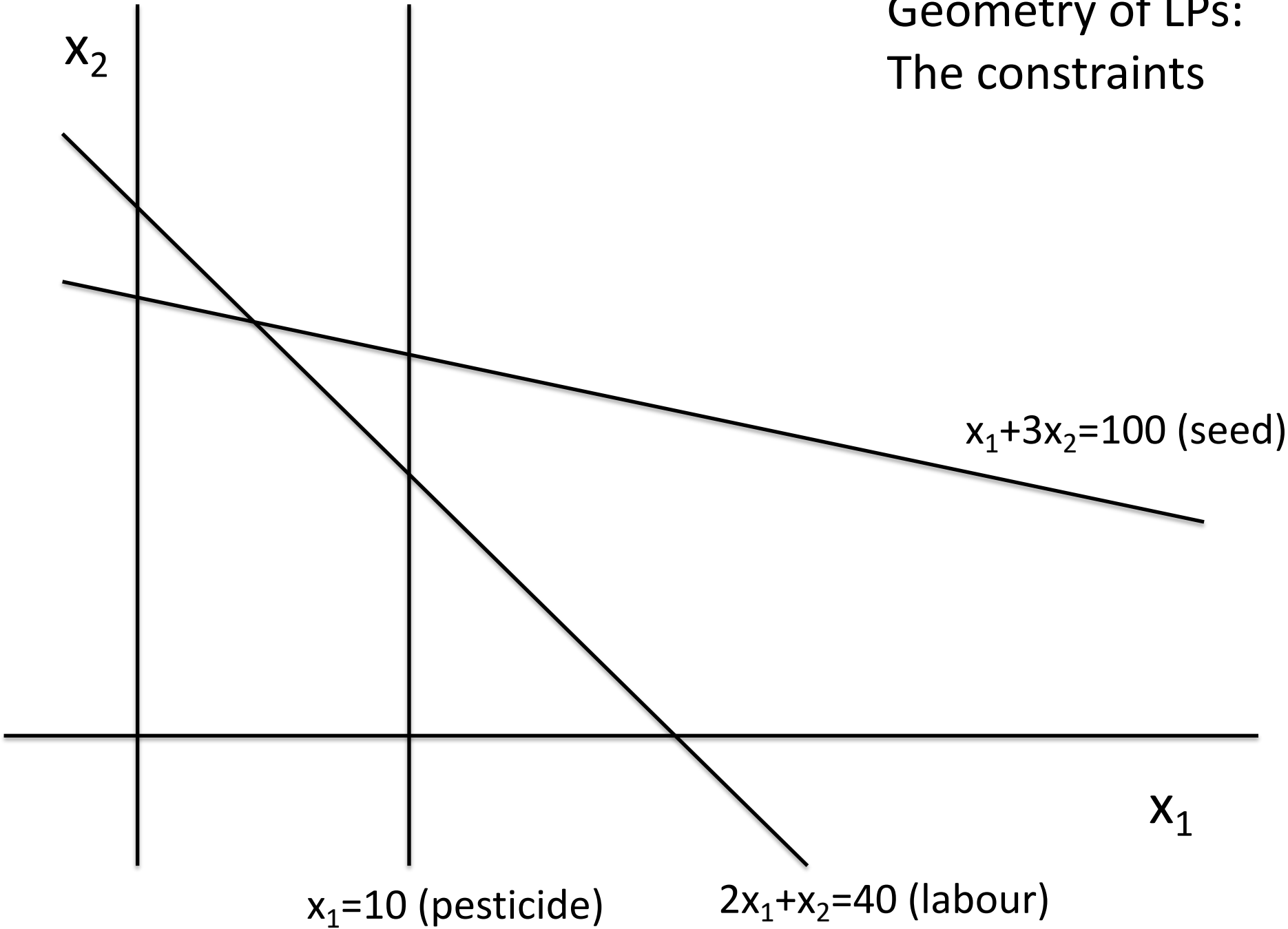
# Geometry of LPs: The constraints



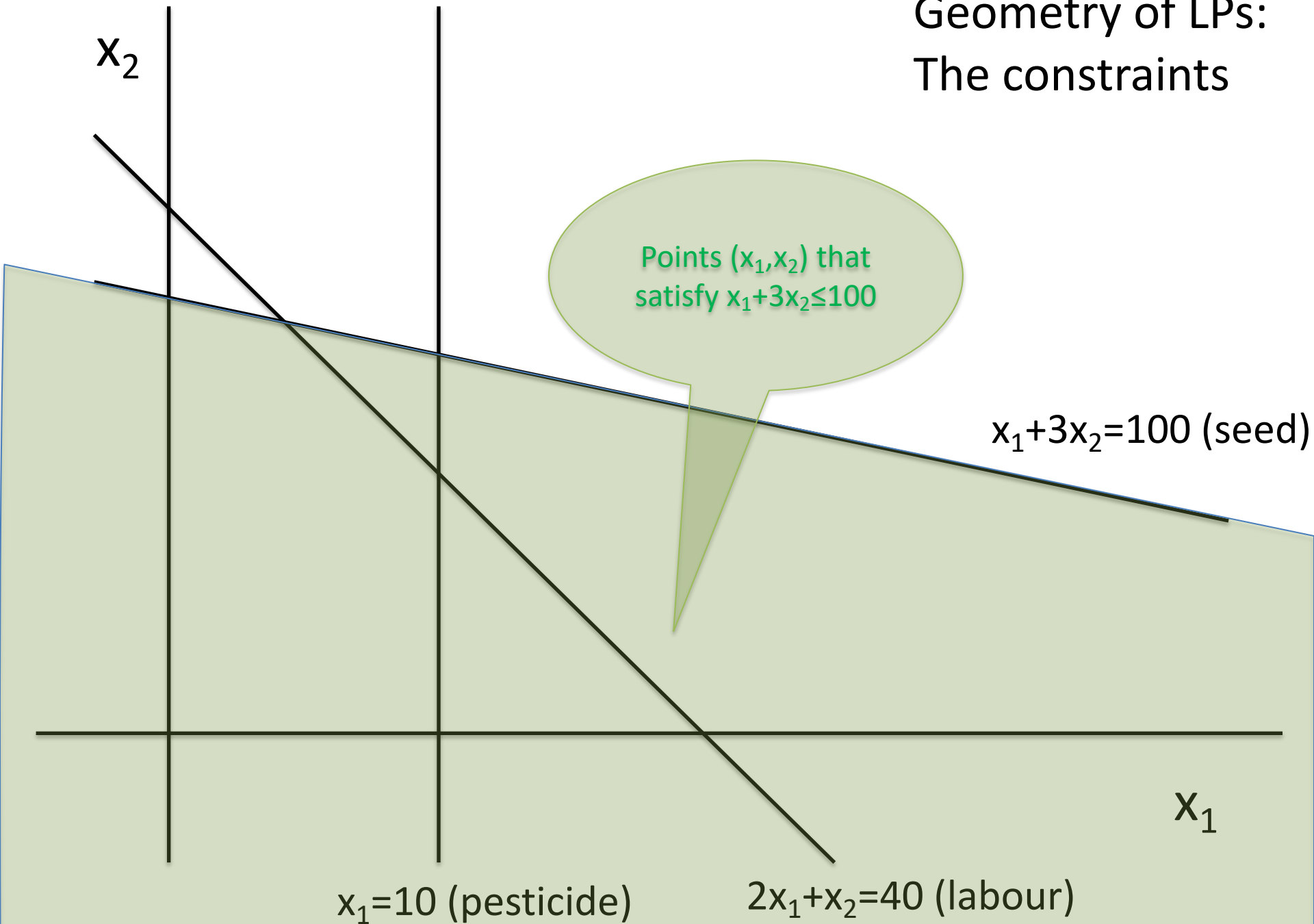
# Geometry of LPs: The constraints



# Geometry of LPs: The constraints

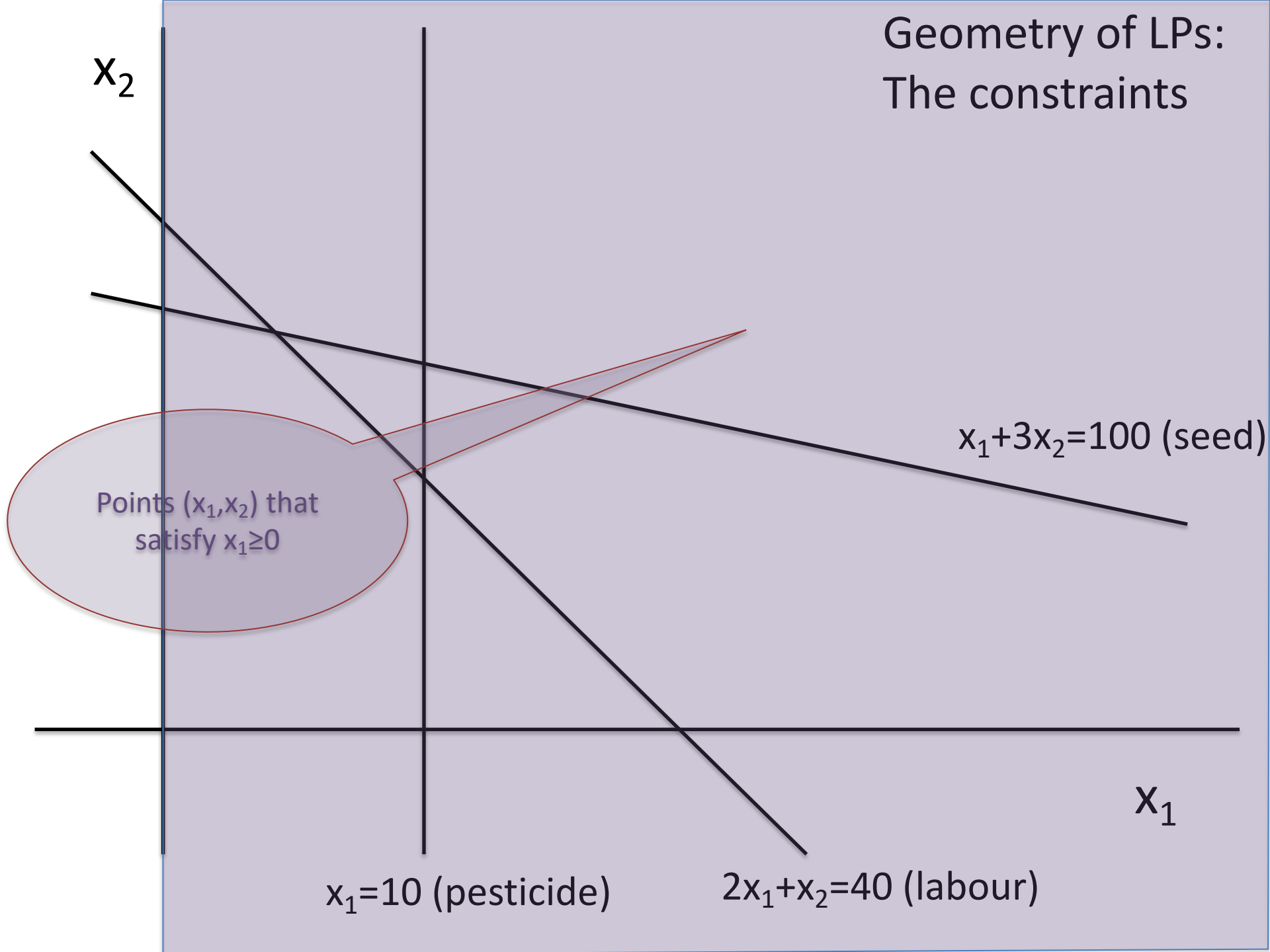


# Geometry of LPs: The constraints





# Geometry of LPs: The constraints



# Geometry of LPs: The constraints

$x_2$

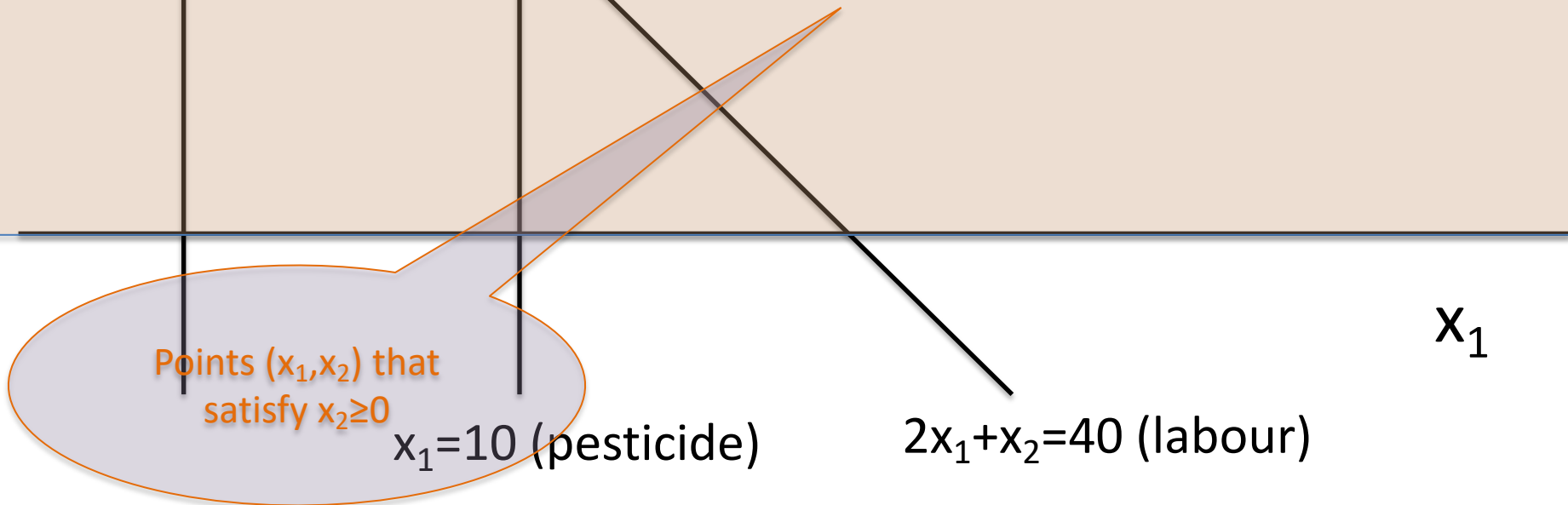
$x_1 + 3x_2 = 100$  (seed)

$x_1$

Points  $(x_1, x_2)$  that  
satisfy  $x_2 \geq 0$

$x_1 = 10$  (pesticide)

$2x_1 + x_2 = 40$  (labour)



# Geometry of LPs: The constraints

$x_2$

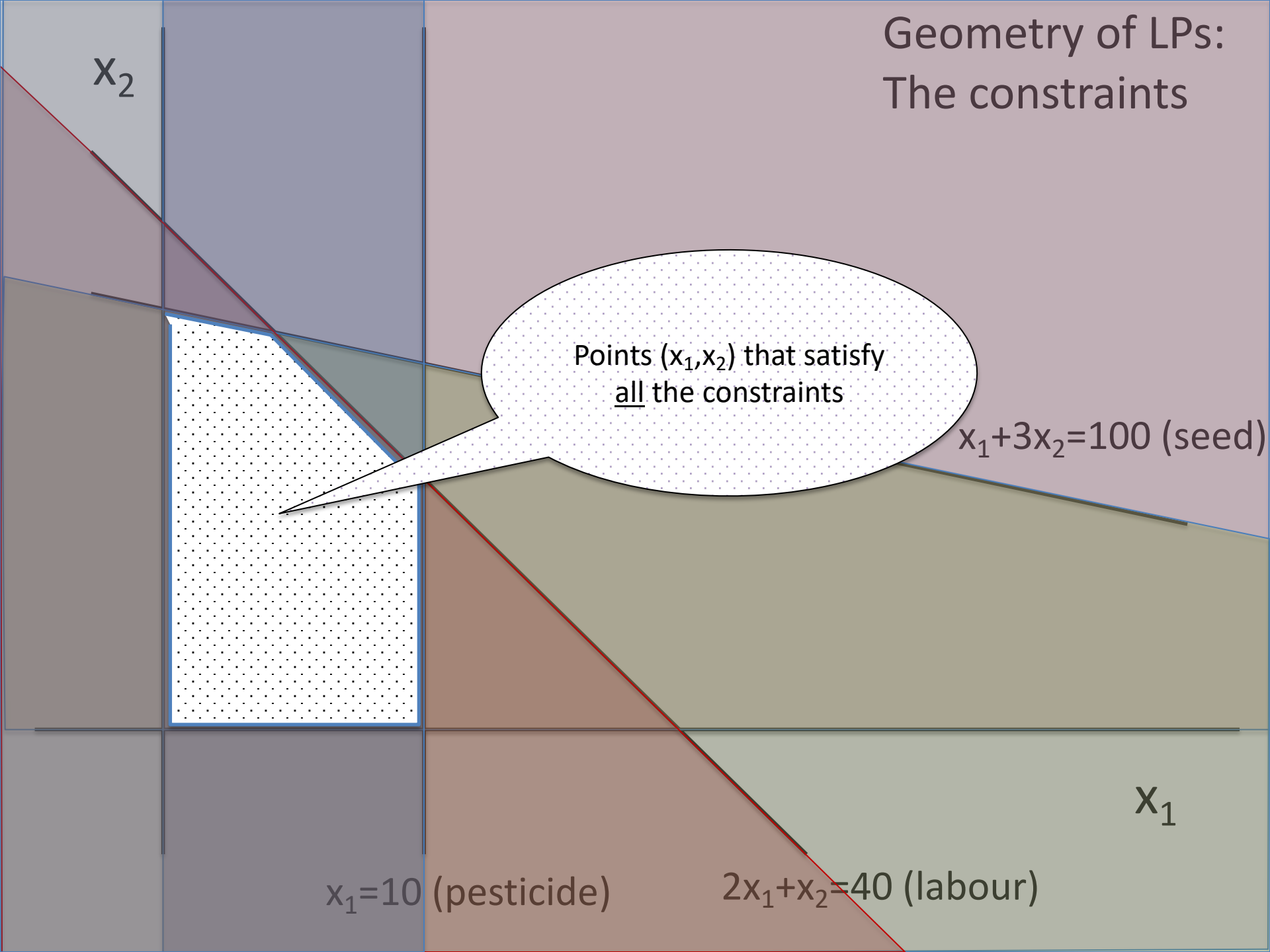
Points  $(x_1, x_2)$  that satisfy  
all the constraints

$x_1 + 3x_2 = 100$  (seed)

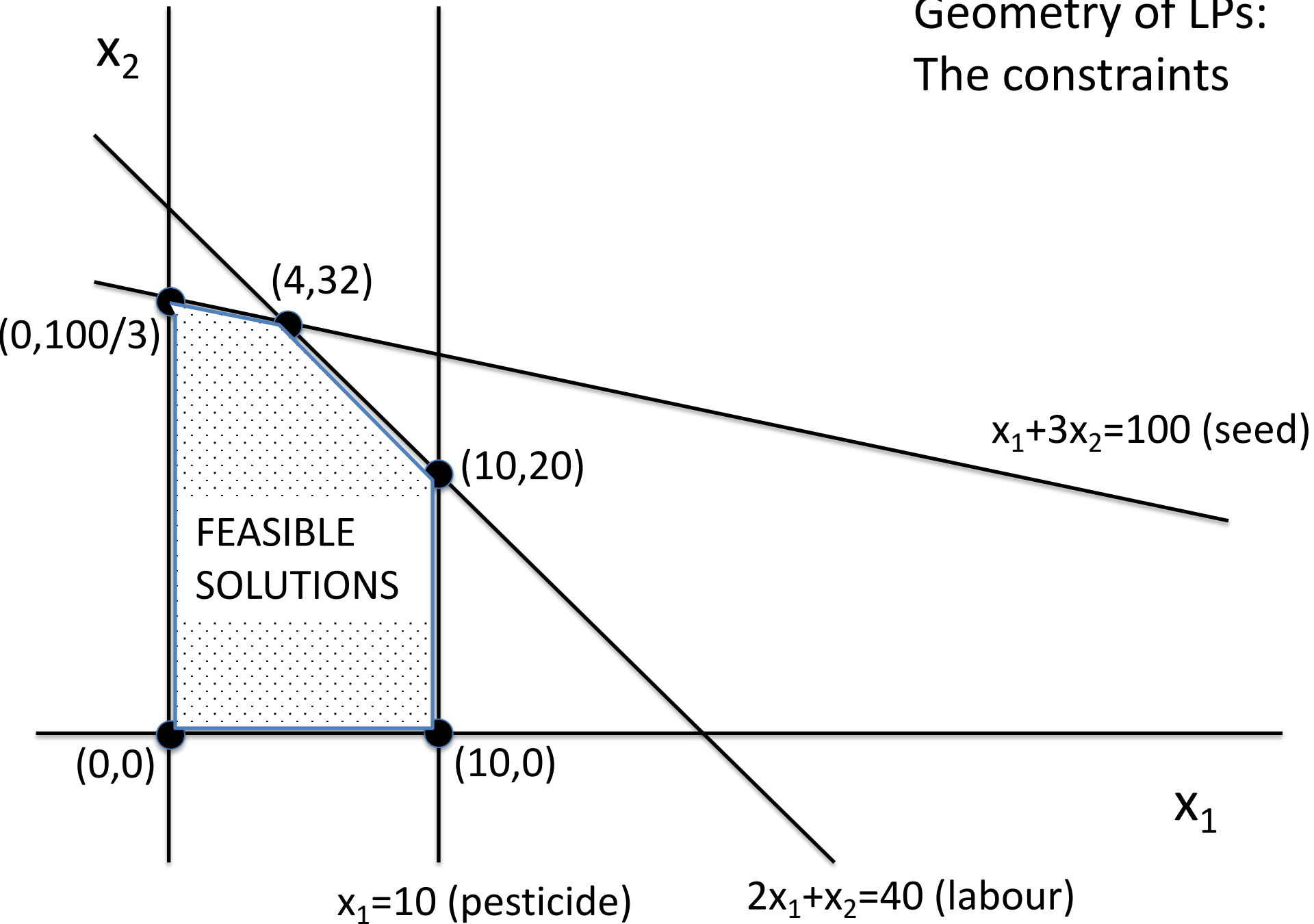
$x_1 = 10$  (pesticide)

$2x_1 + x_2 = 40$  (labour)

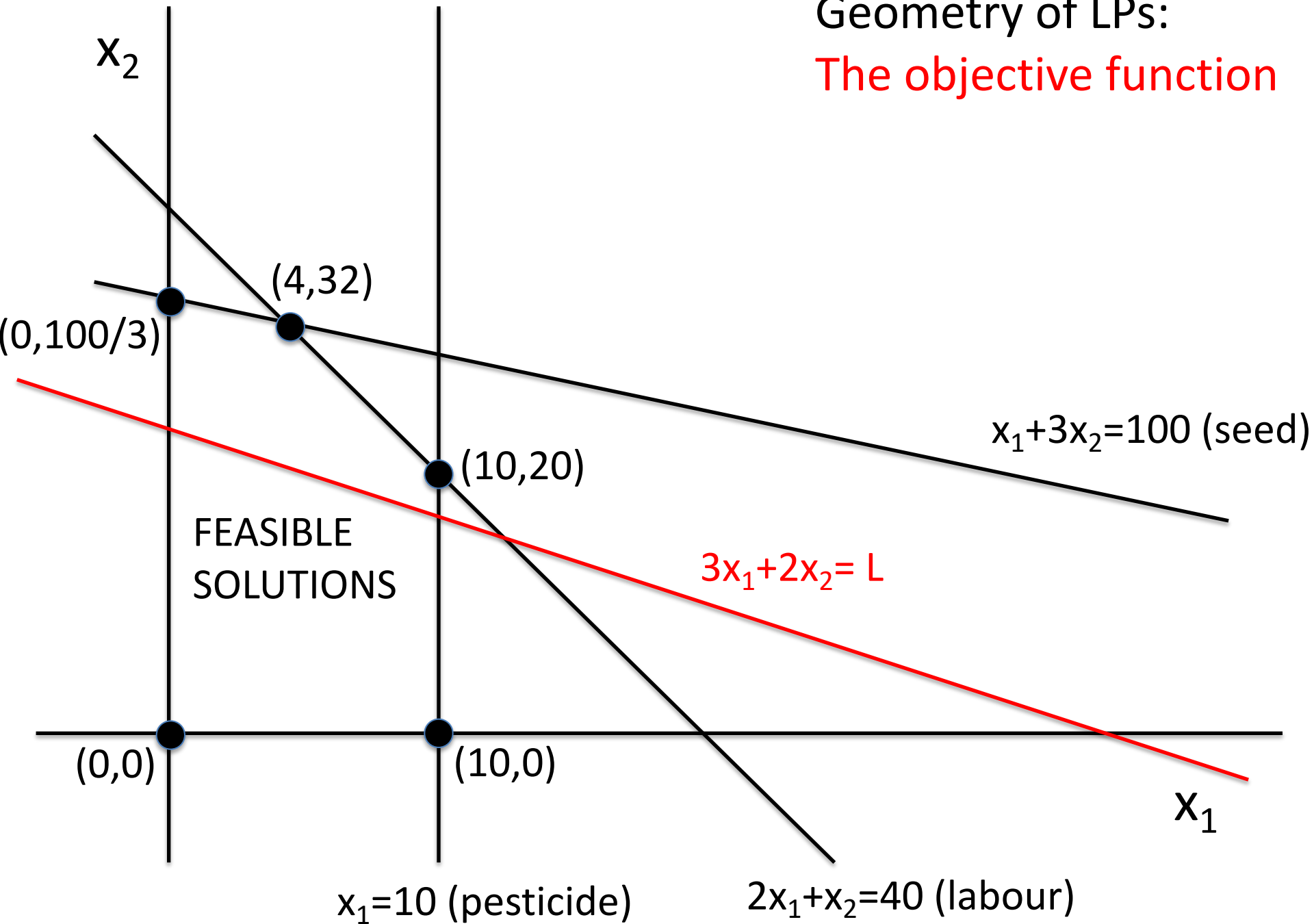
$x_1$



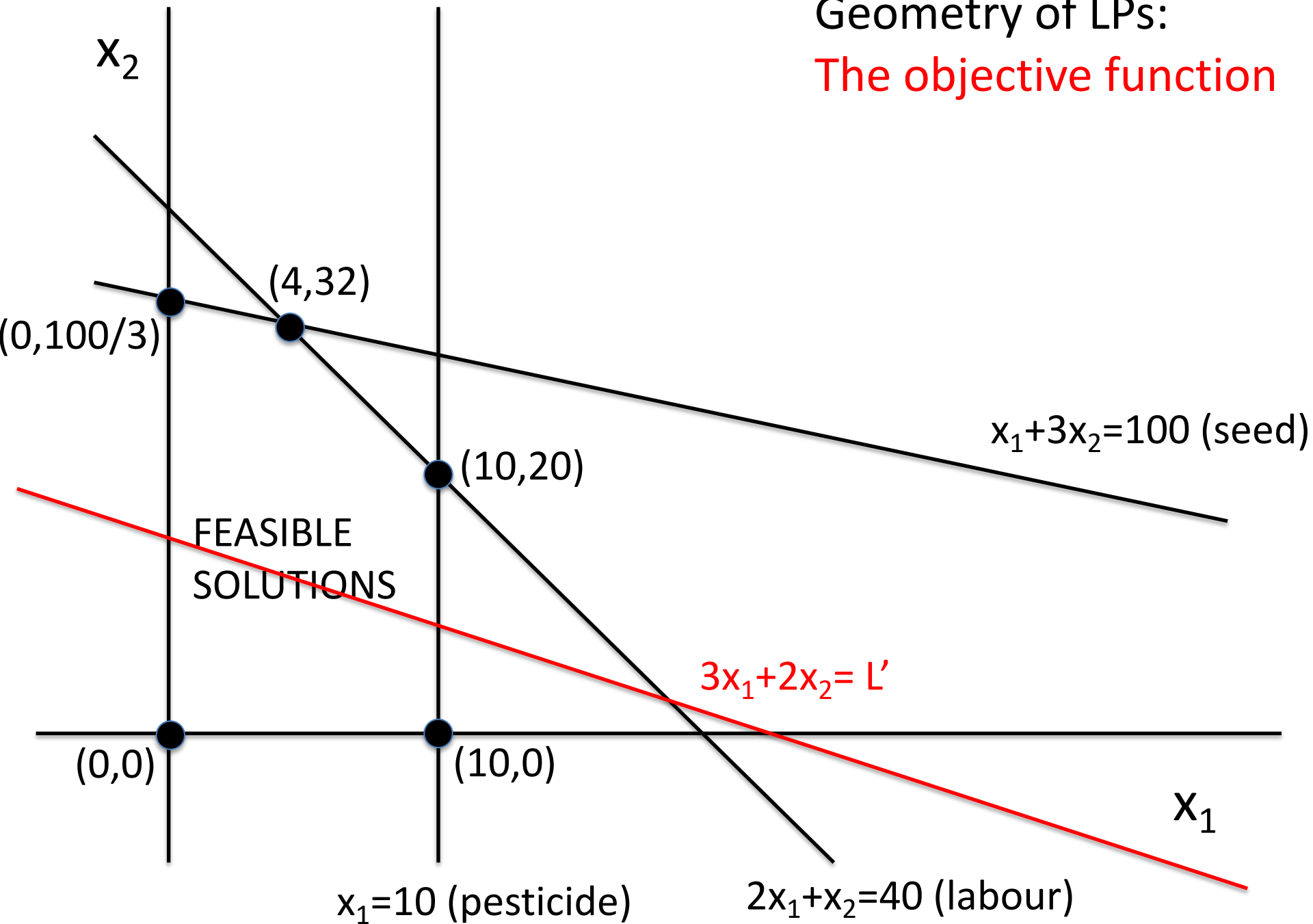
# Geometry of LPs: The constraints



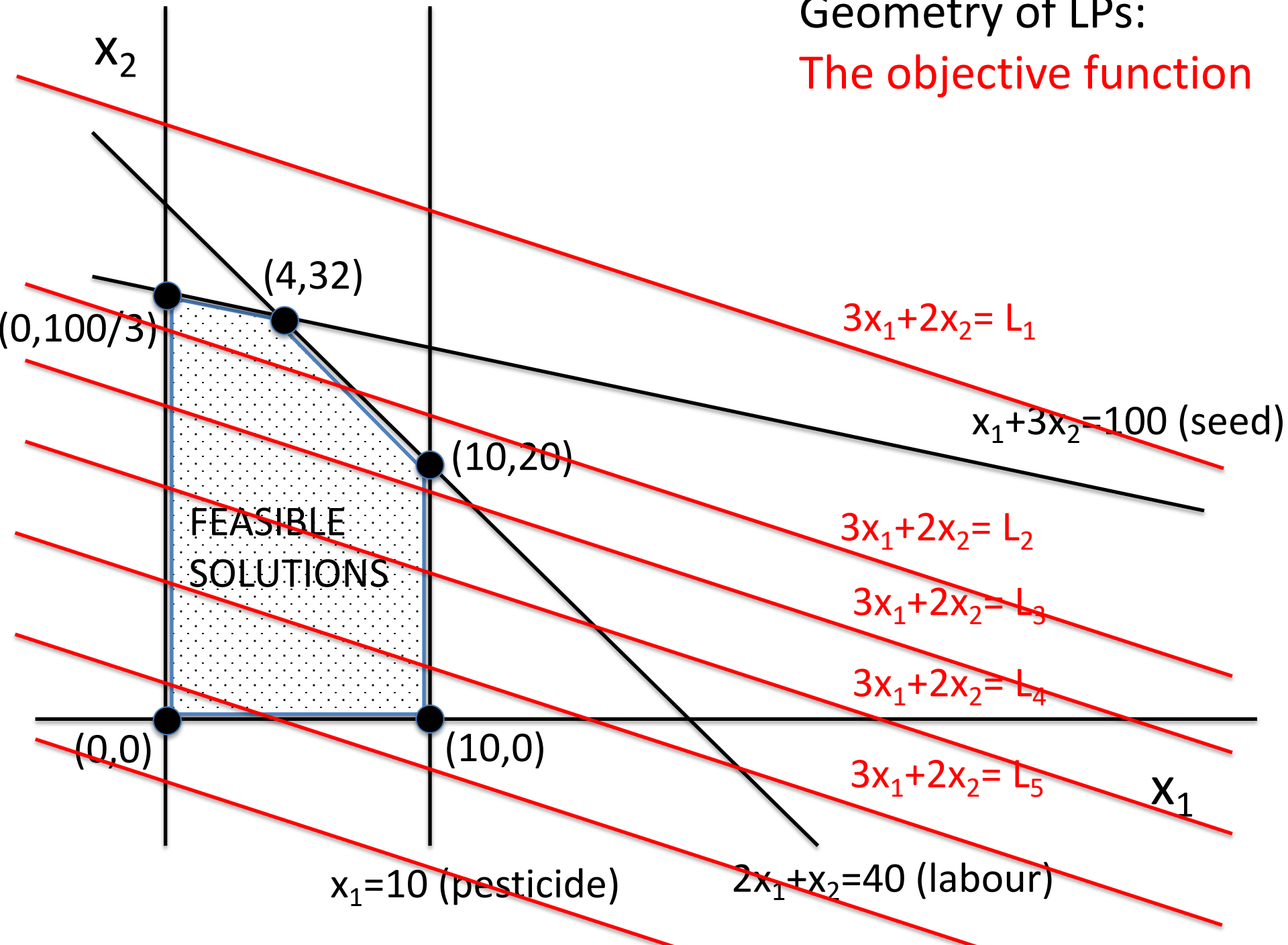
# Geometry of LPs: The objective function



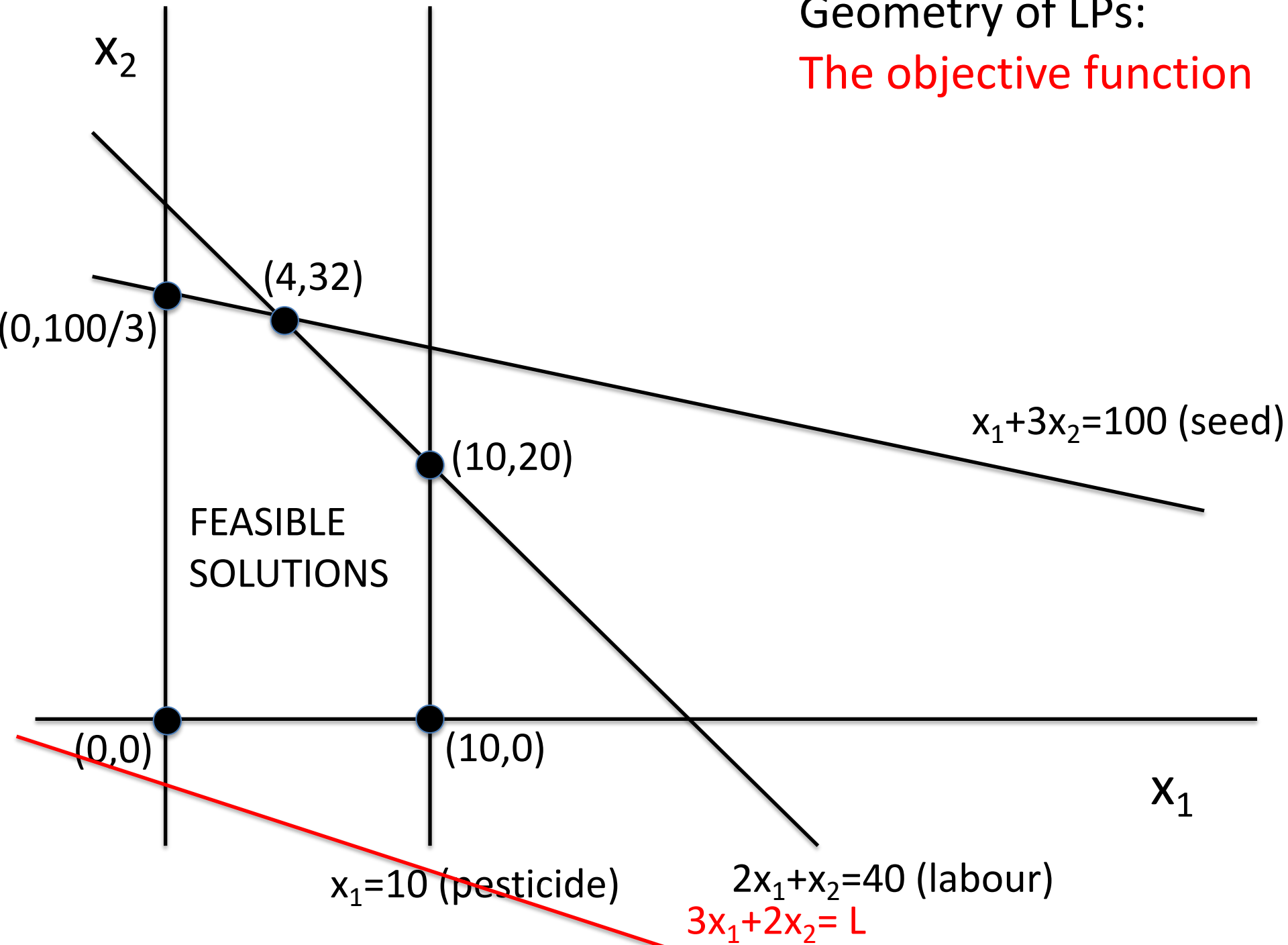
# Geometry of LPs: The objective function



# Geometry of LPs: The objective function

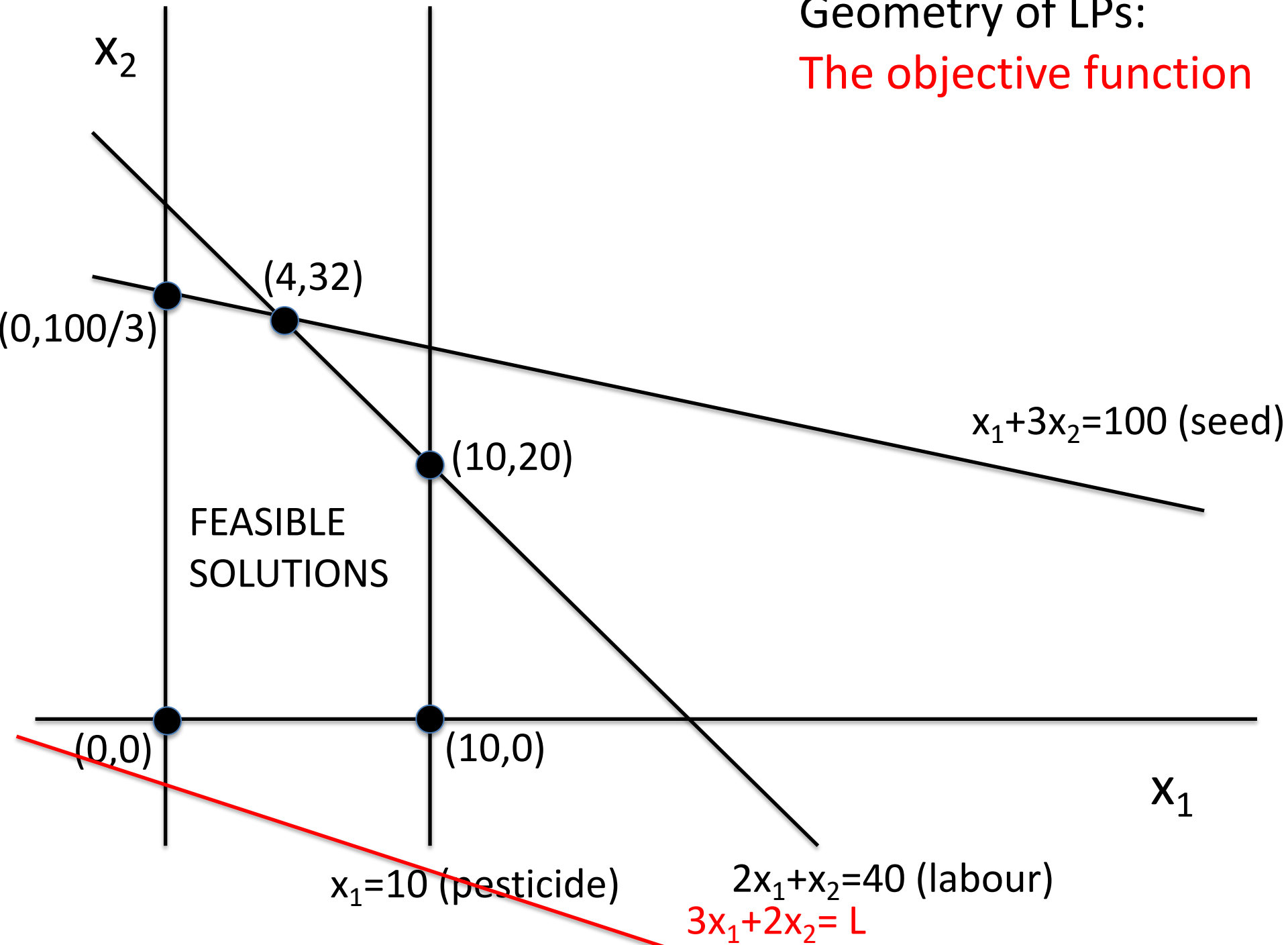


# Geometry of LPs: The objective function

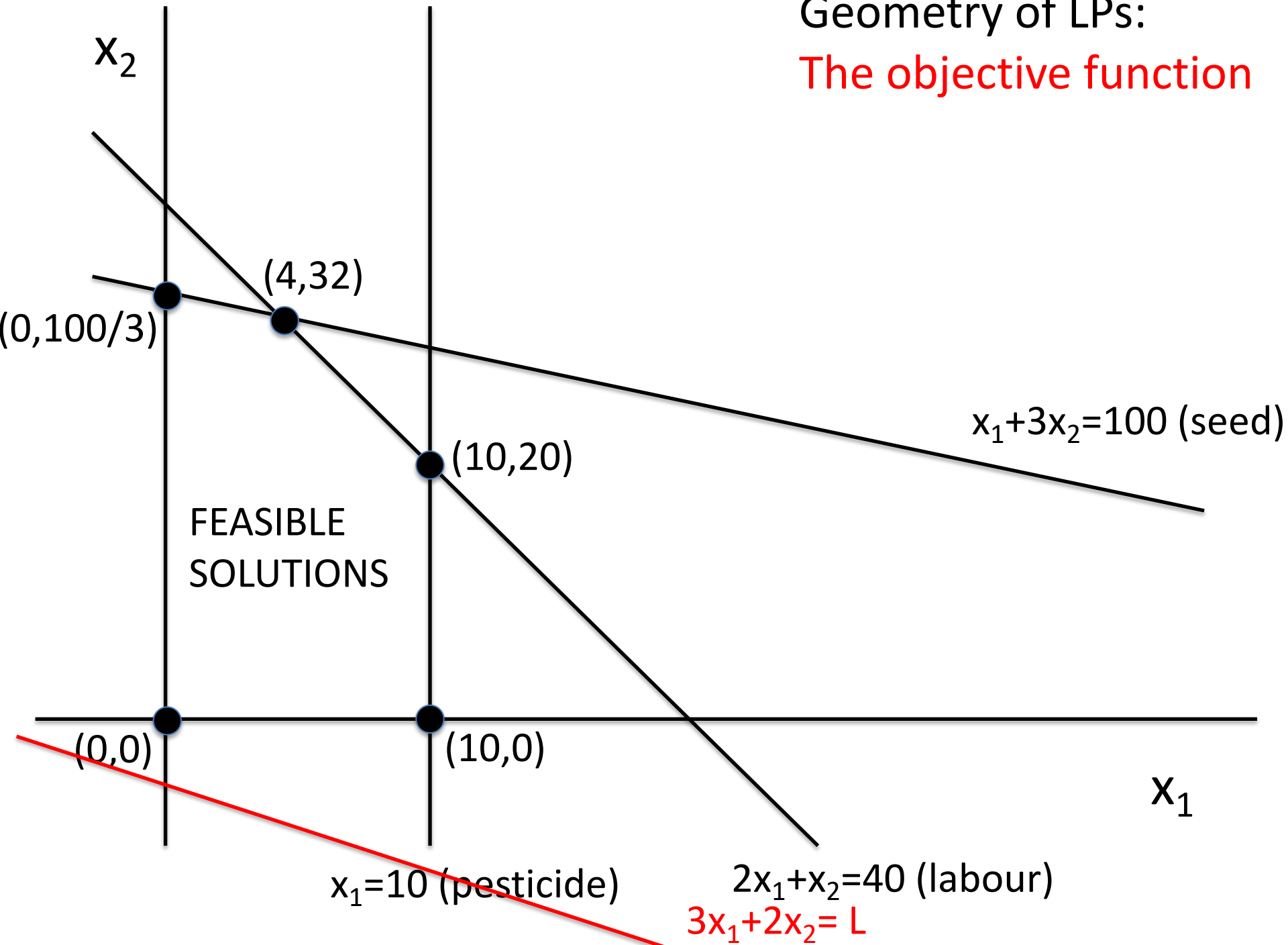




# Geometry of LPs: The objective function

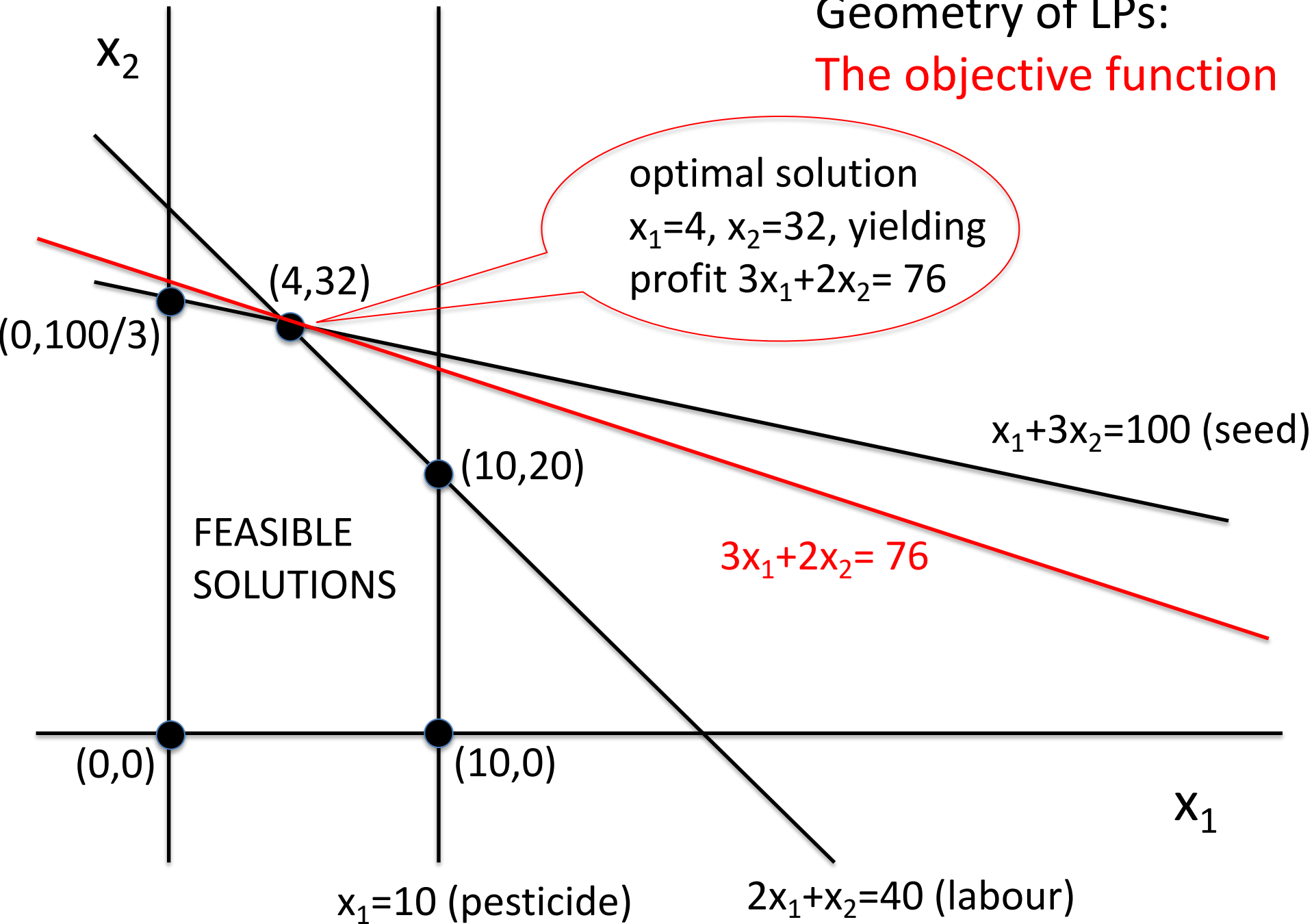


# Geometry of LPs: The objective function



# Geometry of LPs:

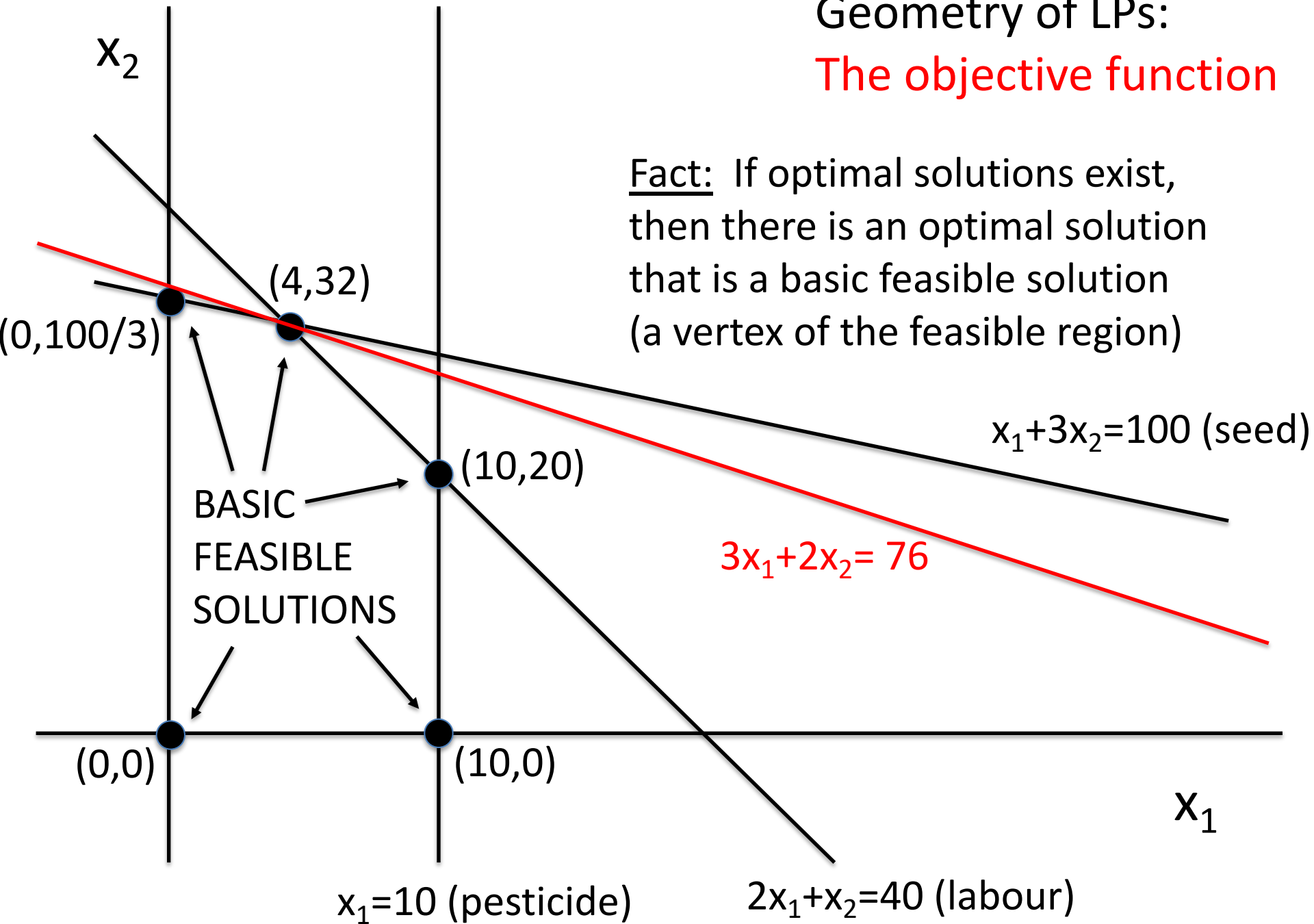
## The objective function



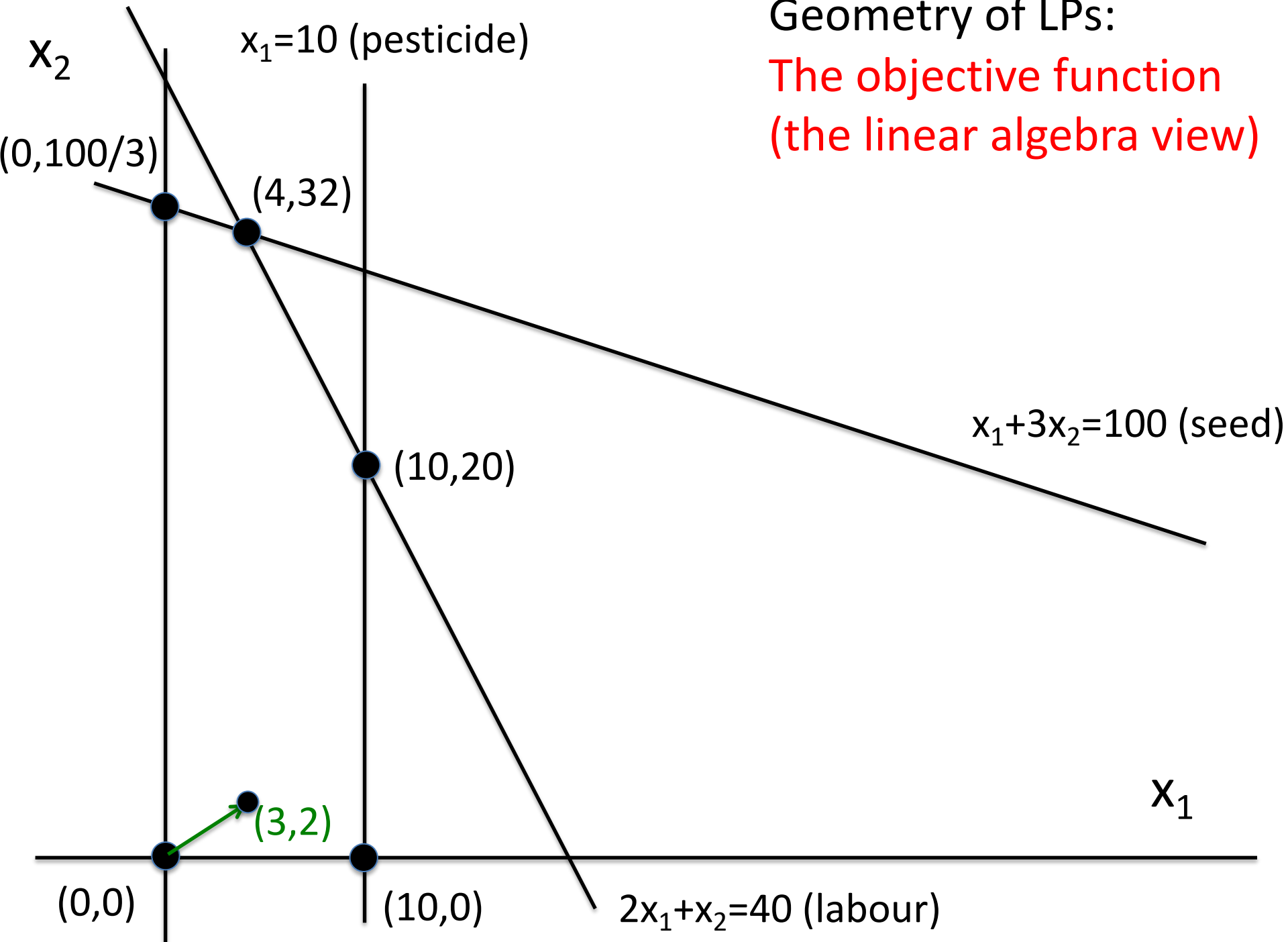
# Geometry of LPs:

## The objective function

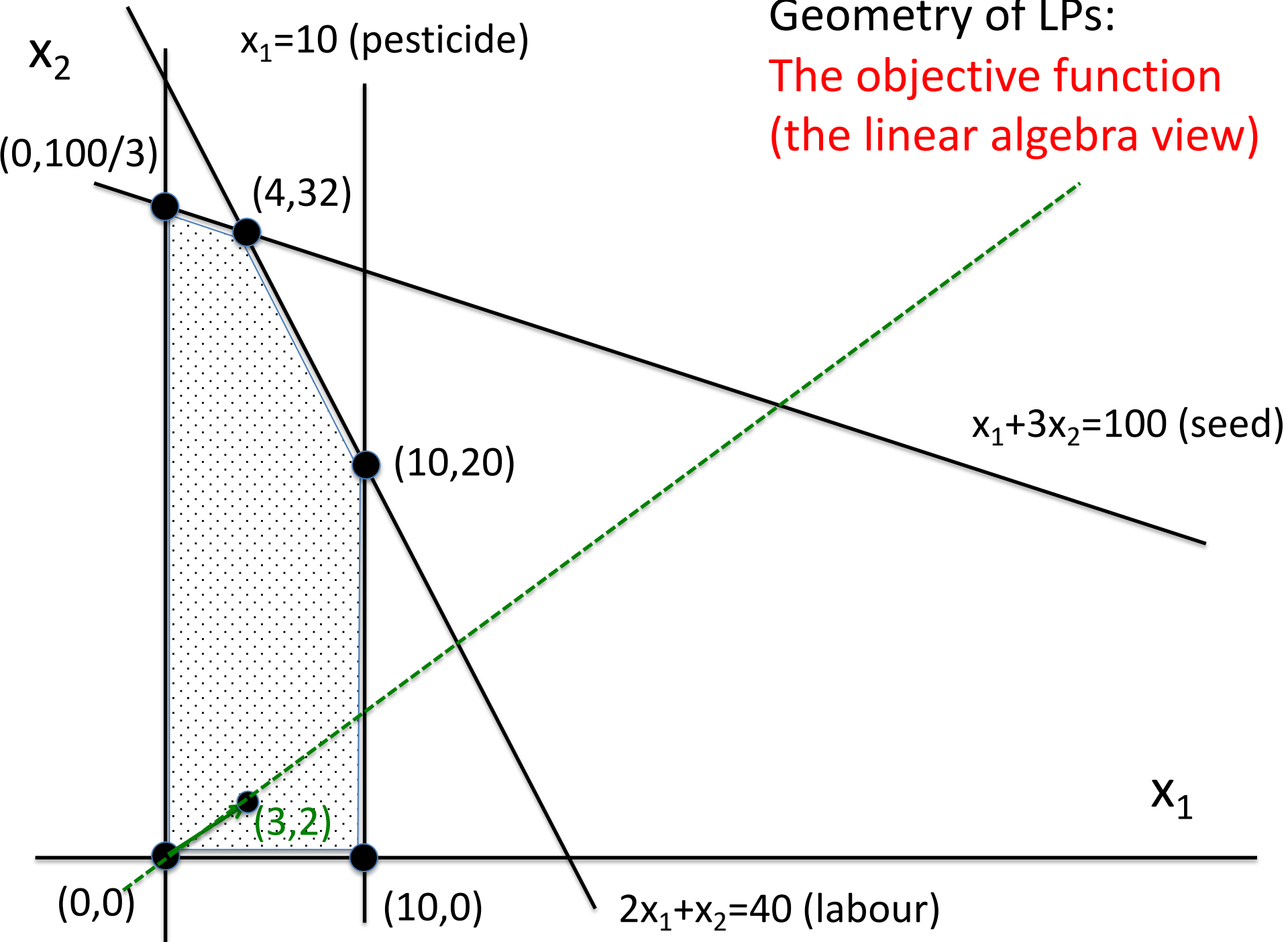
Fact: If optimal solutions exist, then there is an optimal solution that is a basic feasible solution (a vertex of the feasible region)



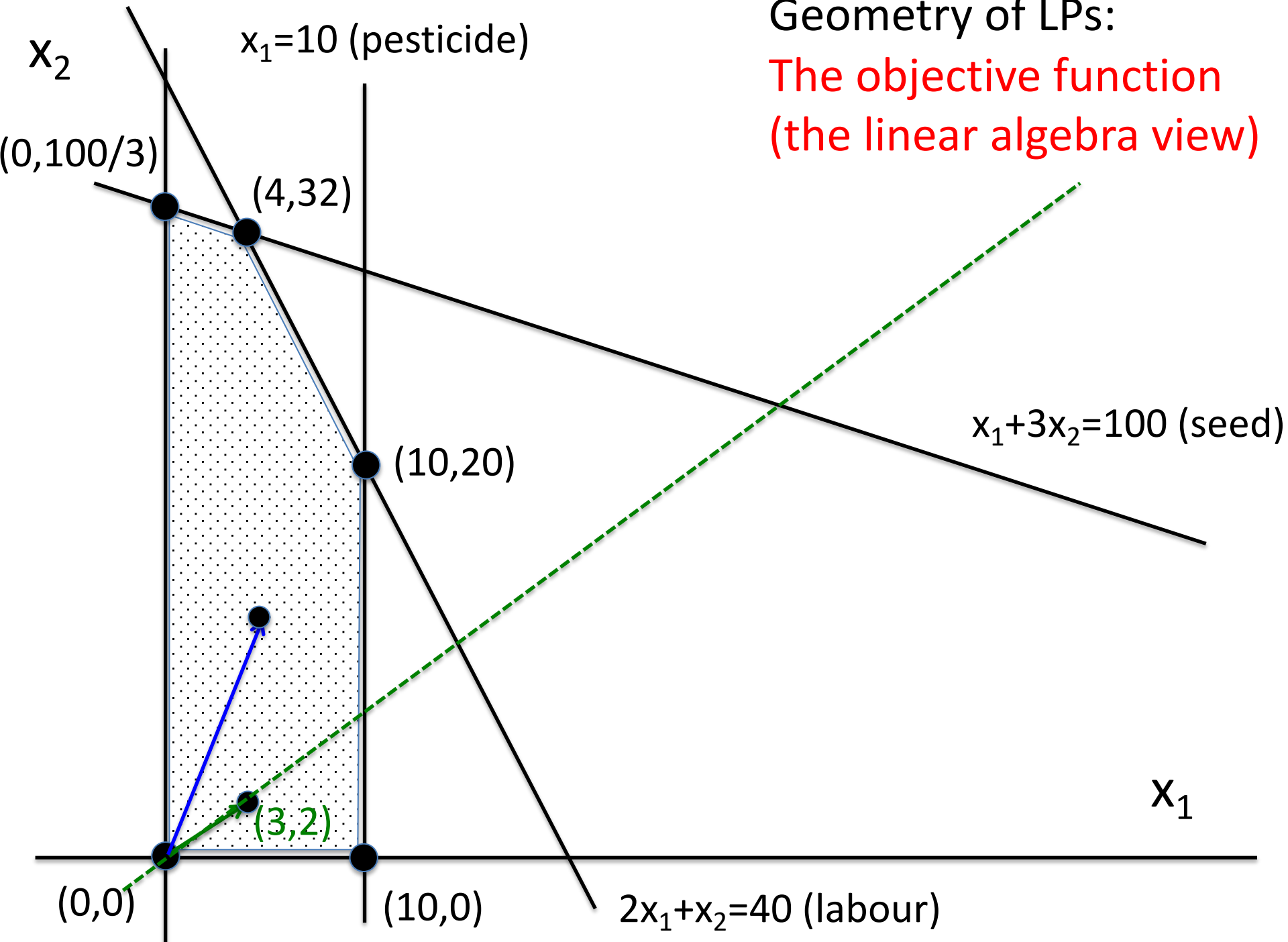
Geometry of LPs:  
The objective function  
(the linear algebra view)



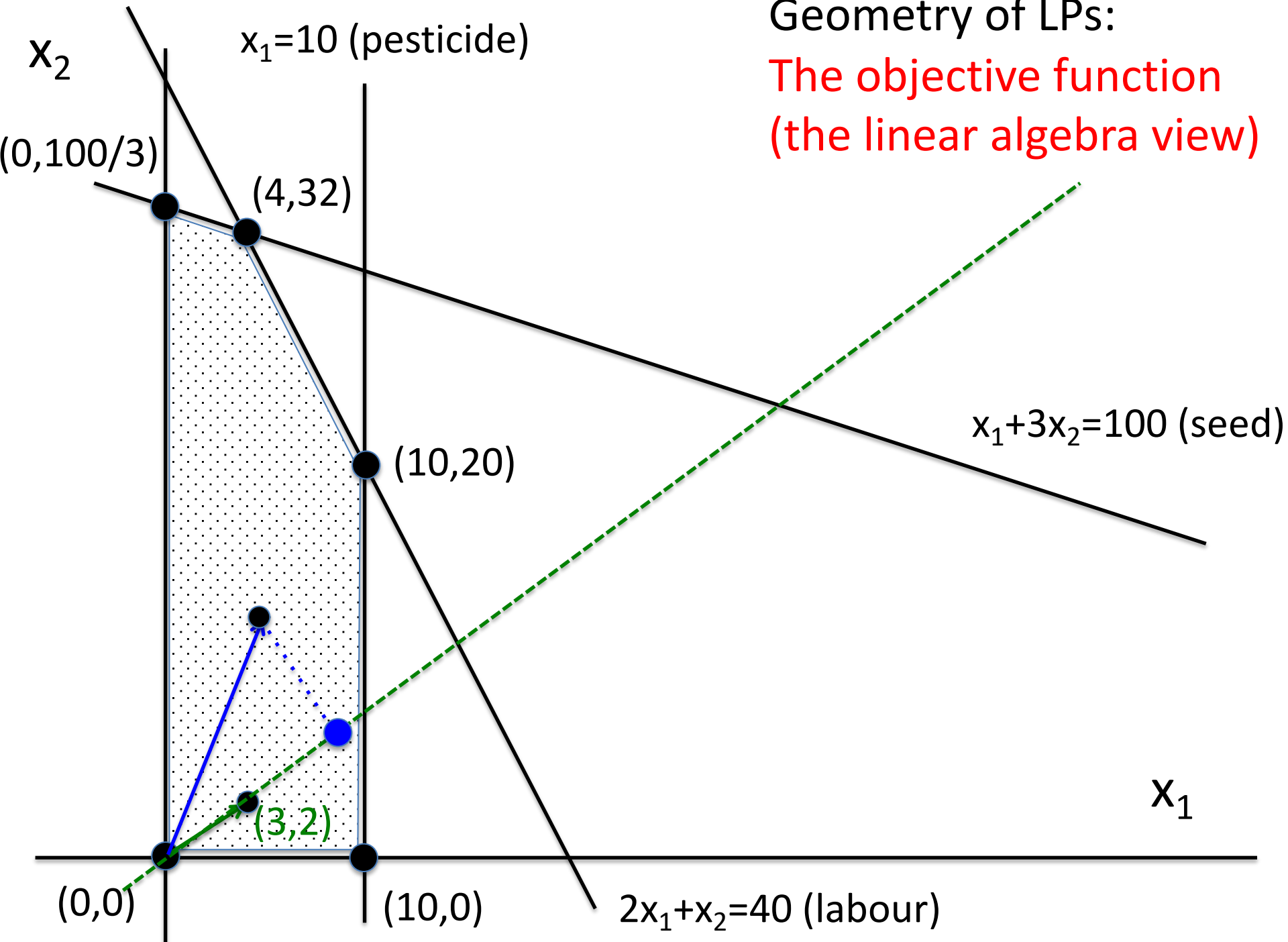
Geometry of LPs:  
The objective function  
(the linear algebra view)



Geometry of LPs:  
The objective function  
(the linear algebra view)

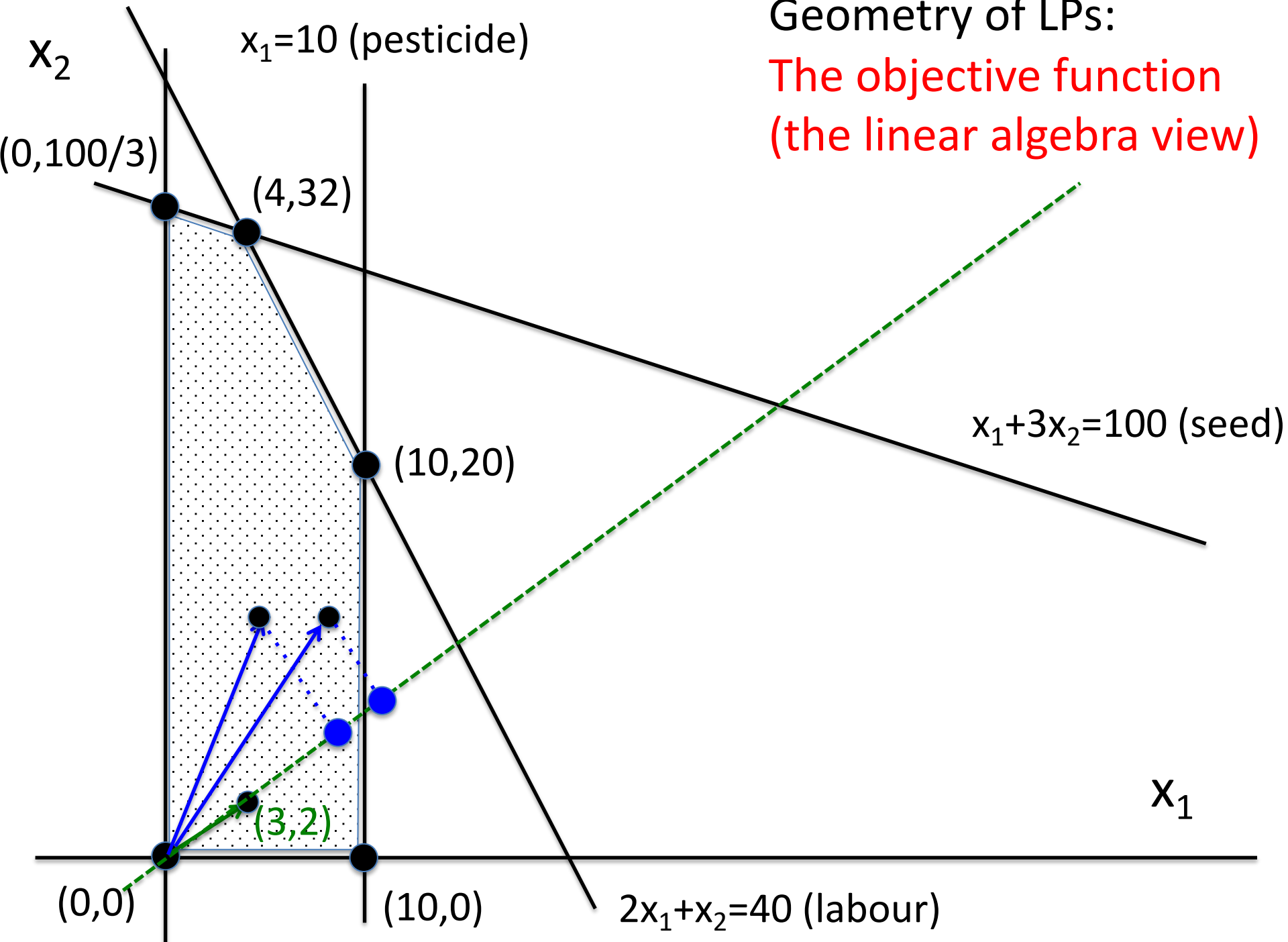


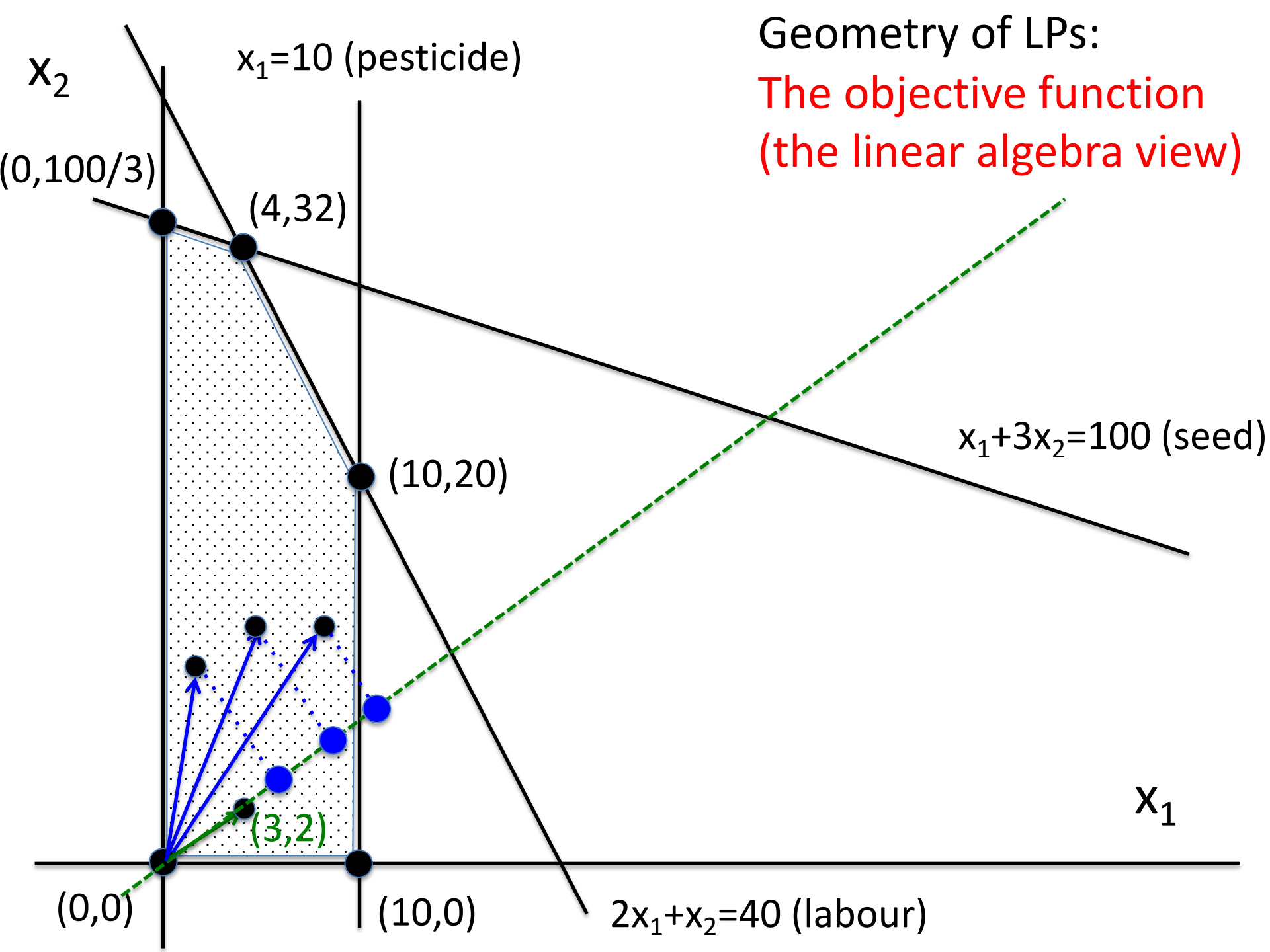
Geometry of LPs:  
The objective function  
(the linear algebra view)

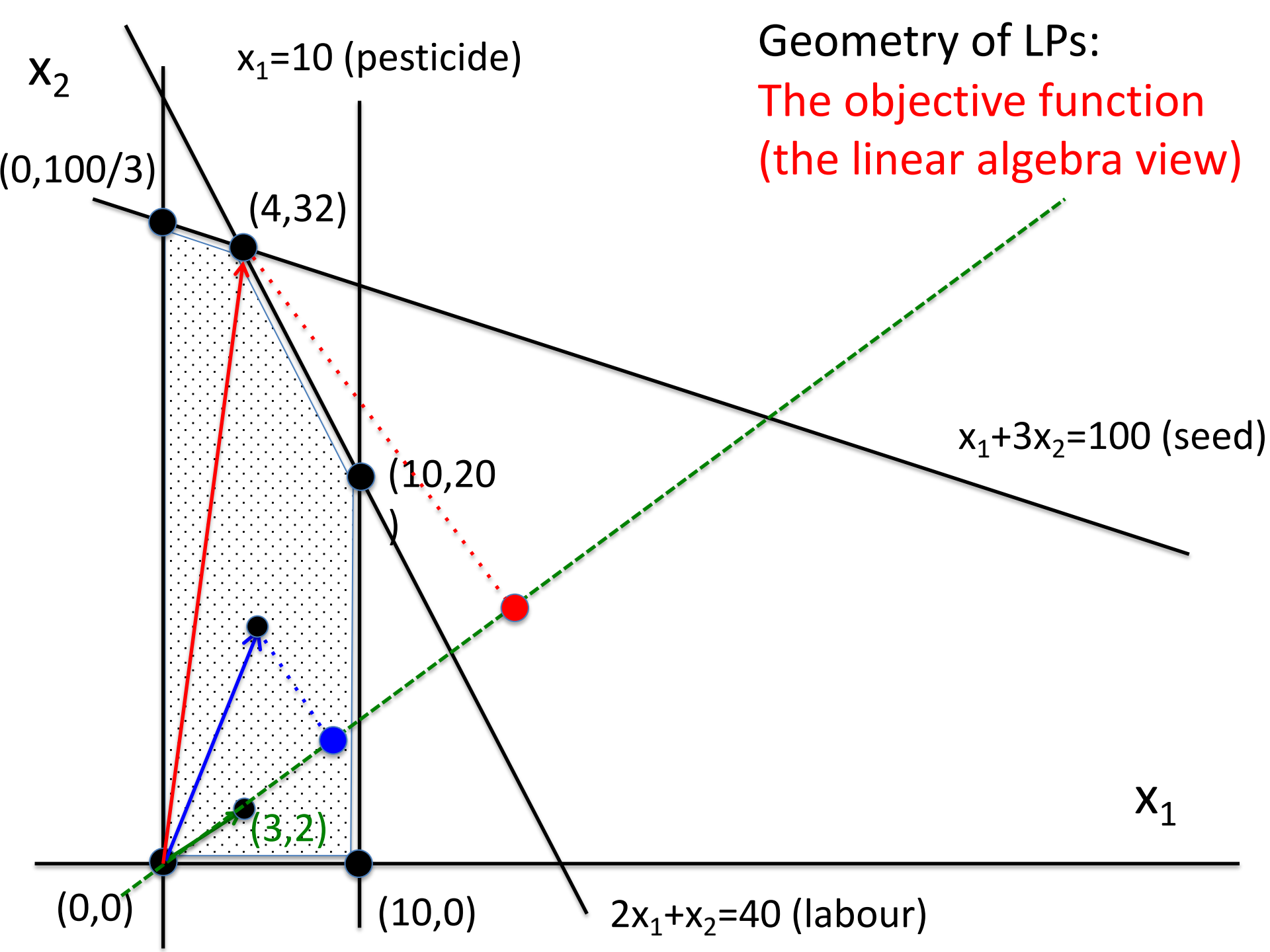




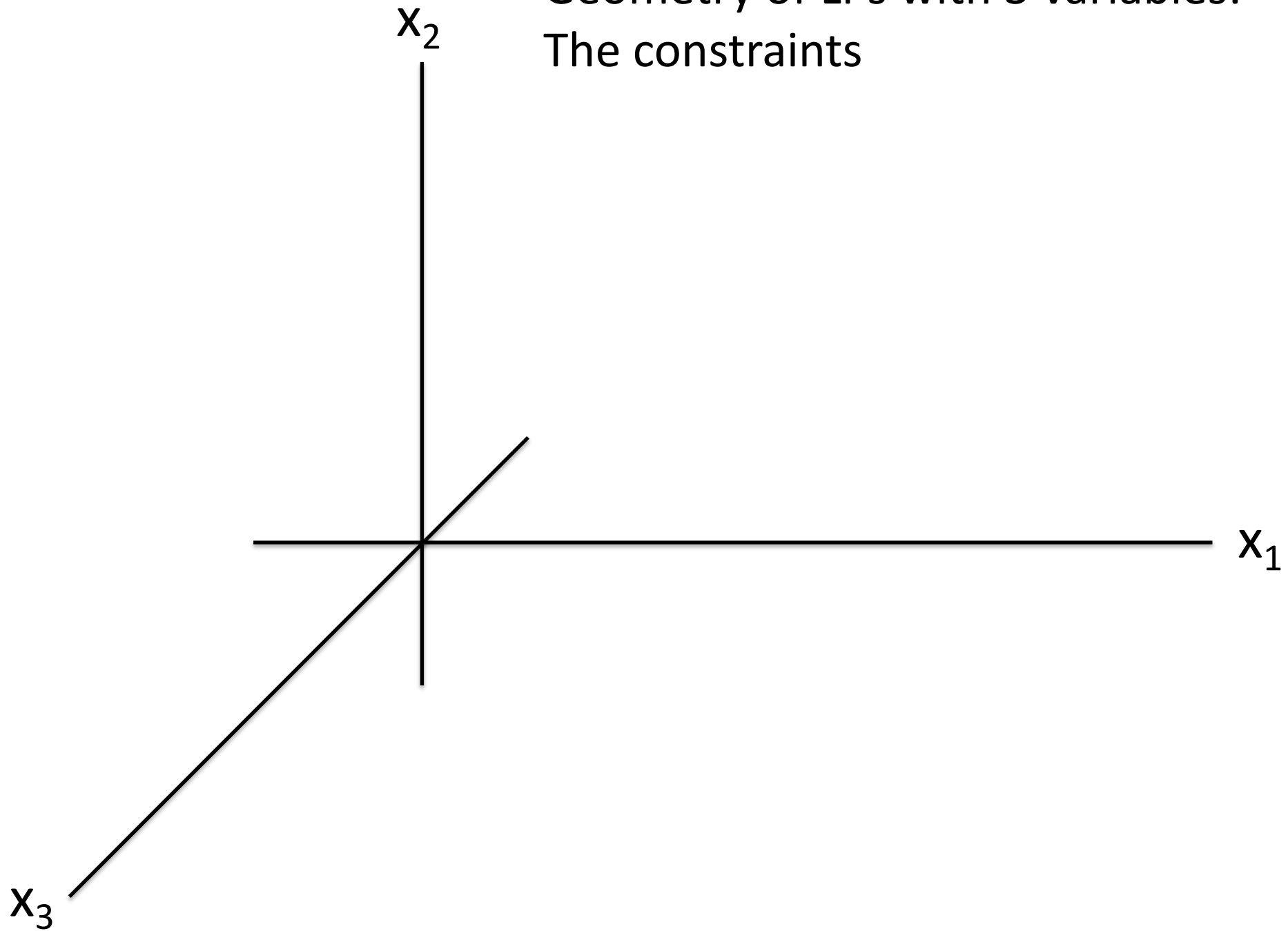
Geometry of LPs:  
The objective function  
(the linear algebra view)



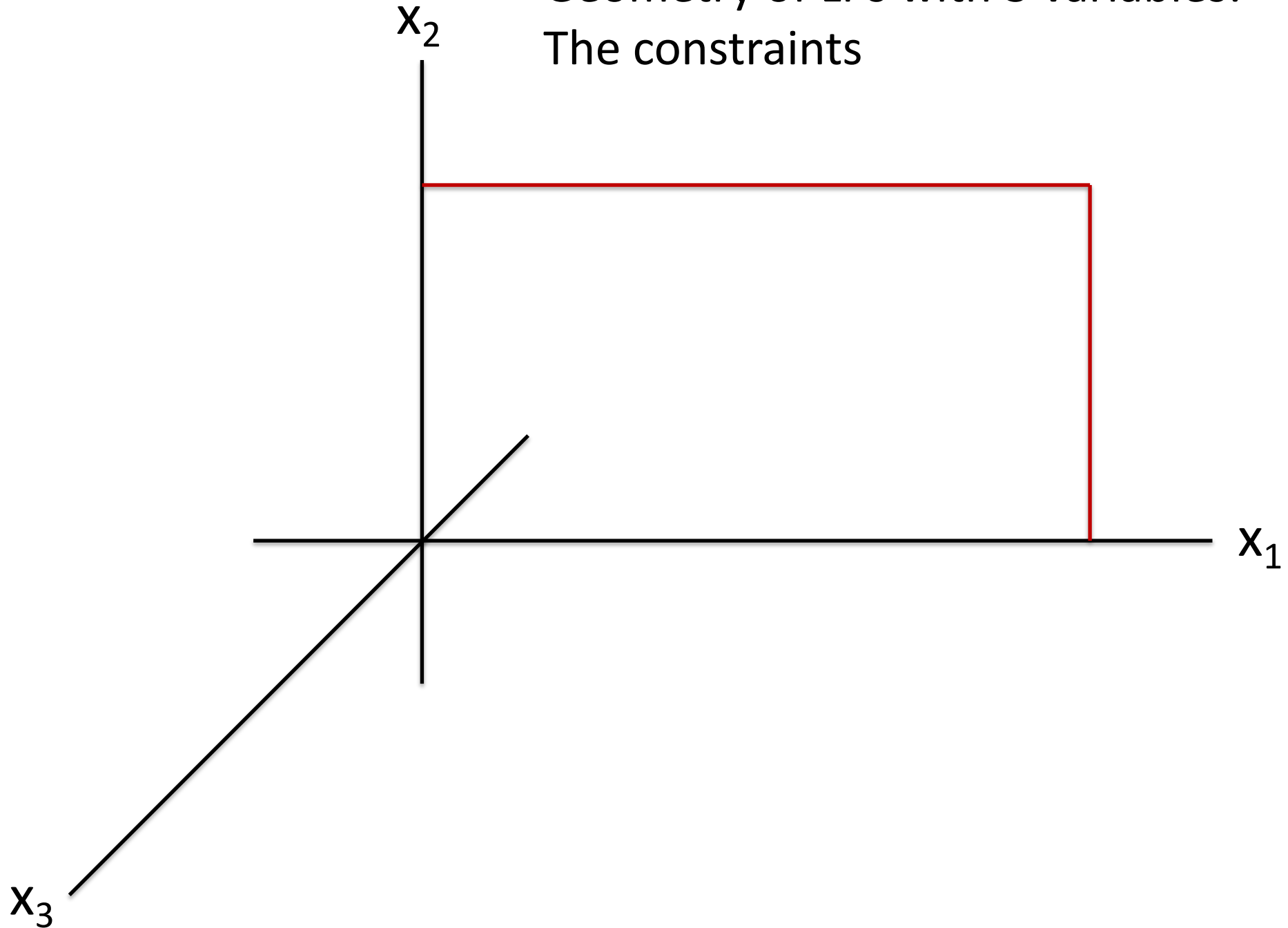




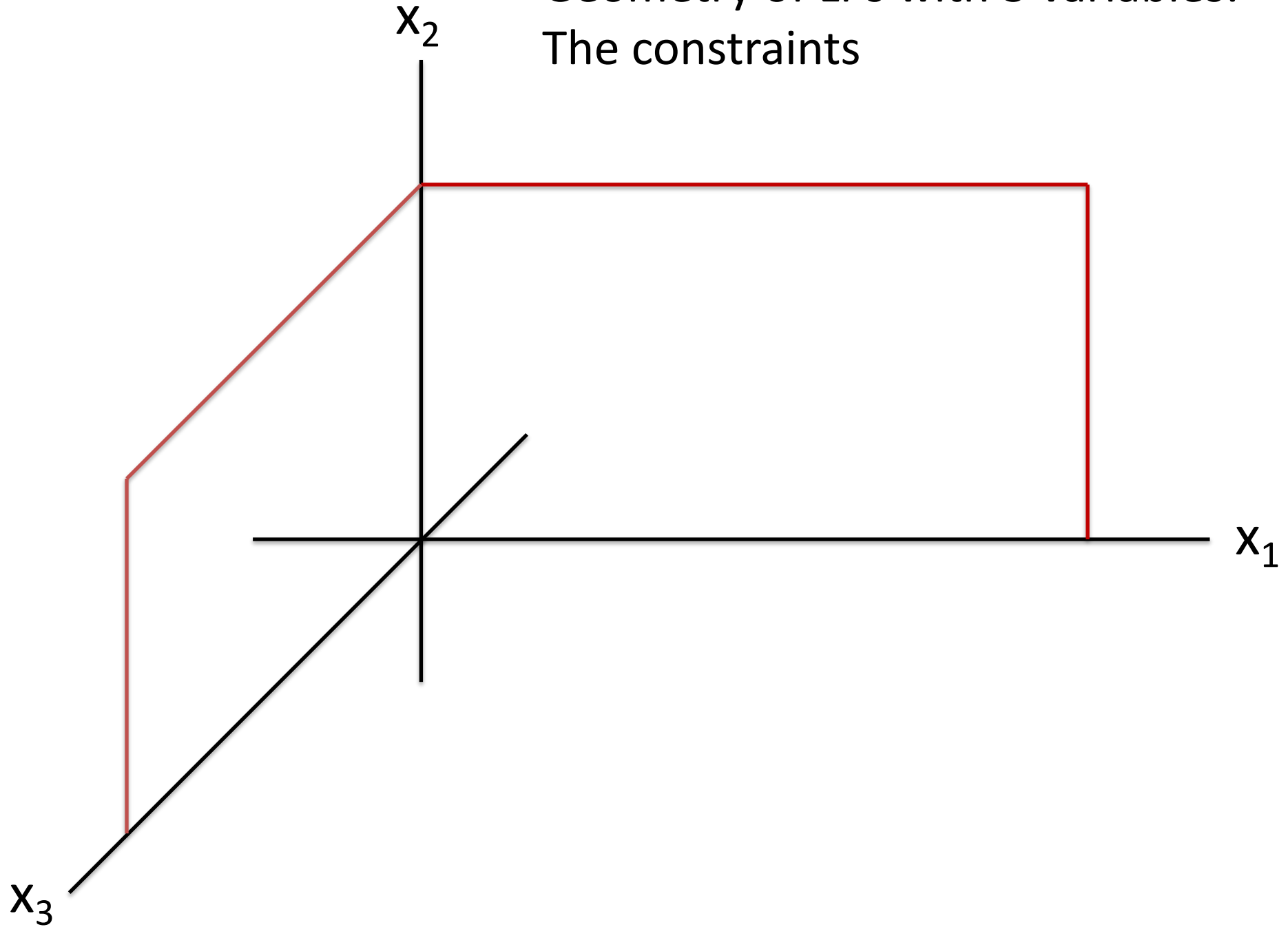
# Geometry of LPs with 3 variables: The constraints



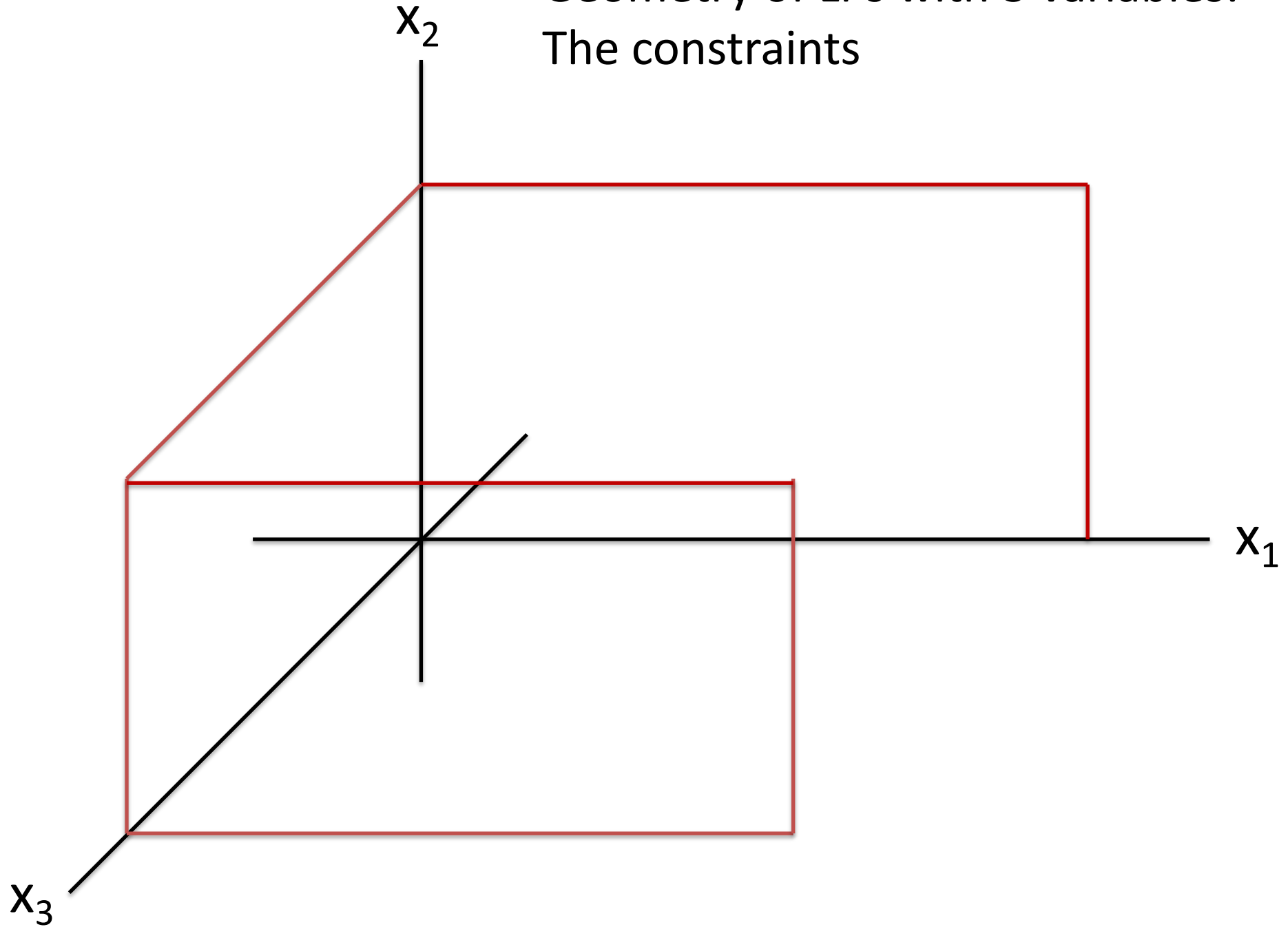
# Geometry of LPs with 3 variables: The constraints



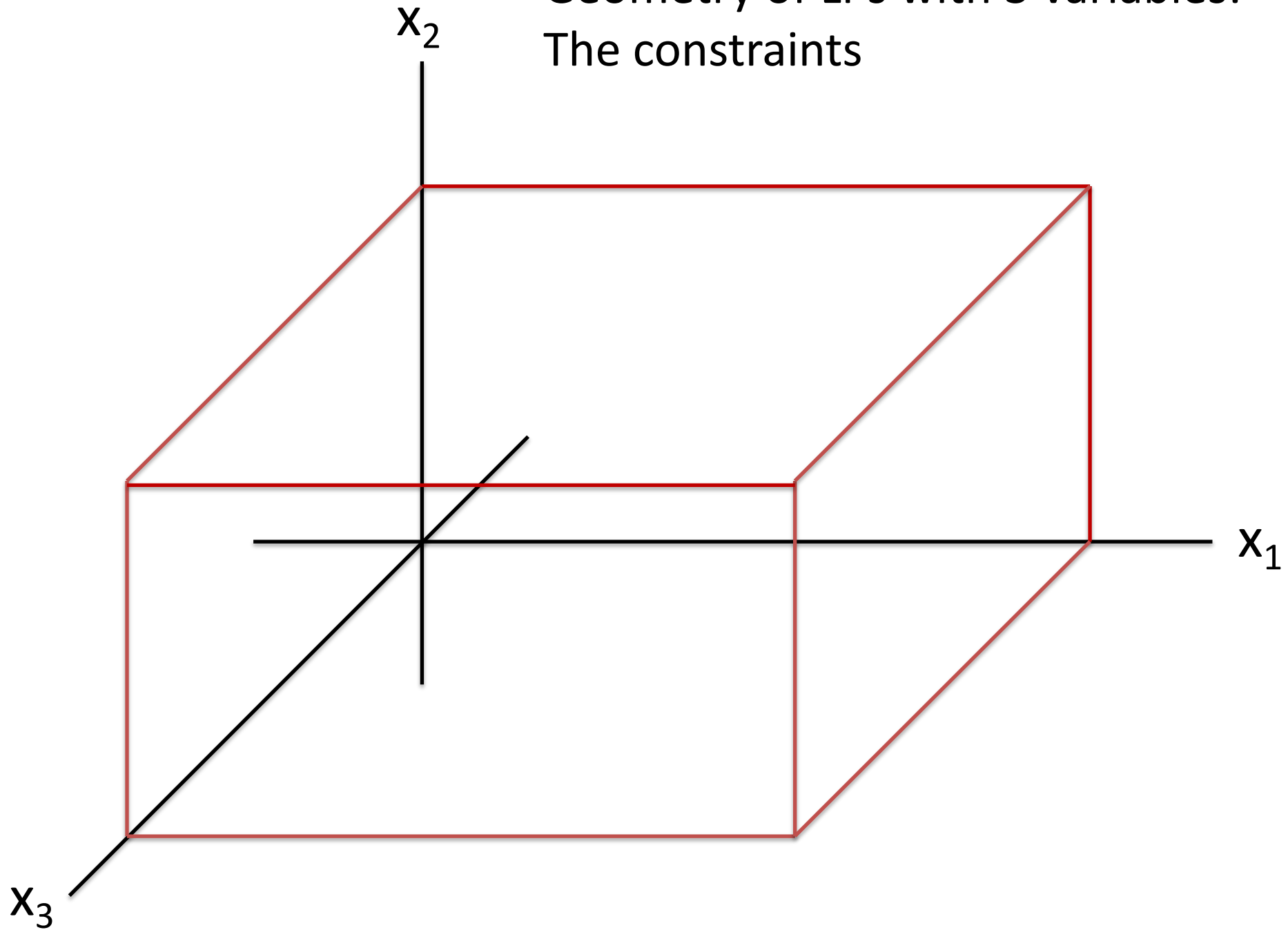
# Geometry of LPs with 3 variables: The constraints



# Geometry of LPs with 3 variables: The constraints

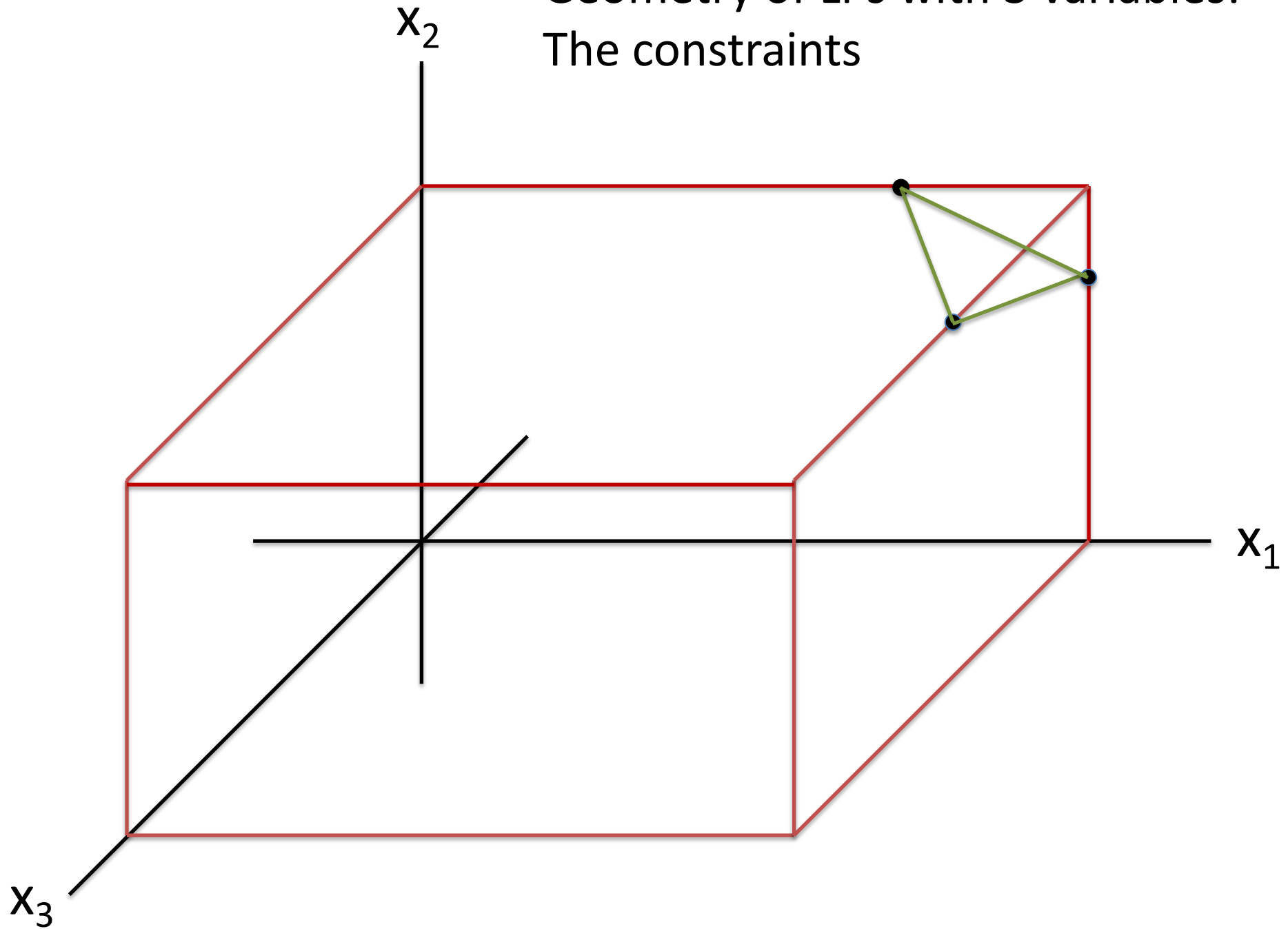


# Geometry of LPs with 3 variables: The constraints

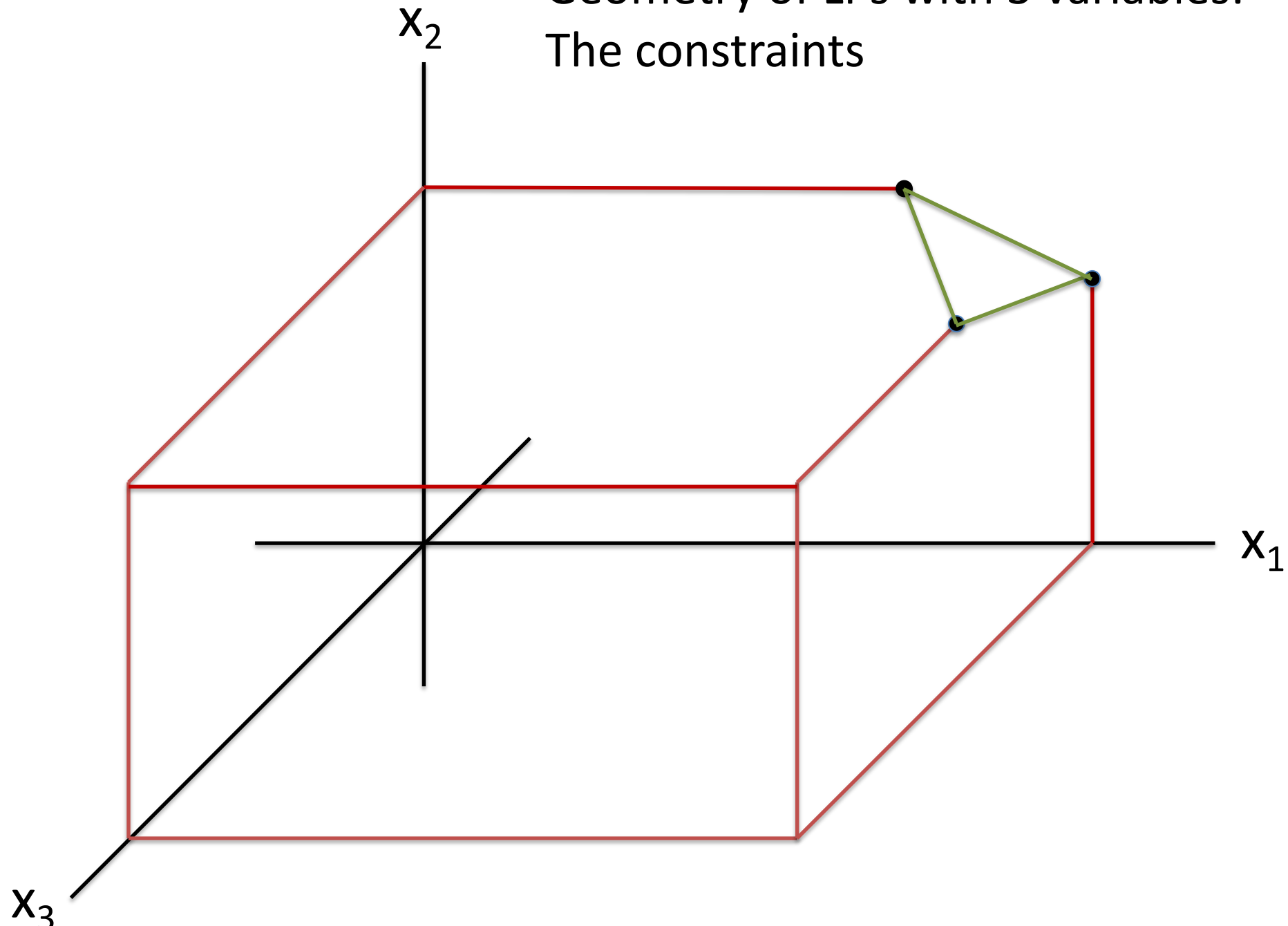




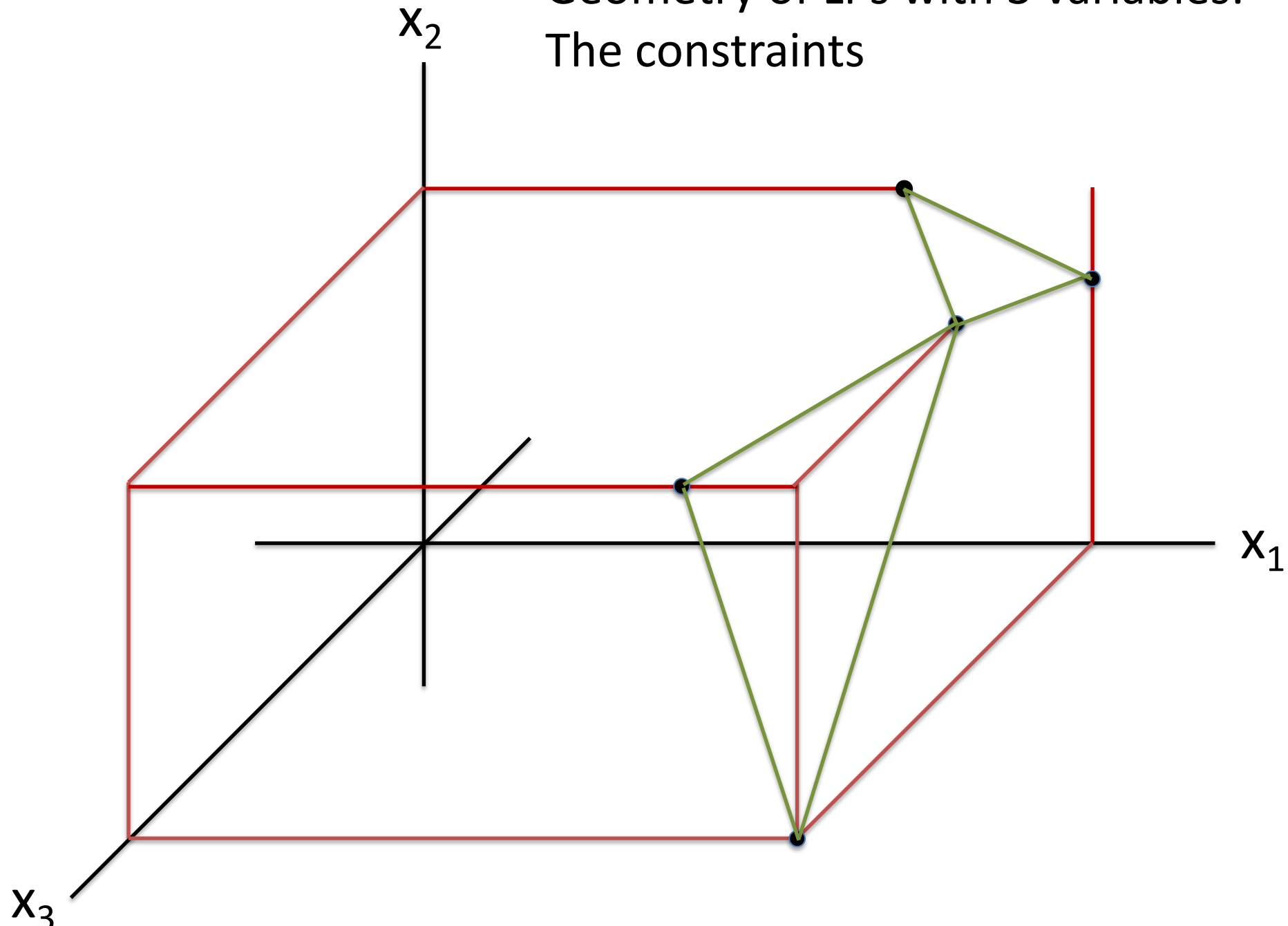
# Geometry of LPs with 3 variables: The constraints



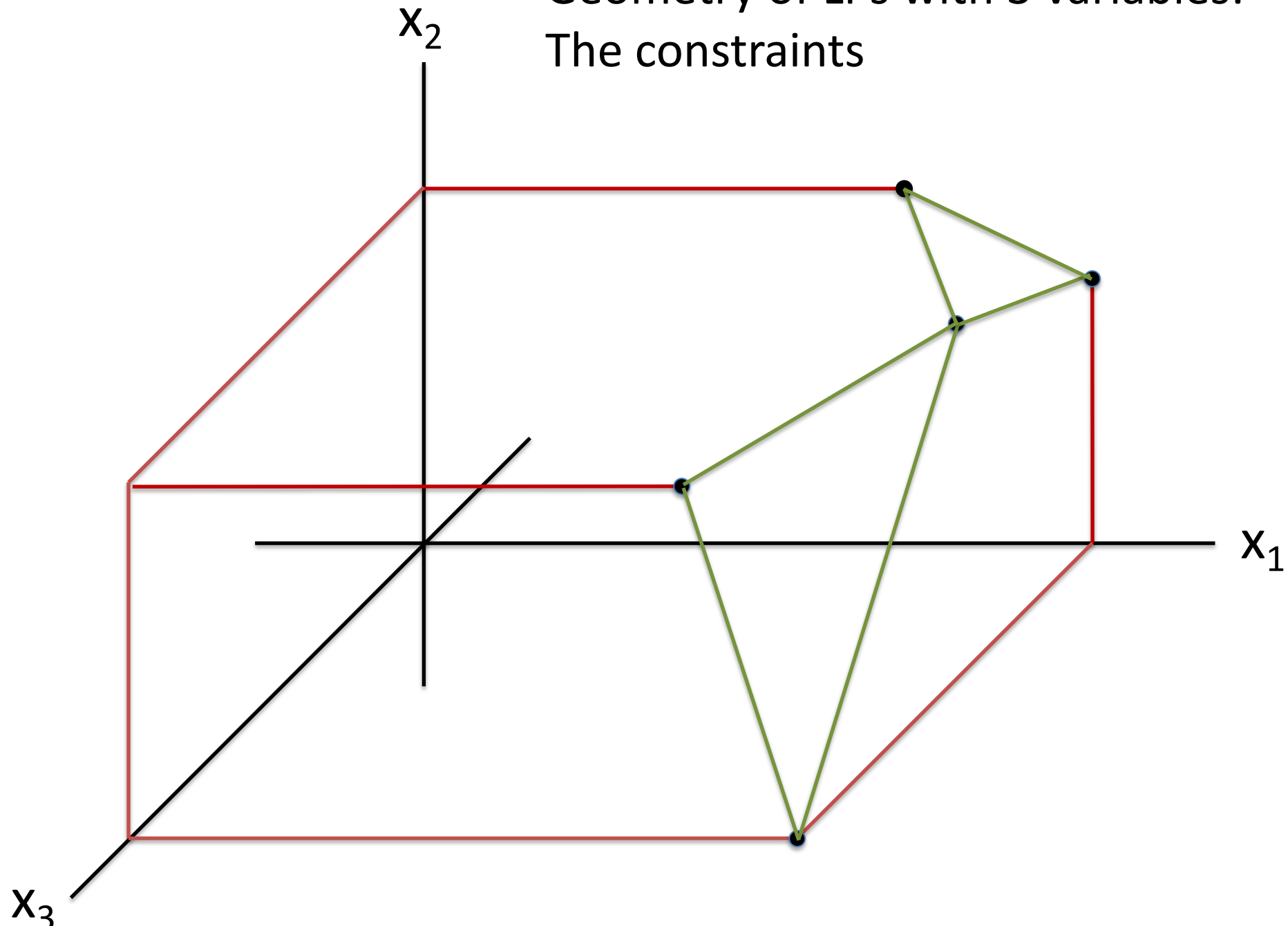
# Geometry of LPs with 3 variables: The constraints



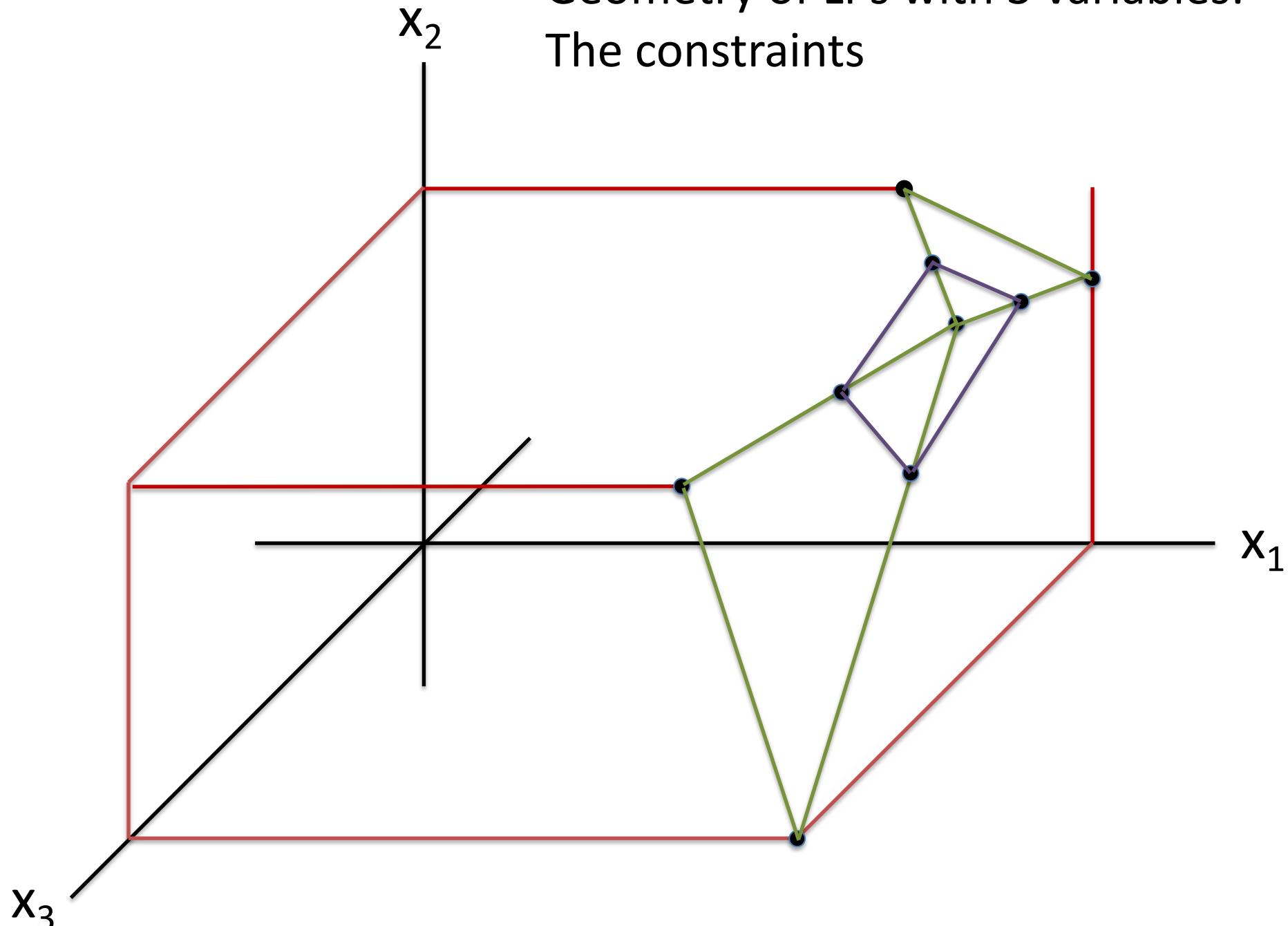
# Geometry of LPs with 3 variables: The constraints



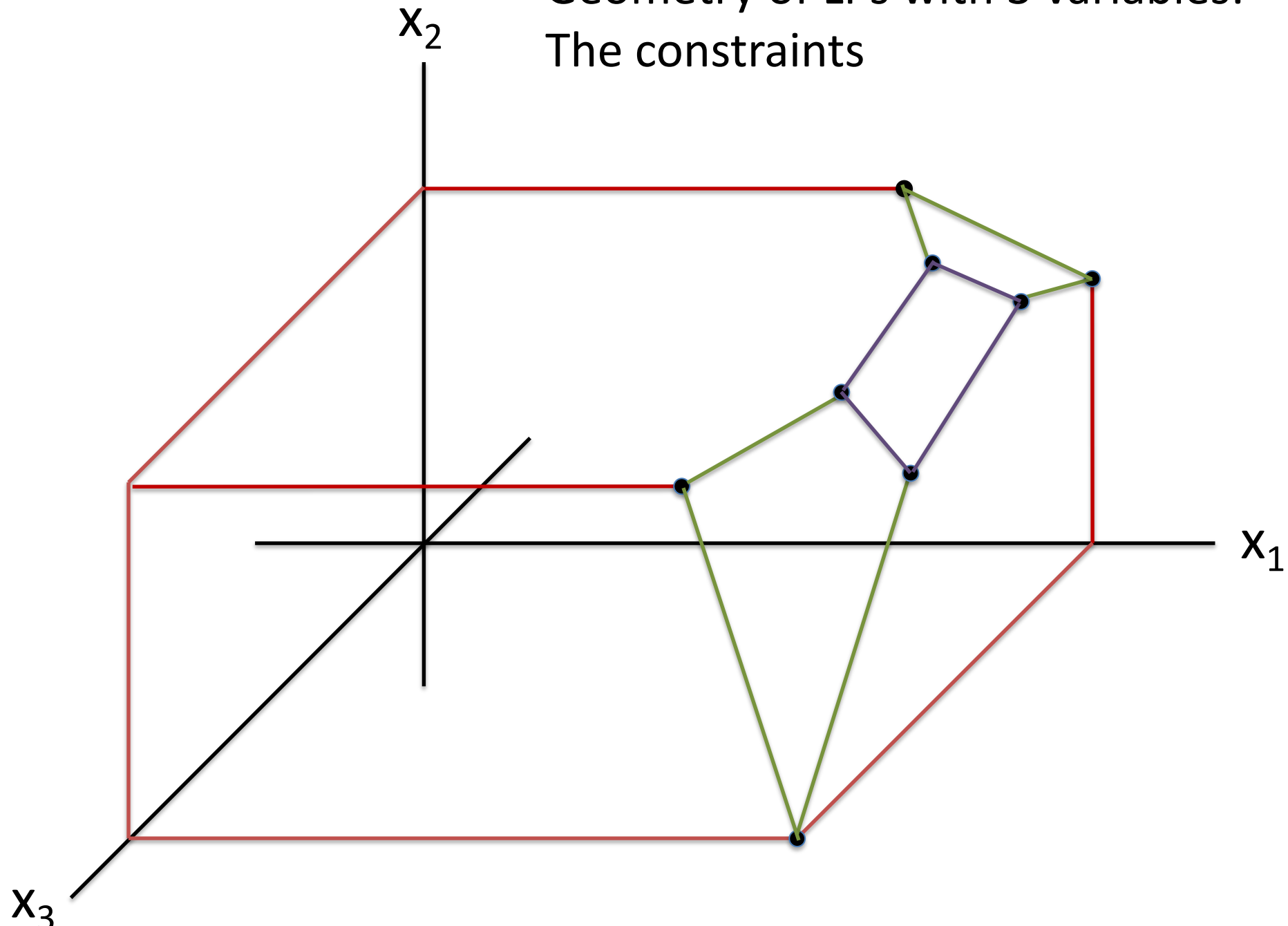
# Geometry of LPs with 3 variables: The constraints



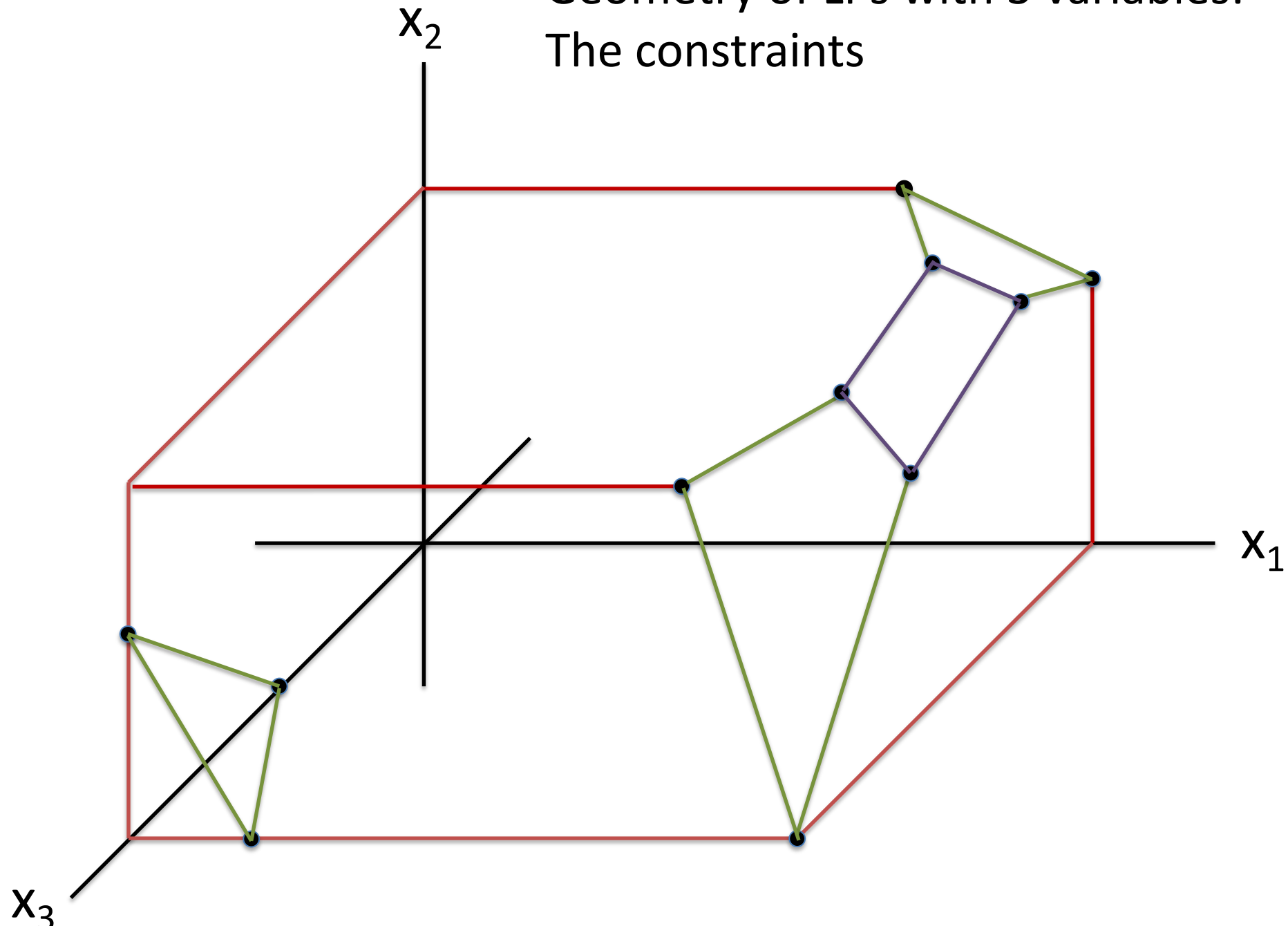
# Geometry of LPs with 3 variables: The constraints



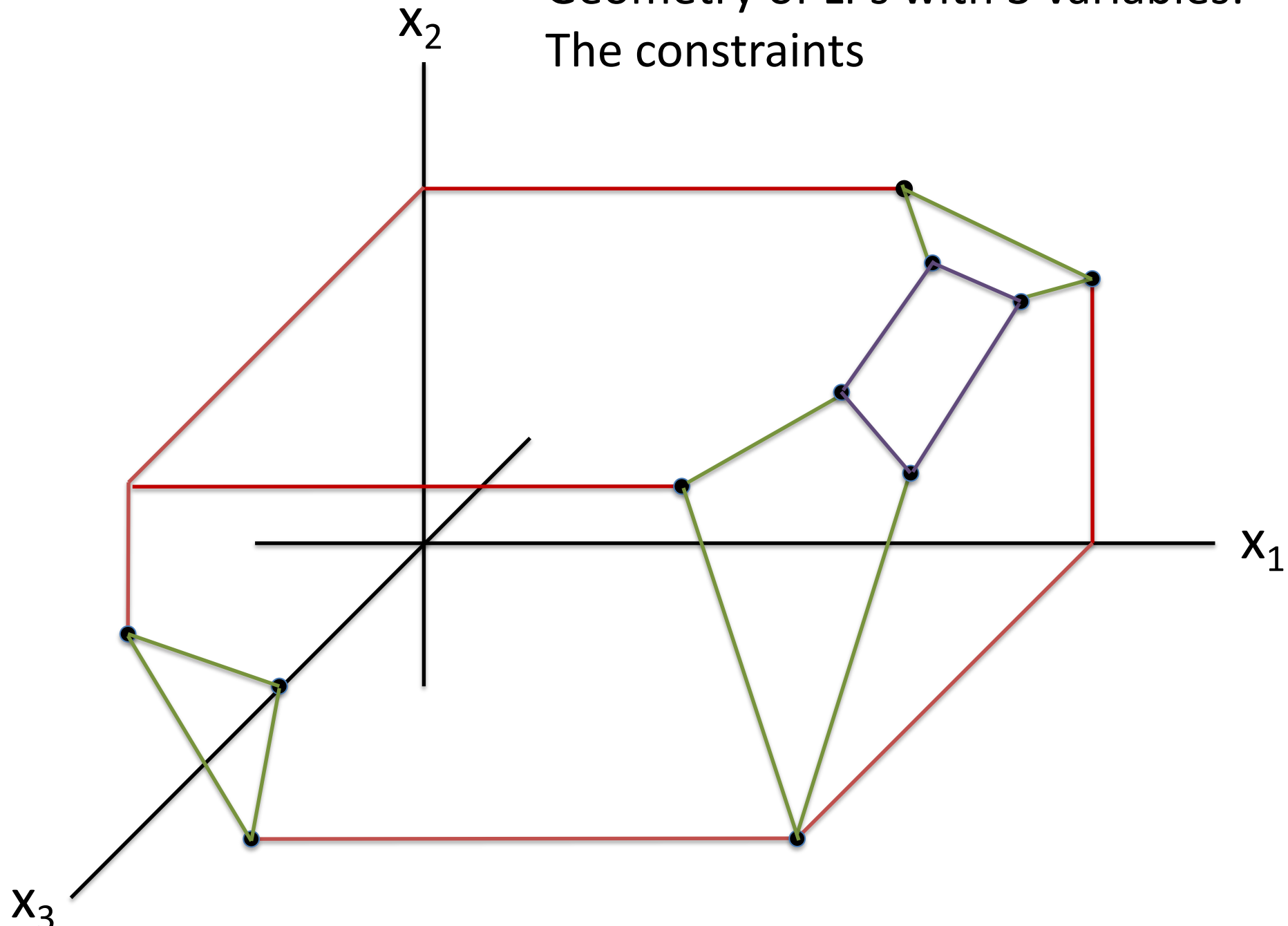
# Geometry of LPs with 3 variables: The constraints



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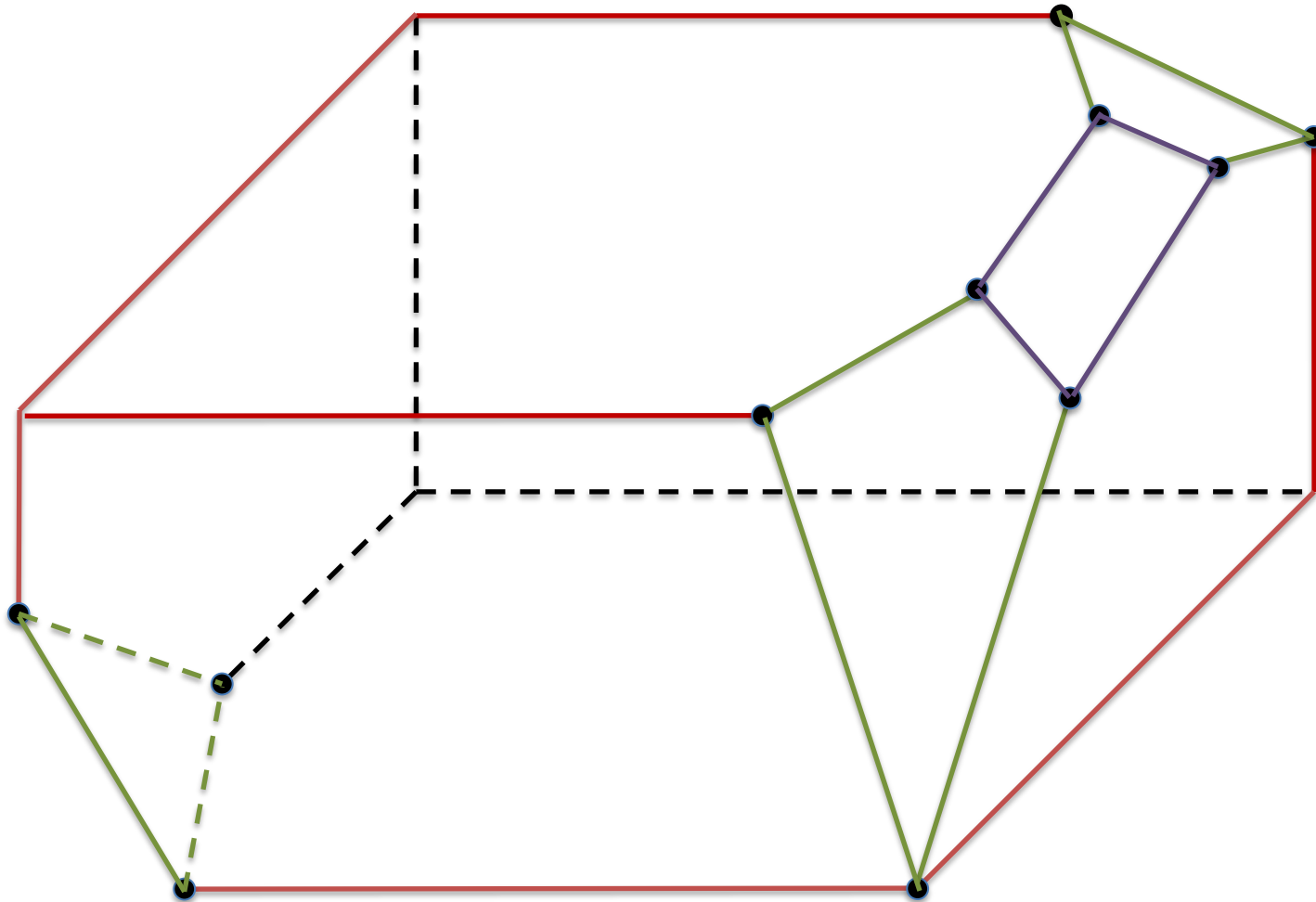


# Geometry of LPs with 3 variables: The constraints



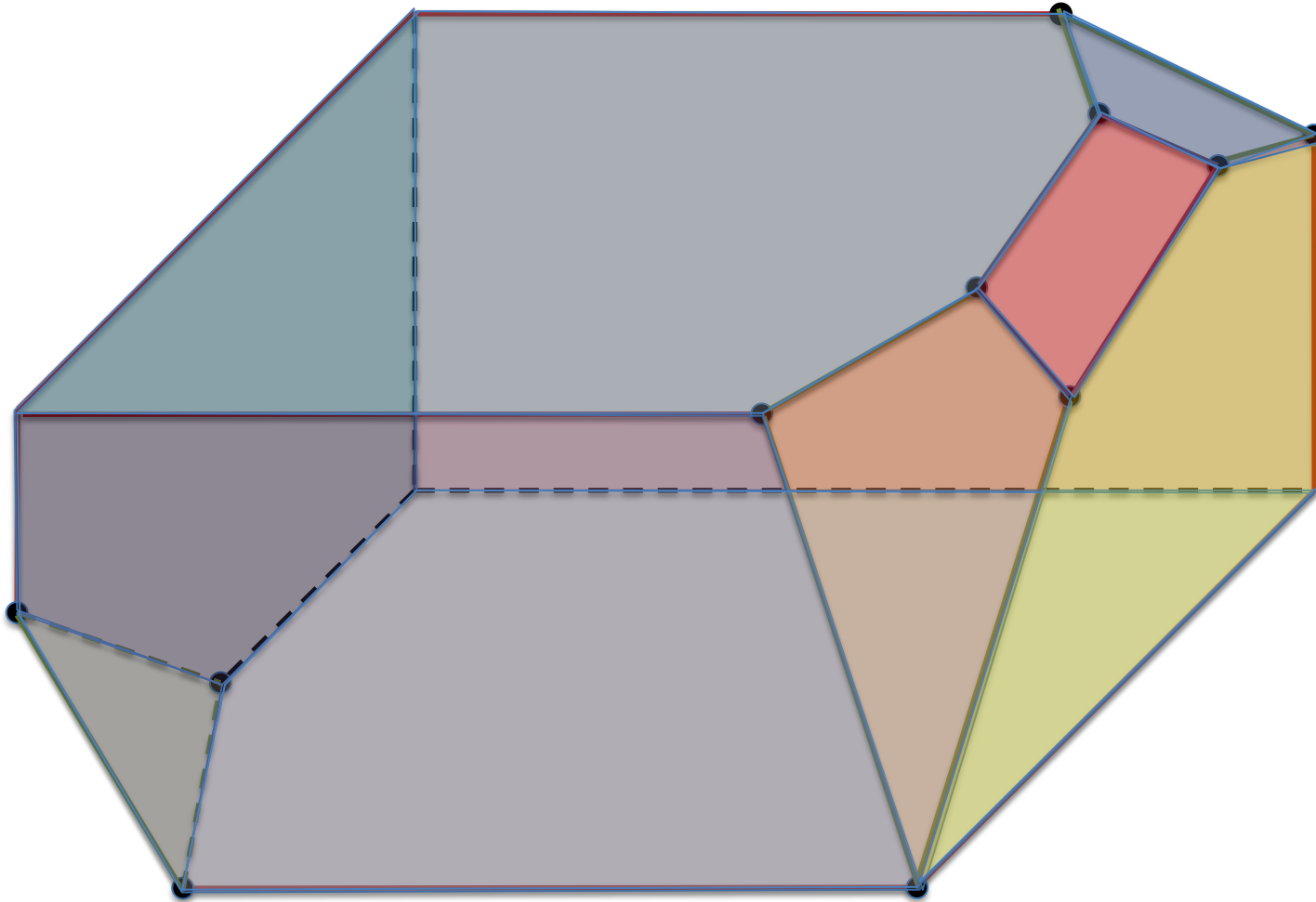


# Geometry of LPs with 3 variables: The constraints



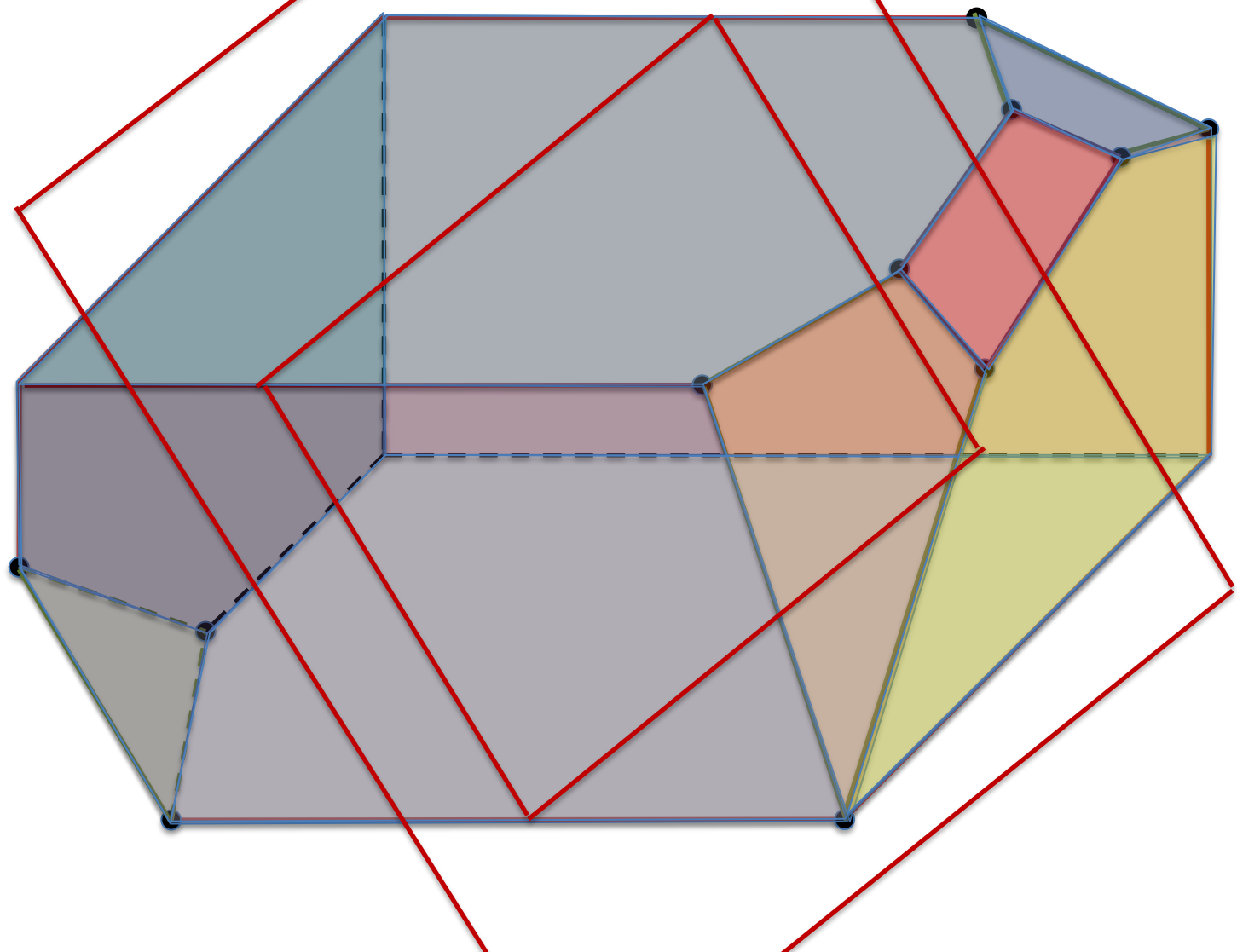
# Geometry of LPs with 3 variables: The constraints

Feasible region:  
a convex polyhedron



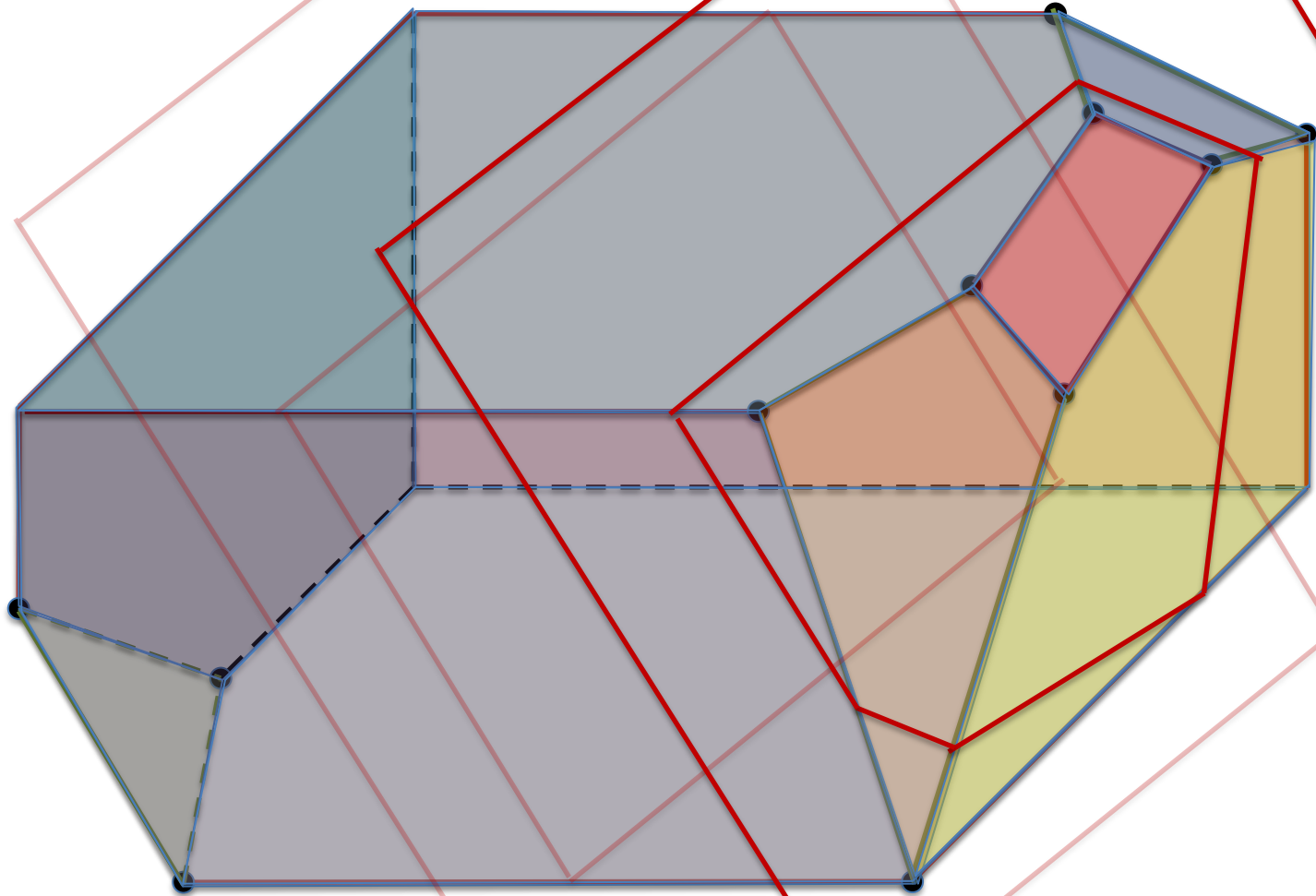
Geometry of LPs with 3 variables:

The objective function: set of parallel planes



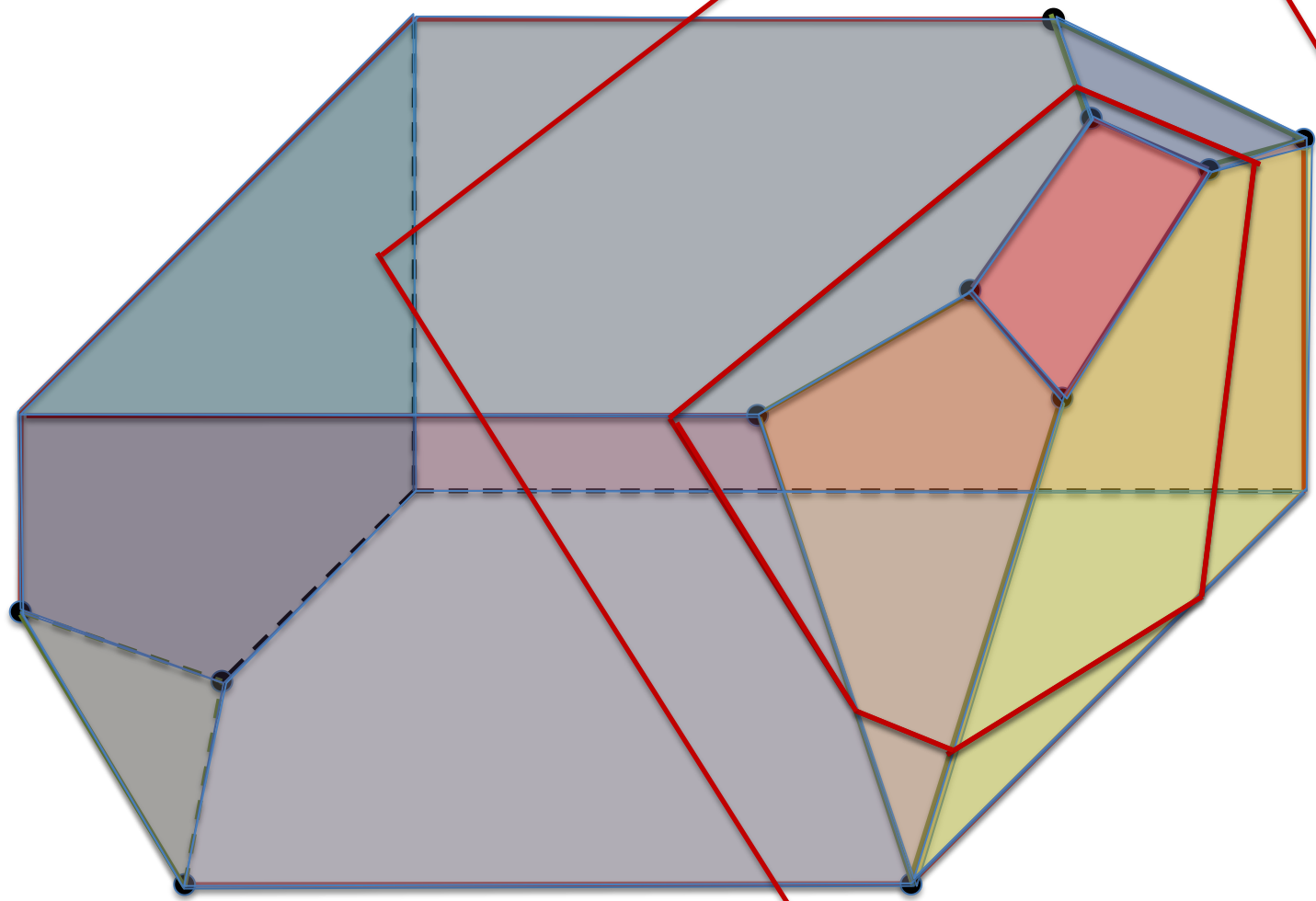
Geometry of LPs with 3 variables:

The objective function: set of parallel planes



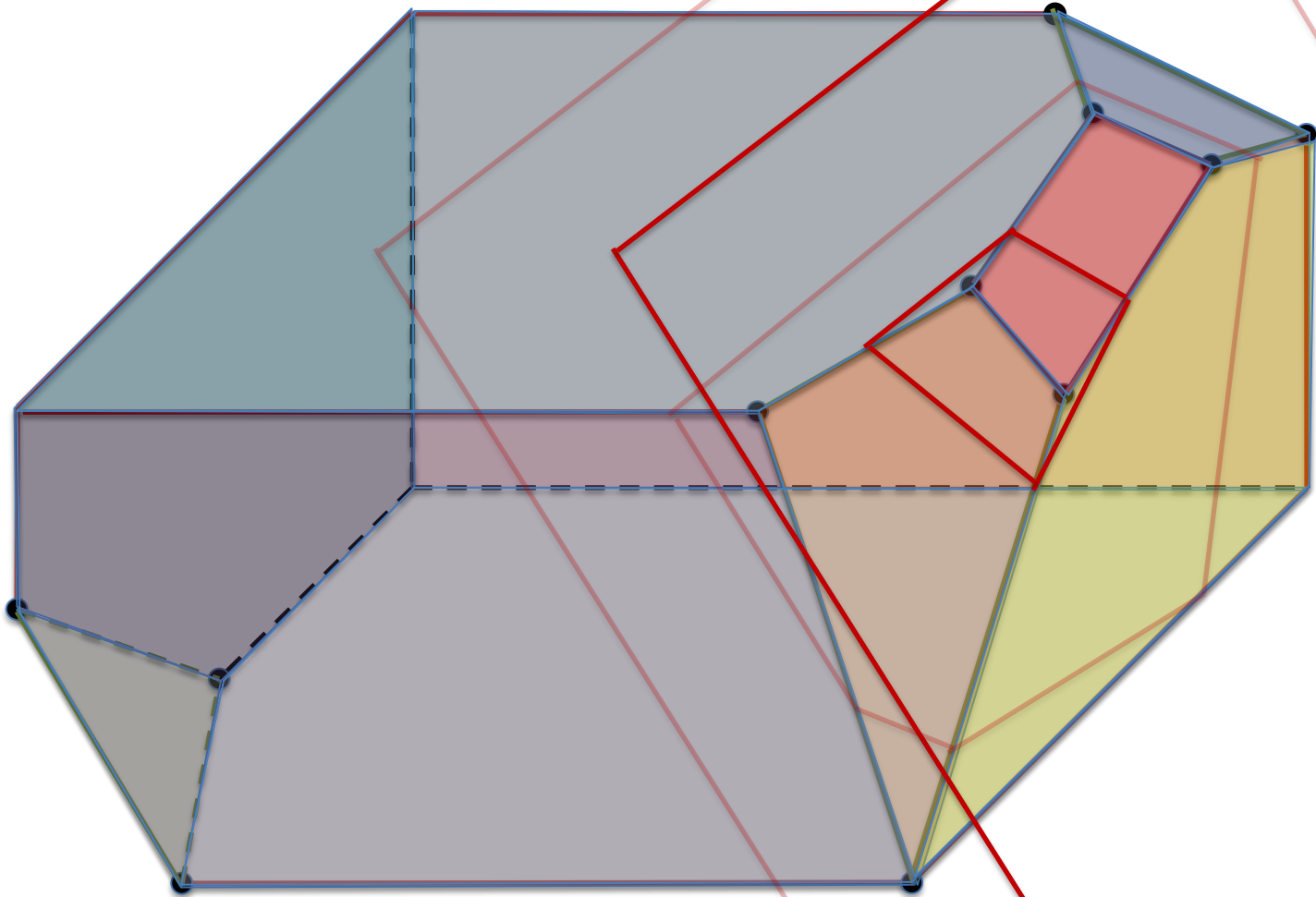
Geometry of LPs with 3 variables:

The objective function: set of parallel planes



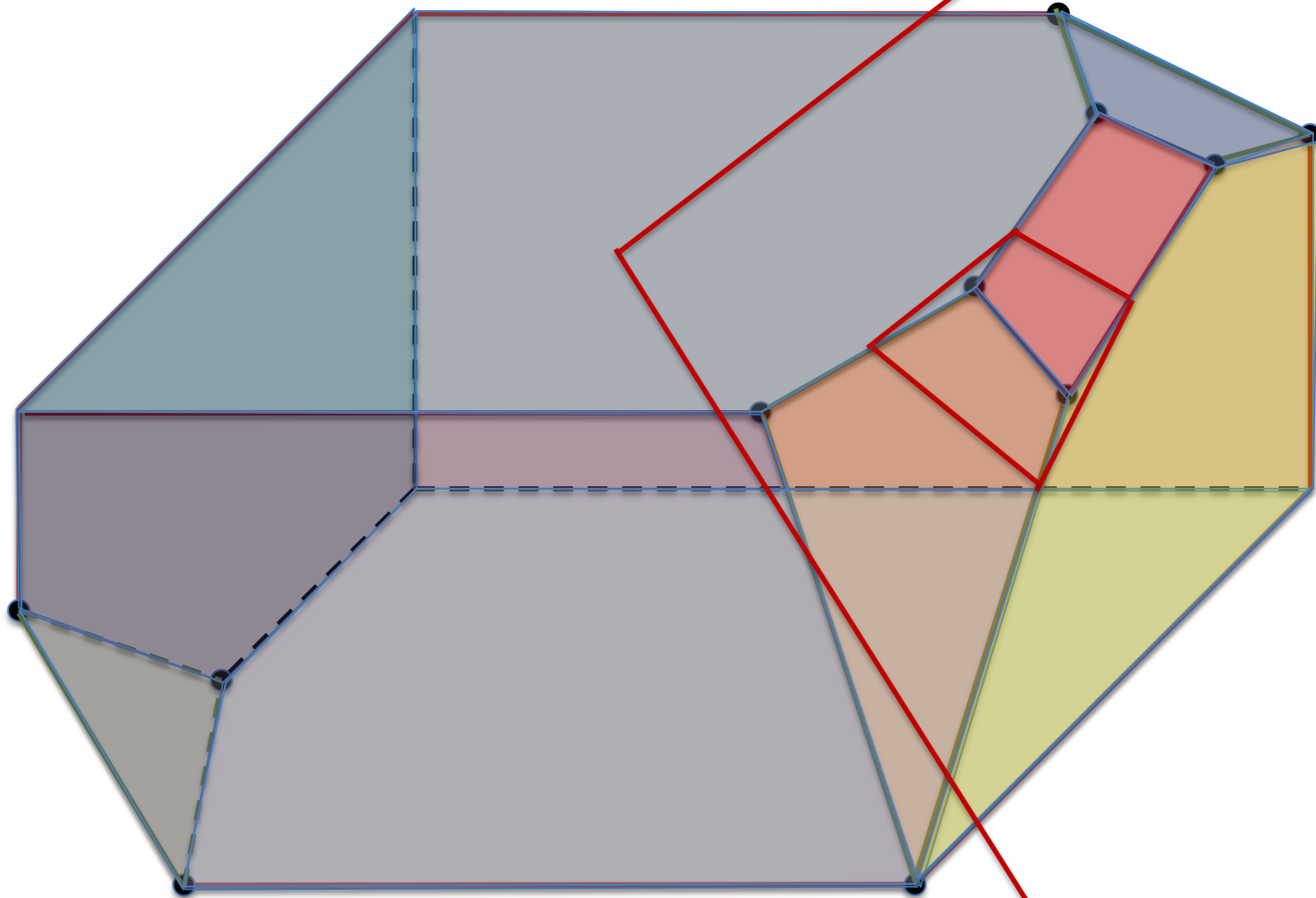
Geometry of LPs with 3 variables:

The objective function: set of parallel planes



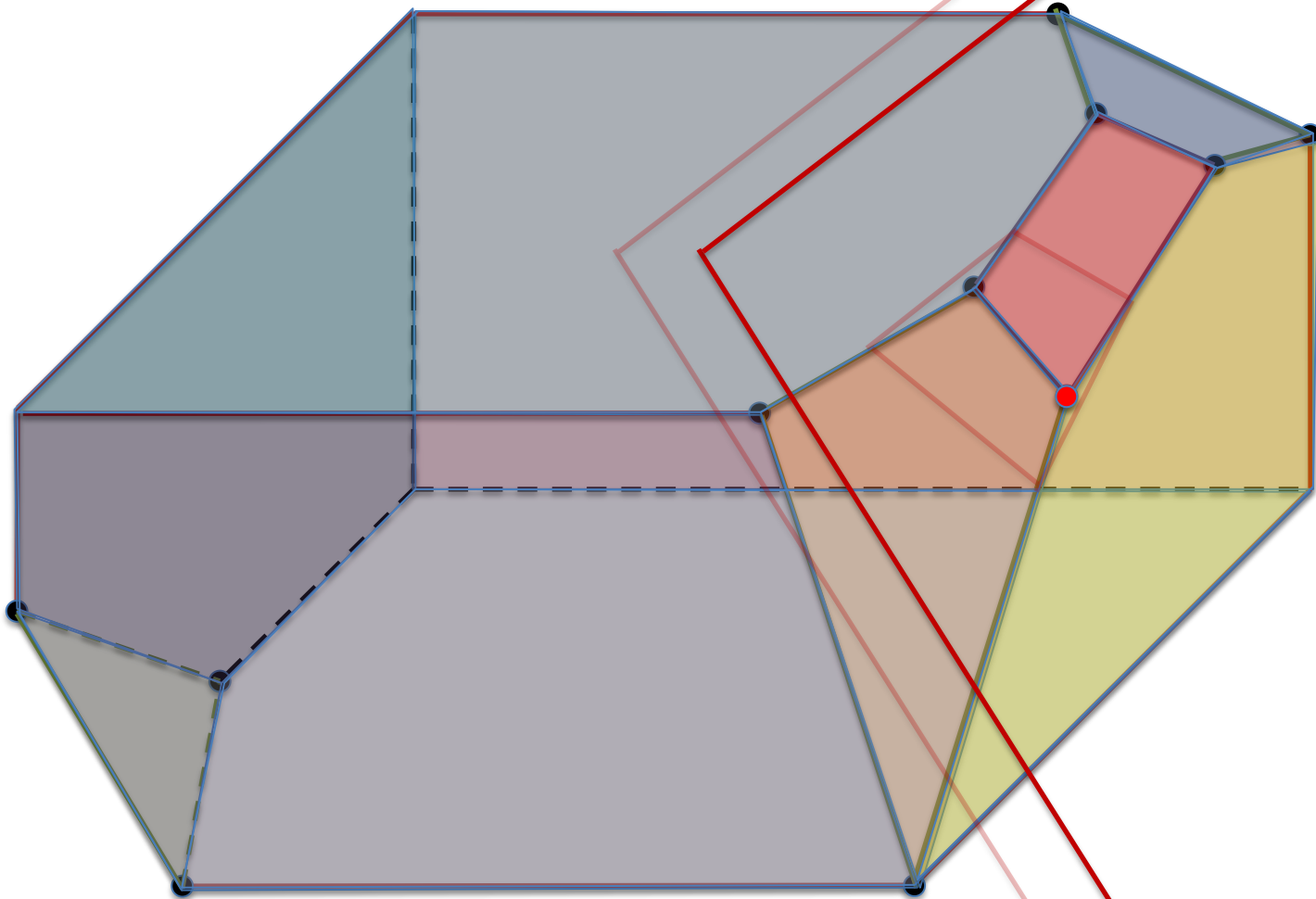
Geometry of LPs with 3 variables:

The objective function: set of parallel planes



Geometry of LPs with 3 variables:

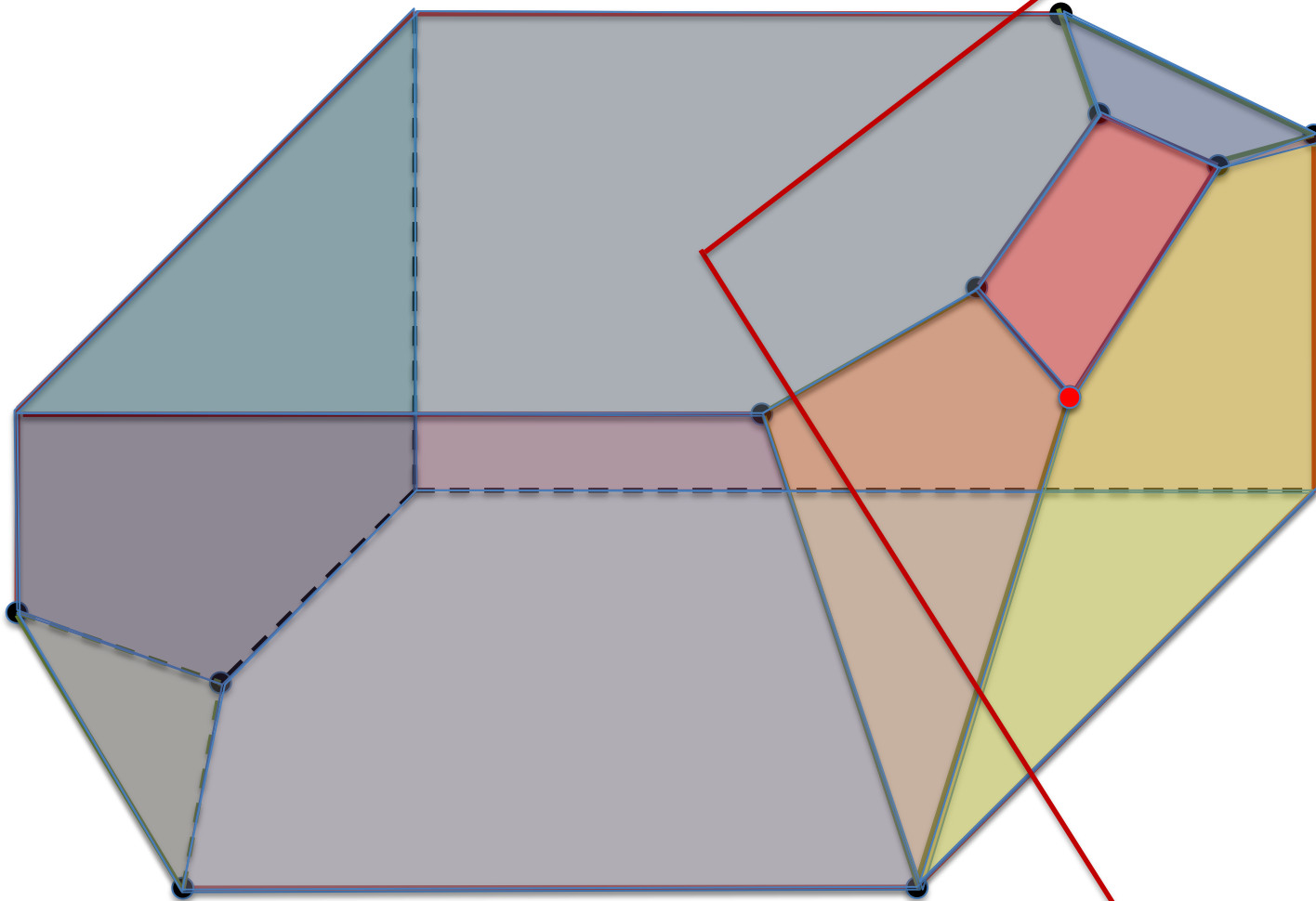
The objective function: set of parallel planes





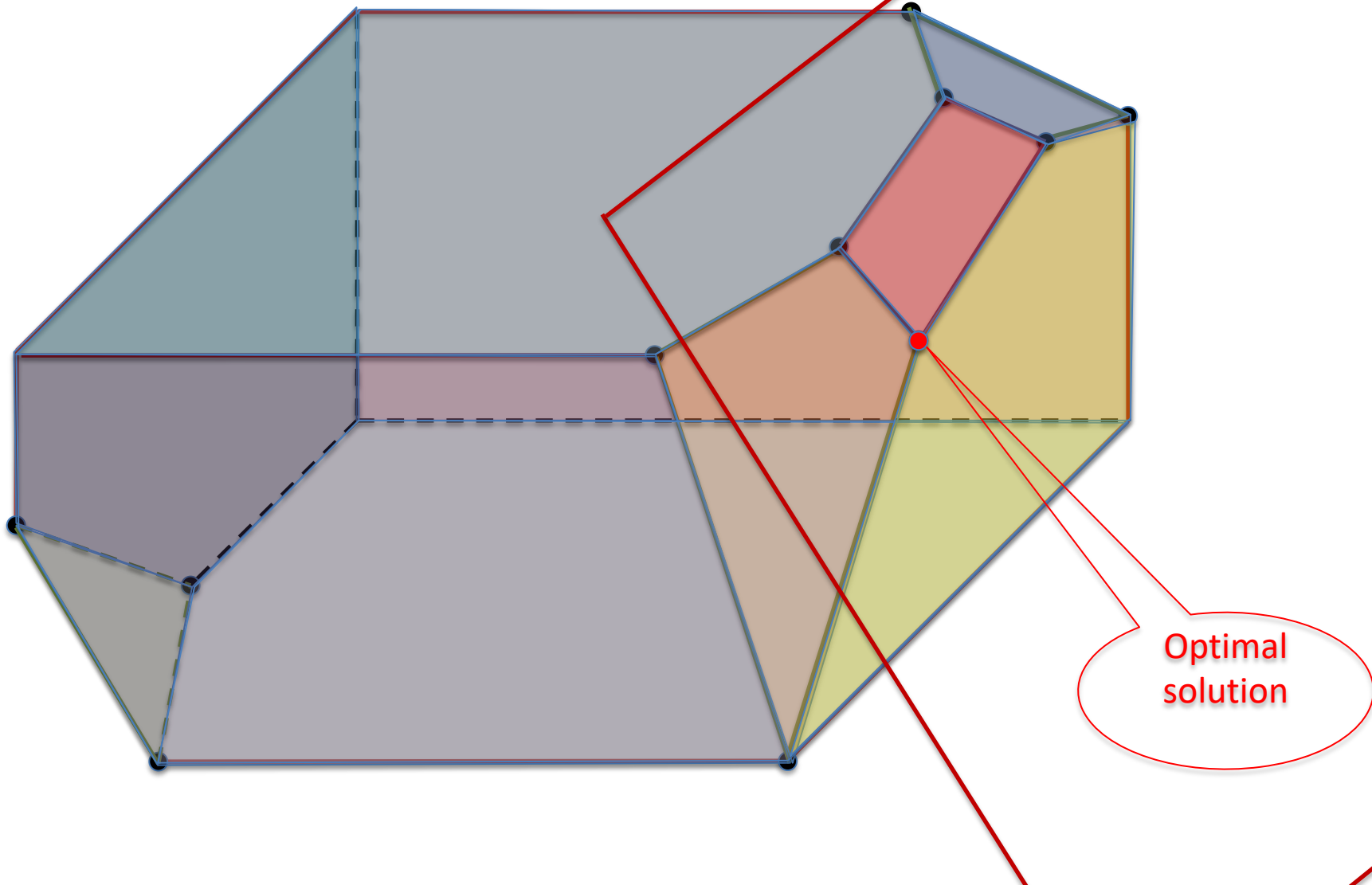
Geometry of LPs with 3 variables:

The objective function: set of parallel planes



Geometry of LPs with 3 variables:

The objective function: set of parallel planes



# Geometric interpretation of LPs

	<b>2 variables</b>	<b>3 variables</b>	<b>n variables</b>
constraint	half-plane	half-space	half-hyperspace
feasible region	convex polygon	convex polyhedron	convex polytope
objective function	parallel lines	parallel planes	parallel hyperplanes
basic feasible solution	polygon vertex	polyhedron vertex	polytope vertex

# LP terminology

Objective function: Function being minimized or maximized.

Solution: Assignment of real values to the variables.

Feasible solution: Solution that satisfies all the constraints.

Feasible region: The set of feasible solutions; a convex polytope.

Basic feasible solution: A vertex of the feasible solution polytope.

Optimal solution: A feasible solution that minimizes or maximizes the objective function; it is not necessarily unique and it might not exist (see below: infeasible and unbounded LP).

Value of a solution: Value of objective function at a solution; sometimes called "cost" of a solution (for minimization problems)

Optimal value: Value of optimal solution; sometimes called "optimal cost" (for minimization problems).

Feasible LP: LP that has feasible solutions.

Infeasible LP: LP with no feasible solutions.

Bounded LP: LP with an optimal solution.

Unbounded LP: LP that is feasible but has no optimal solution.