Computer Science C63 — March 5, 2025
Scarborough Campus — University of Toronto

Homework Assignment #5

Due: March 12, 2025, by 11:59 pm

• ***You must submit your assignment through the Crowdmark system.*** *You will receive by email an invitation through which you can submit your work in the form of separate PDF docucments with your answers to each question of the assignment. To work with a partner, you and your partner must form a group on Crowdmark. Crowdmark does not enforce a limit on the size of groups.* ***The course policy that limits the size of each group to at most two remains in effect:*** *submissions by groups of more than two persons will not be graded.*
• *To minimize bias in grading, Crowdmark does not reveal your name to the grader. Please do not subvert this feature by including your name(s) on the files you submit.*
• *It is your responsibility to ensure that the files you submit are legible. To this end, I encourage you to learn and use the LaTex typesetting system, which is designed to produce high-quality documents that contain mathematical notation. You are not required to produce the files you submit using LaTex; you may produce it any way you wish, as long as the resulting document is legible.*
• *By virtue of submitting this assignment you (and your partner, if you have one) acknowledge that you are aware of the policy on homework collaboration for this course.*[a]
• *For any question, you may use facts previously proved in this course, its prerequisites, or in the assigned sections of the textbook.*
• *Unless we explicitly state otherwise, you should justify your answers. Your paper will be graded based on the correctness a of your answers, and the clarity, precision, and conciseness of your presentation.*

[a] "In each homework assignment you may collaborate with at most one other student who is currently taking CSCC63. If you collaborate with another student on an assignment, you and your partner must submit only one copy of your solution, with both of your names. The solution will be graded in the usual way and both partners will receive the same mark. Collaboration involving more than two students is not allowed. **For help with your homework you may consult only the instructor, TAs, your homework partner (if you have one), your textbook, and your class notes. You may not consult any other source.** *Nota Bene:* 'Other source' includes, but is not limited to, the use of AI-based tools, even for (allegedly) 'just improving' your work. What you submit must be entirely your own creation."

Question 1. (20 marks) A propositional formula is called a ***tautology*** if it is satisfied by every truth assignment to its variables.

a. The ***CNF tautology problem***, CNF-TAUT, is the following decision problem:

INSTANCE: $\langle F \rangle$, where F is a propositional formula in conjunctive normal form.

QUESTION: Is F a tautology?

Prove that CNF-TAUT $\in$ **P**. (You can describe an algorithm that solves this problem in pseudocode.)

b. The ***DNF non-tautology problem***, DNF-NONTAUT, is the following decision problem:

INSTANCE: $\langle F \rangle$, where F is a propositional formula in <u>disjunctive</u> normal form.

QUESTION: Is F <u>not</u> a tautology?

Prove that DNF-NONTAUT is **NP**-complete.

Question 2. (25 marks) The Hamiltonian Path problem, HAMPATH, is defined as follows:

INSTANCE: $\langle G \rangle$, where G is a directed graph.

QUESTION: Is there a path of G that visits every node exactly once? Such a path is called a ***Hamiltonian path*** of G.

a. Prove that HAMPATH $\in$ **NP**.

b. Give a polytime mapping-reduction of HAMPATH to CNF-SAT, and prove that it is correct. (Since HAMPATH $\in$ **NP**, the Cook-Levin Theorem implies that such a reduction exists. The question here is not to argue that such a reduction exists, but to show one explicitly.)

Hint: There are various ways of doing this. The simplest one I could think of involves introducing, among other variables, a variable e_{uv} for every pair of nodes u, v of the given graph G, intended to be true if and only if (u, v) is an edge of G. Construct a formula using these and any other variables needed that is satisfiable if and only if G has a Hamiltonian path. Write your formula in a modular way with subformulas expressing specific facts, analogous to (but much simpler than) the way we constructed the formula in the proof of the Cook-Levin Theorem. In your answer be sure to explain the role of each piece of your formula. This will help the grader follow your construction, and possibly gain you some partial marks even if your answer is not quite right.