CSCC63

Hamiltonian cycle problem

Instance: $\langle G \rangle$, where G is a directed graph.
Question: Does G have a Hamiltonian cycle?

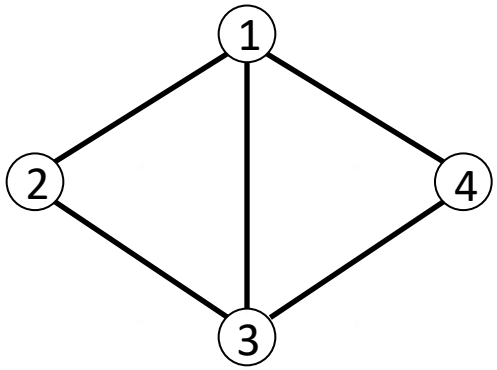
Hamiltonian cycle: Visits every node exactly once.

Prove:

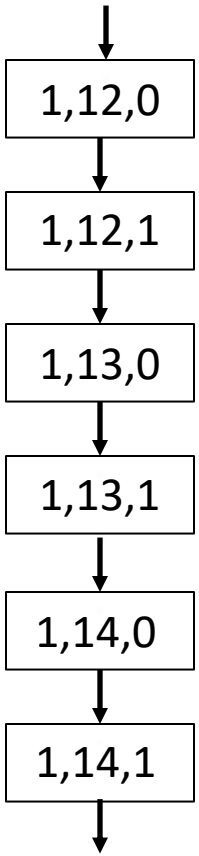
Vertex Cover $\leq_m^p$ Directed Hamiltonian Cycle

Given $\langle G, k \rangle$ where G is an undirected graph and $k \in Z^+$, construct a directed graph G_D s.t.

G has a vertex cover of size k
$\Longleftrightarrow$ G_D has a directed Hamiltonian cycle



G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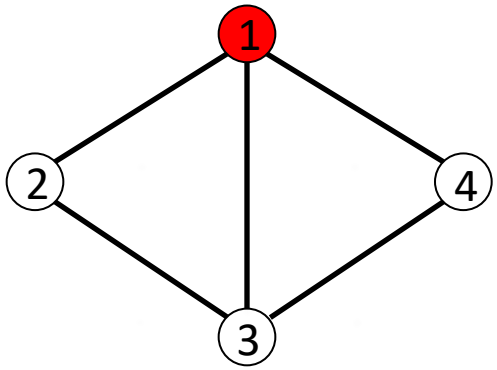
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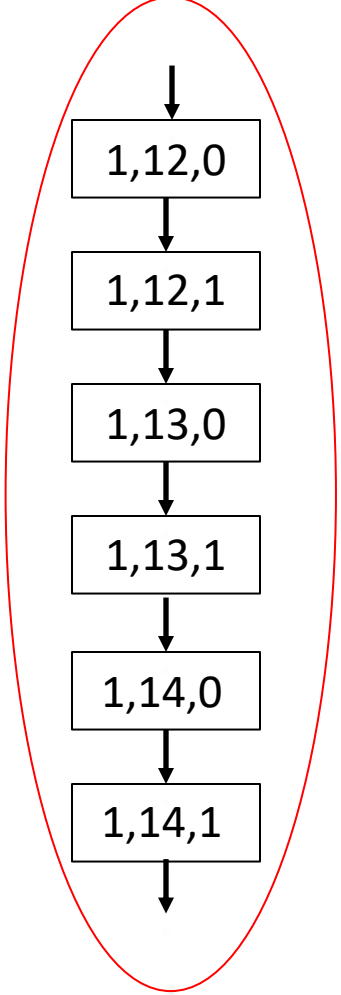
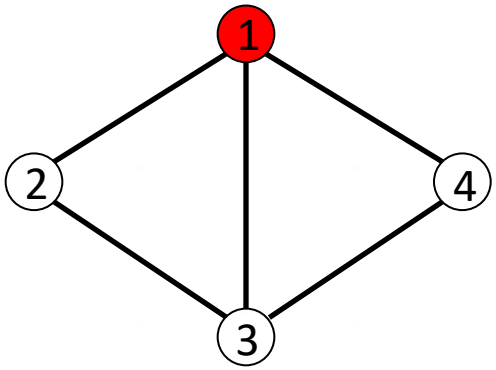
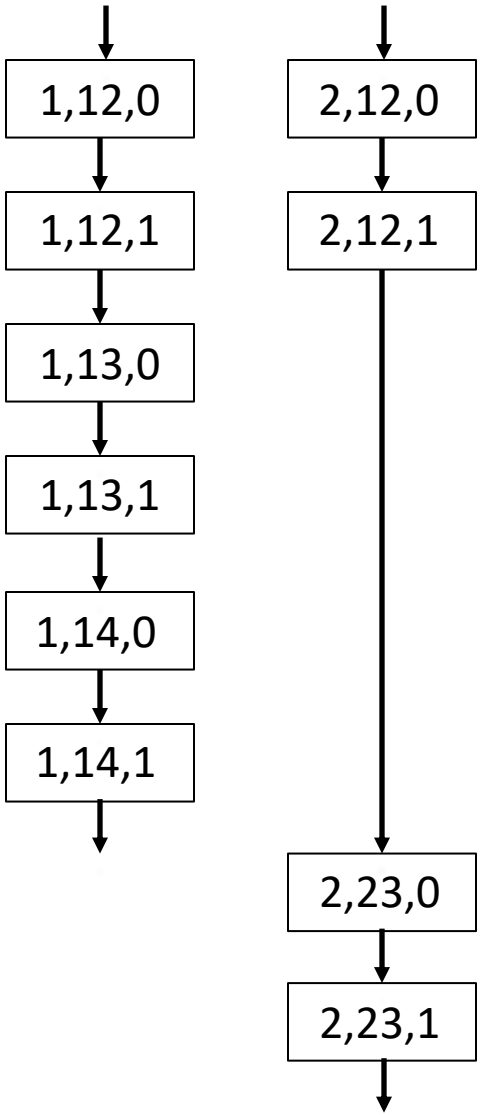


nodes of G_D corresponding to edges incident on node 1 of G

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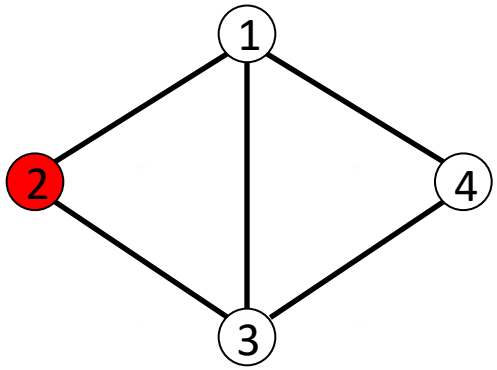
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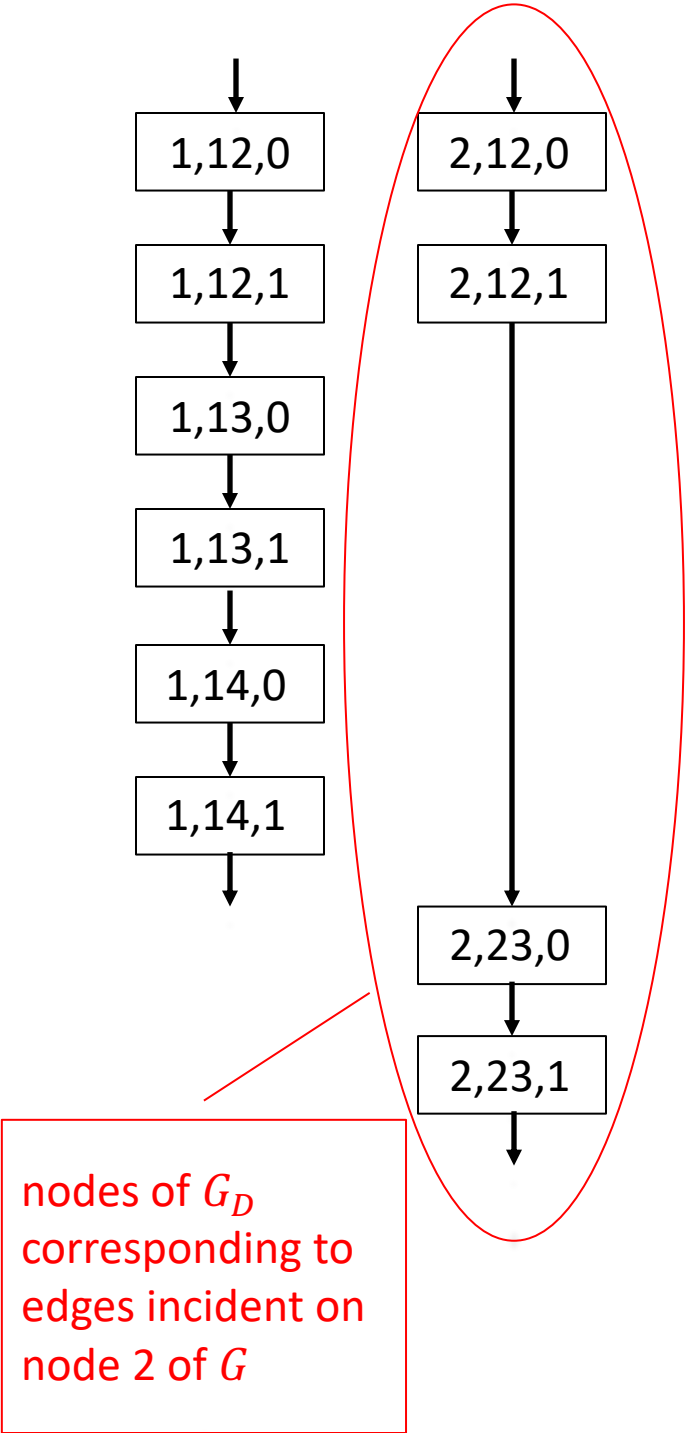
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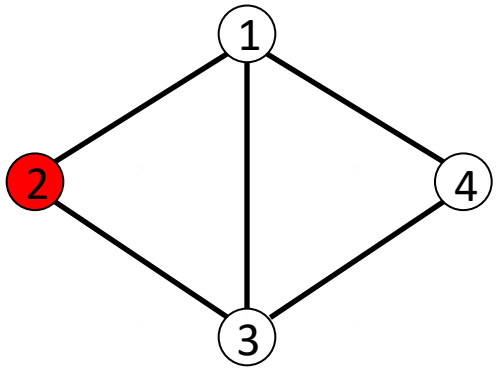
nodes of G_D corresponding to edges incident on node 2 of G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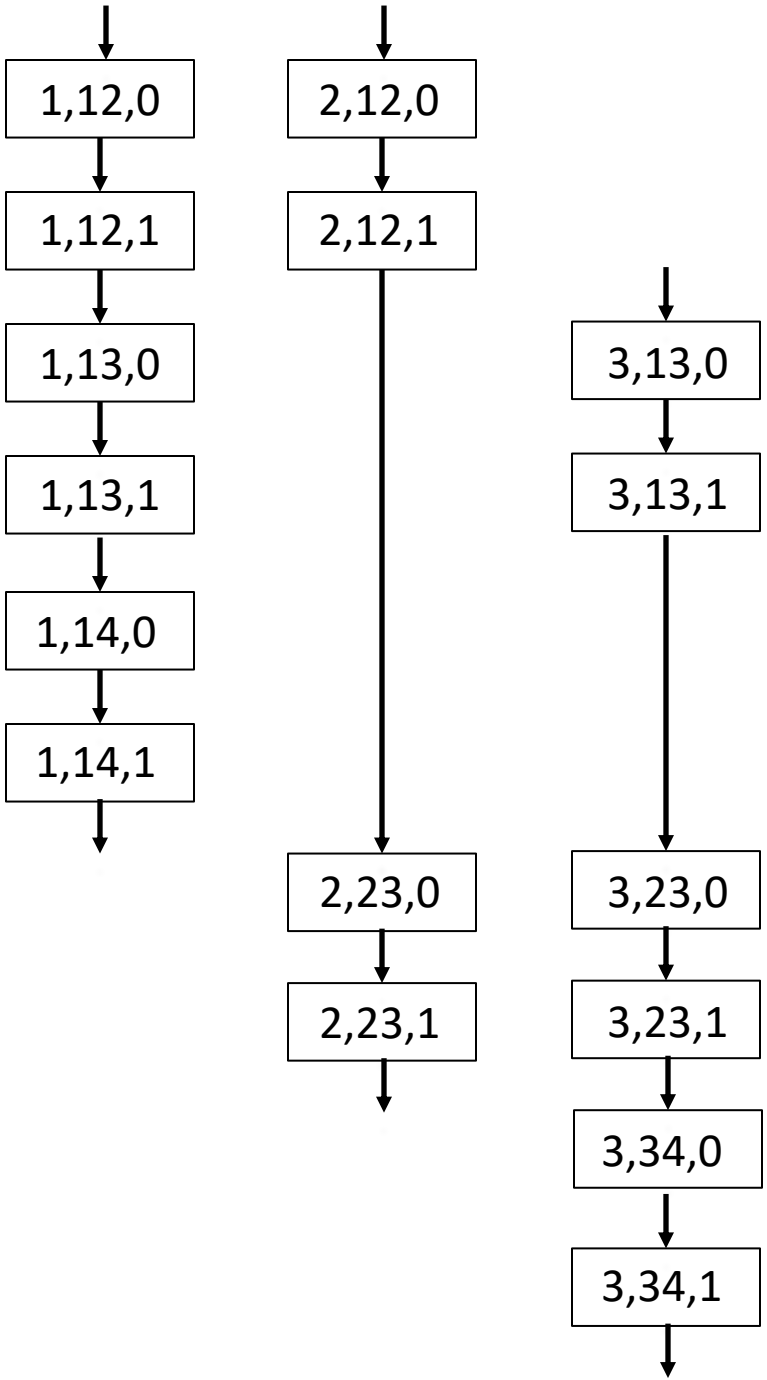
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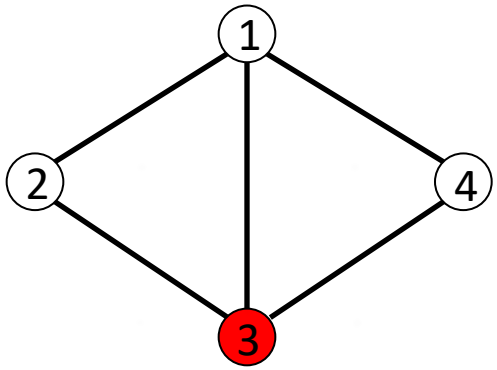
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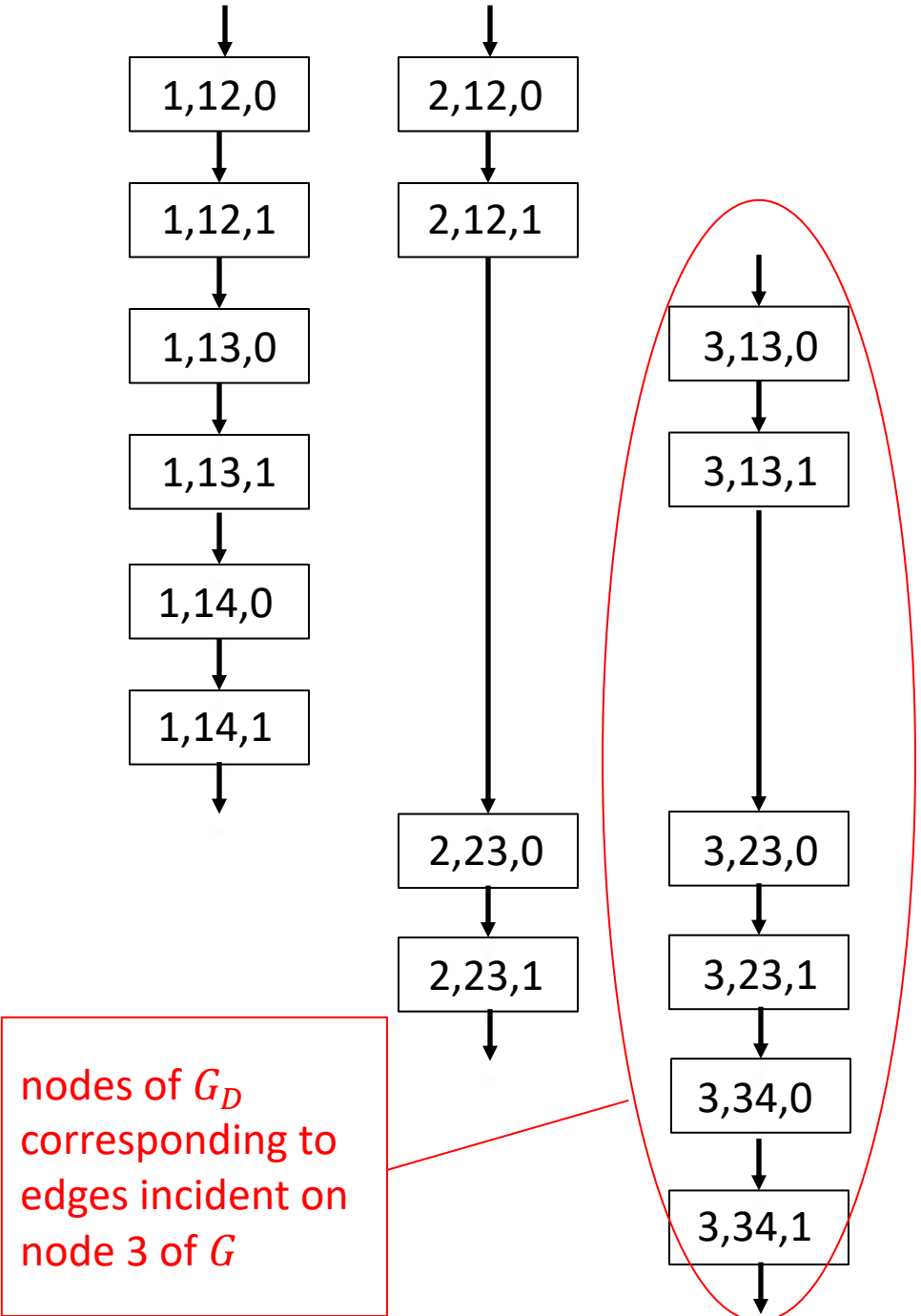
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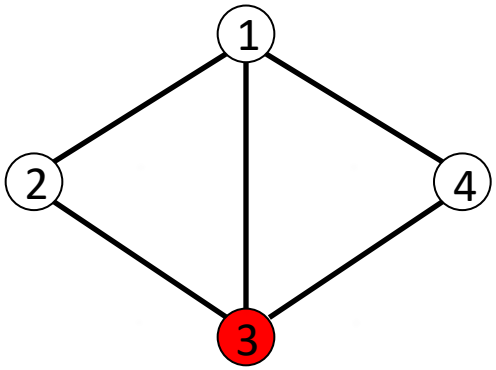
nodes of G_D corresponding to edges incident on node 3 of G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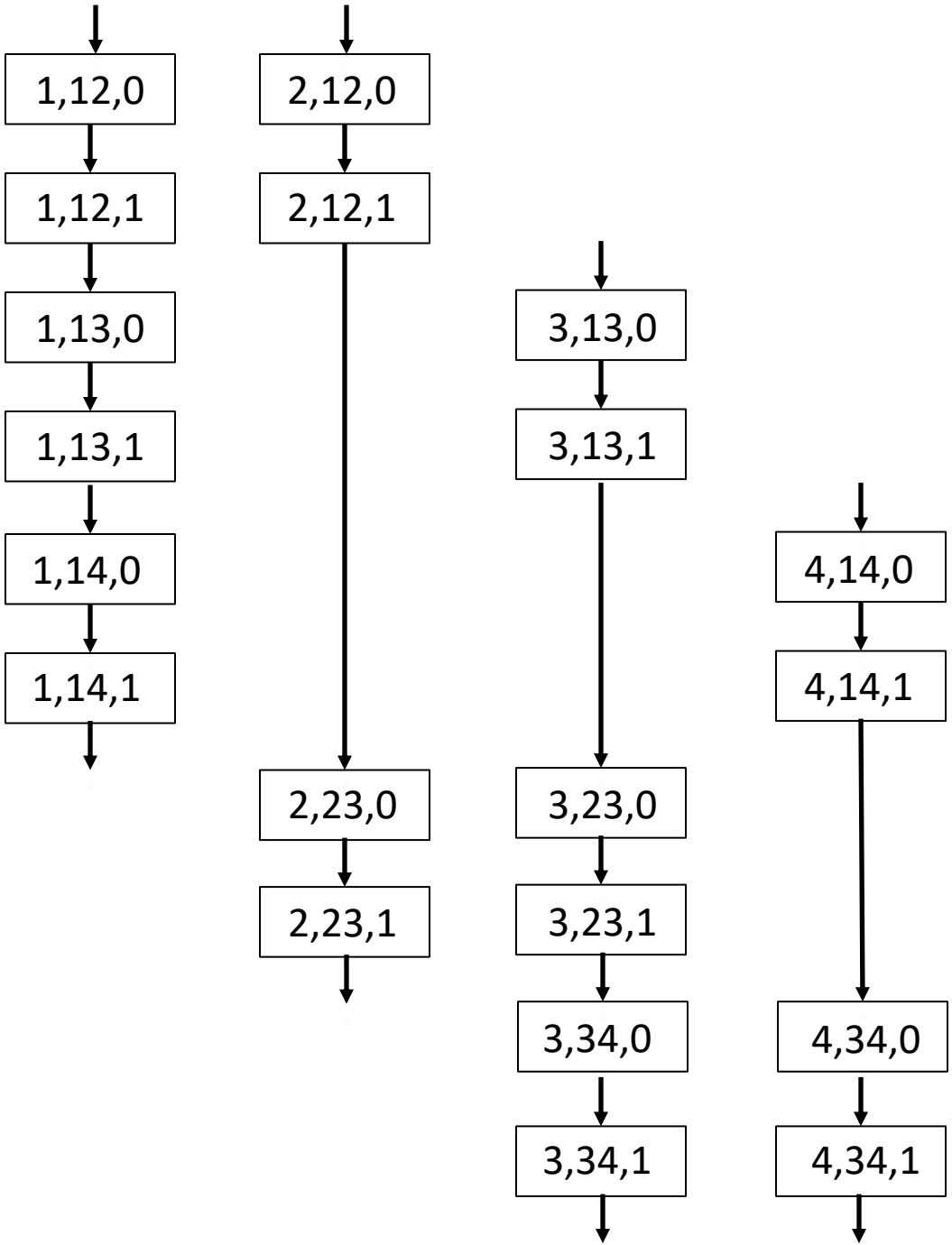
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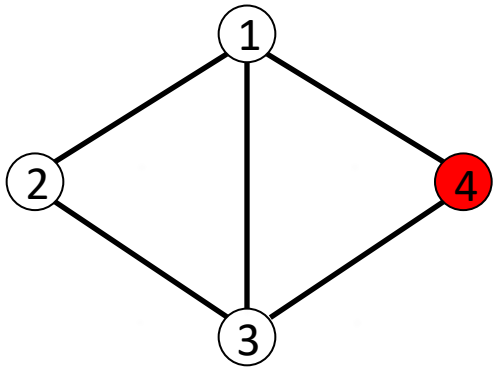
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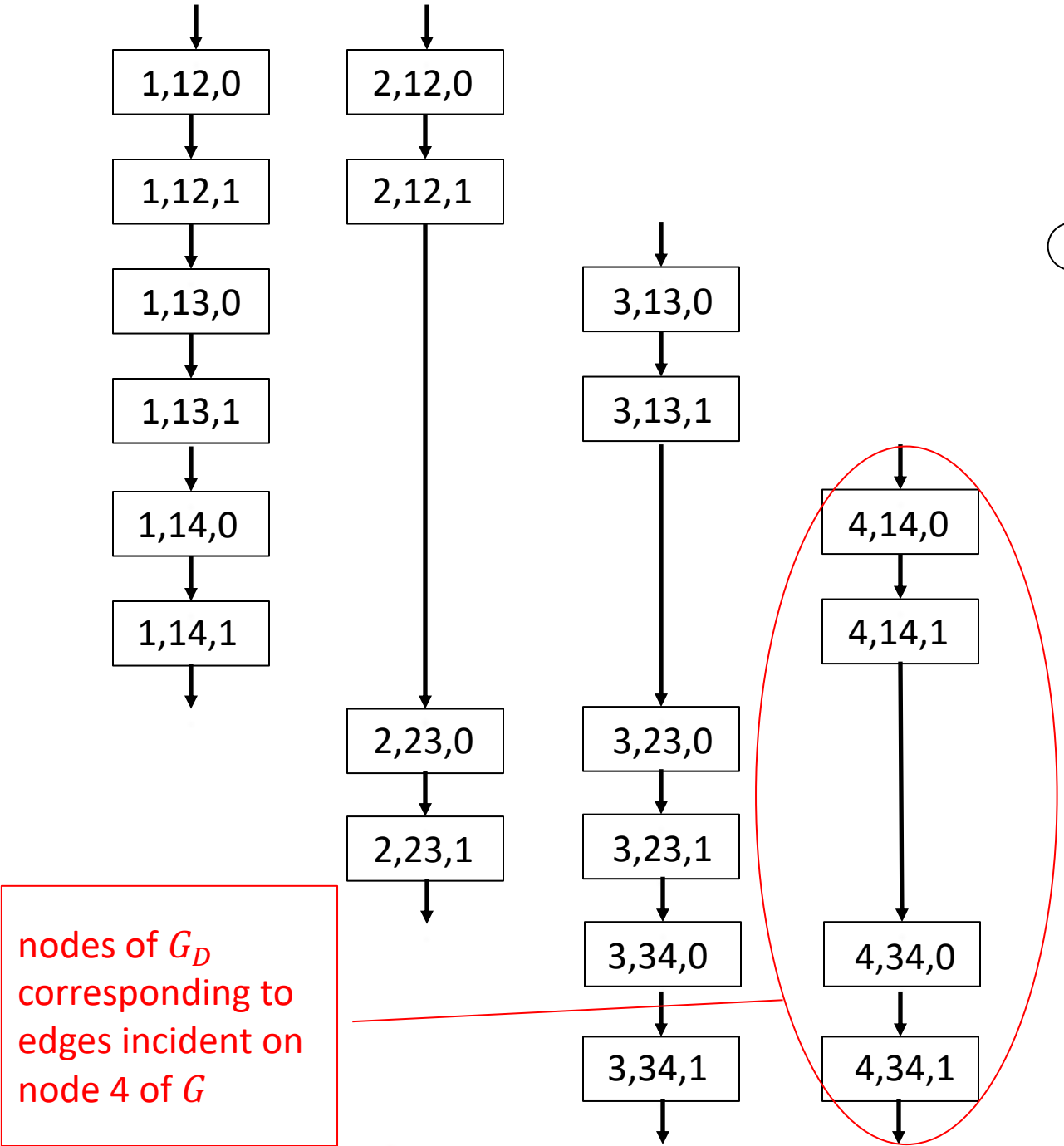
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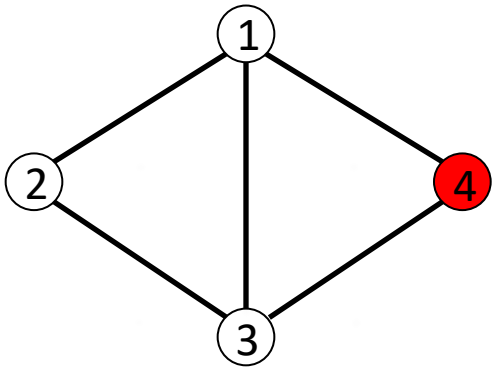
nodes of G_D corresponding to edges incident on node 4 of G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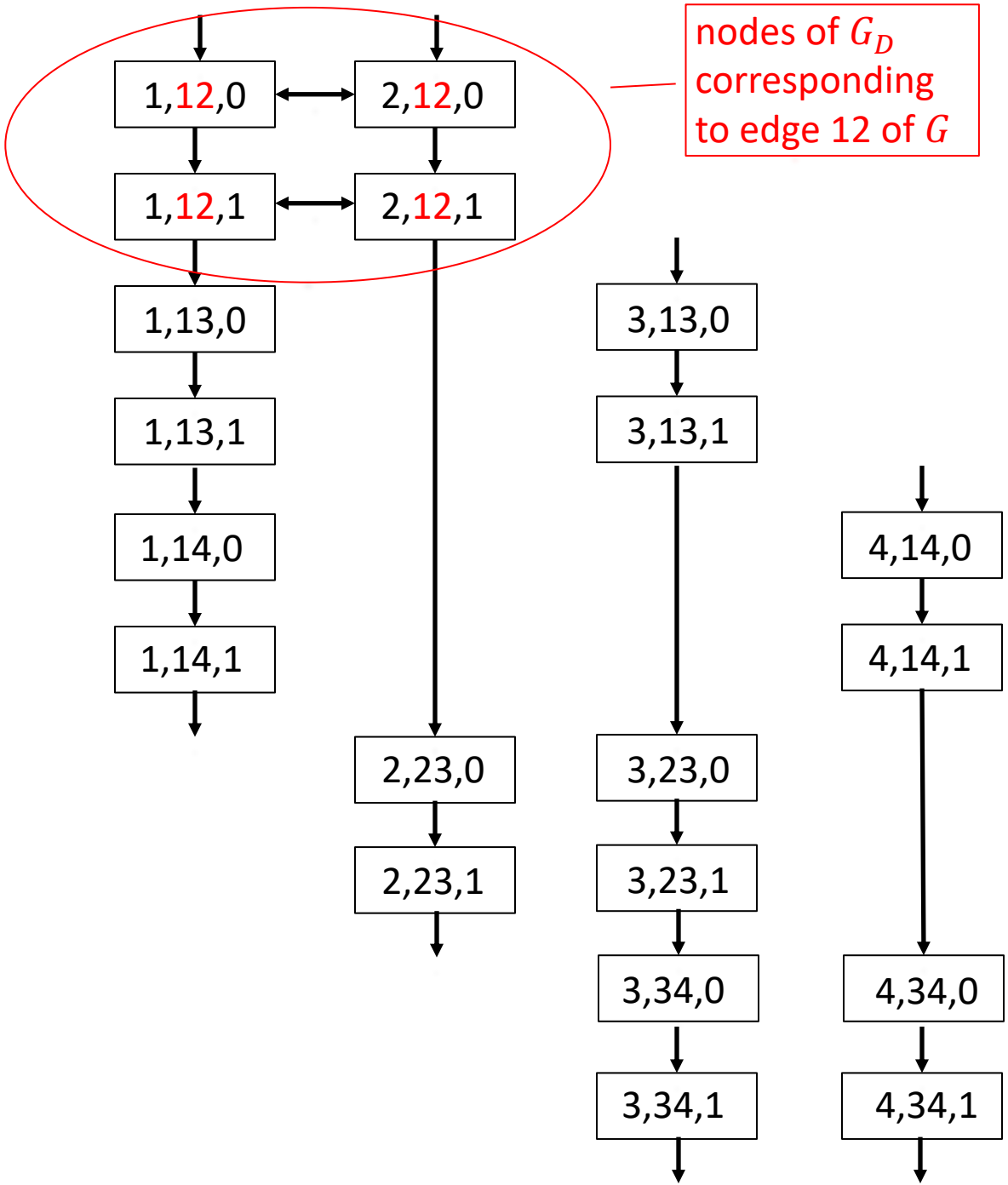
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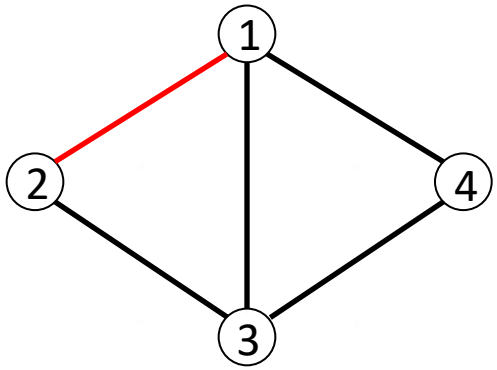
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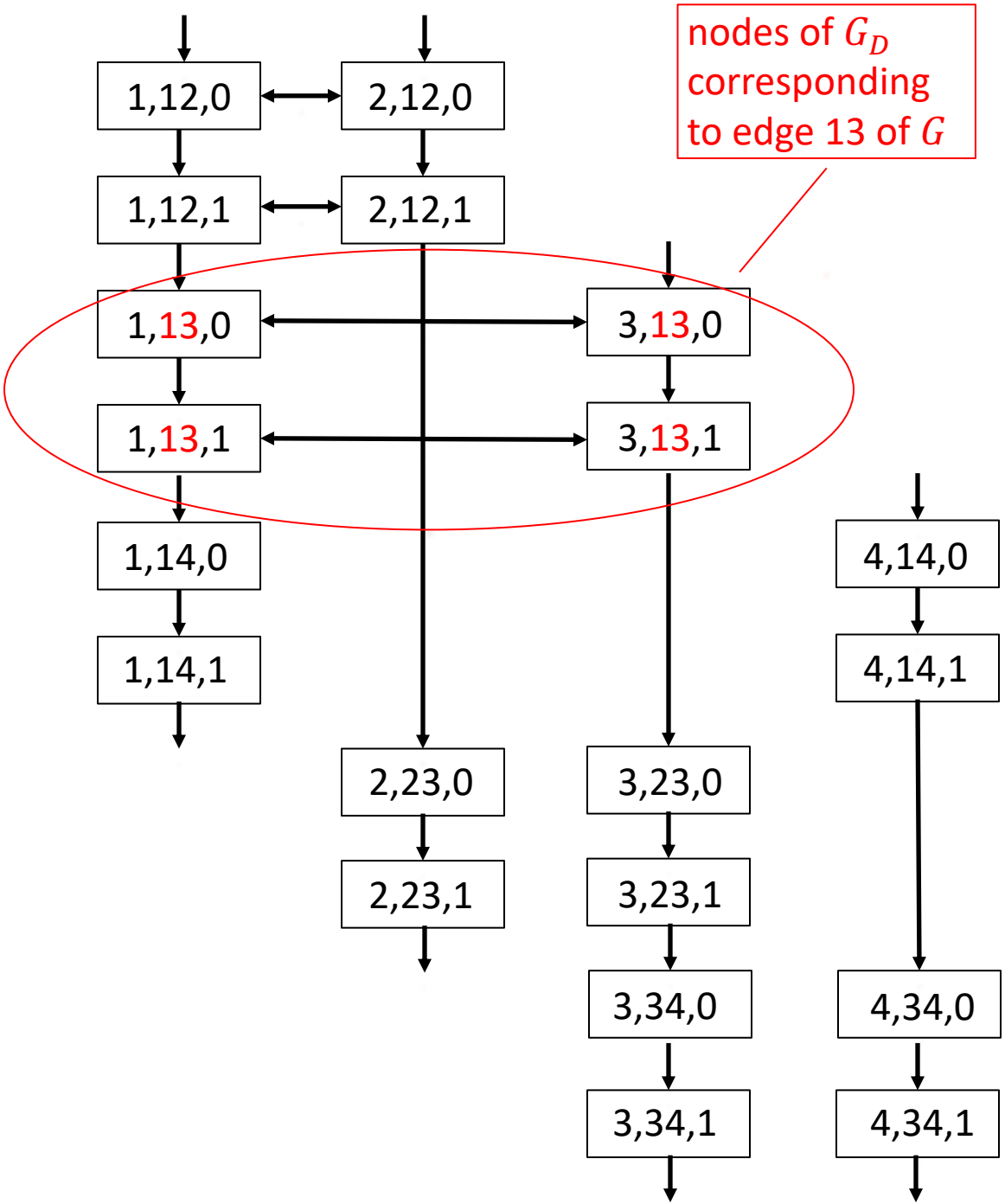
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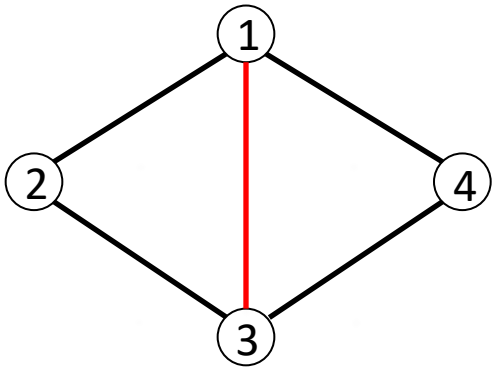
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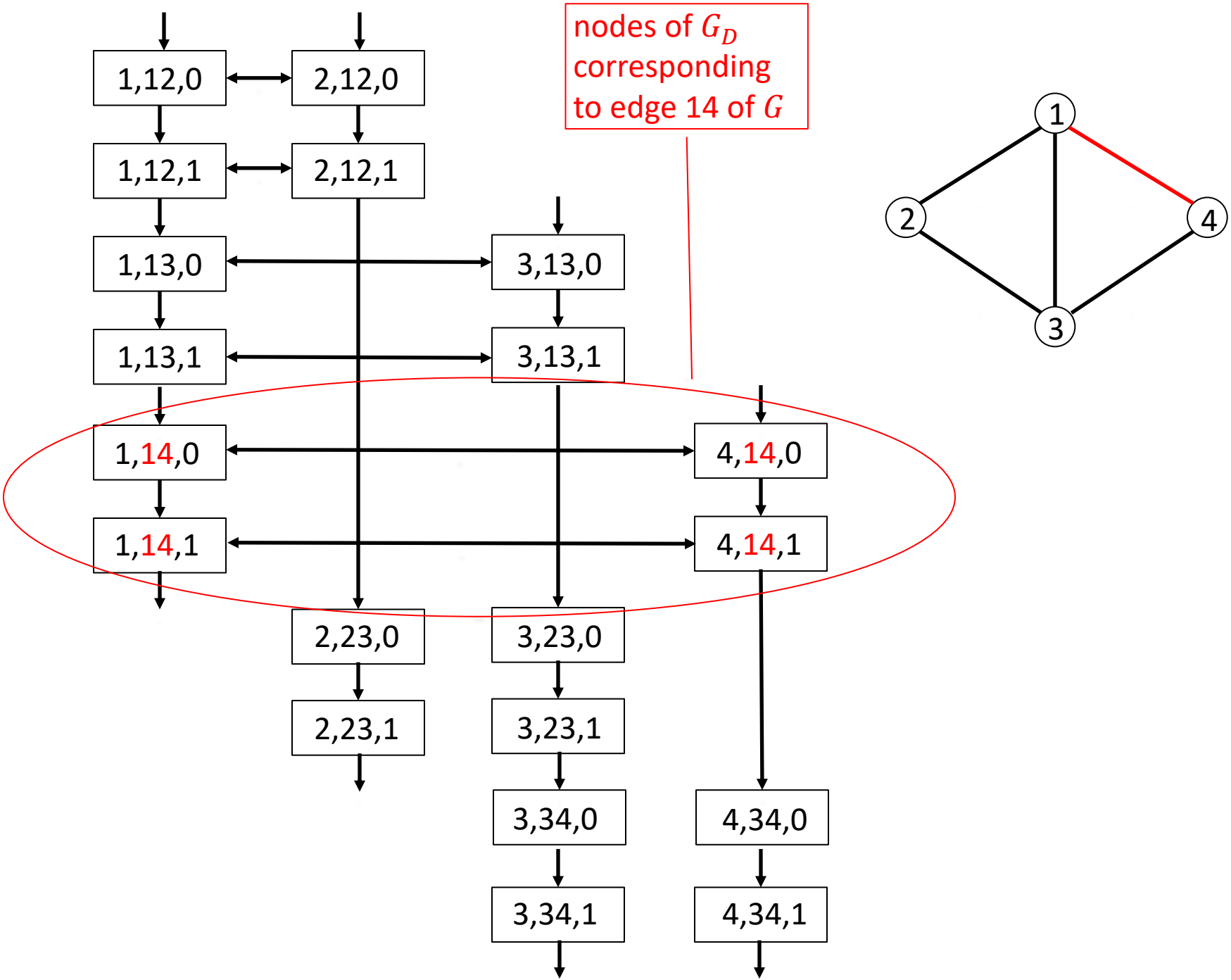
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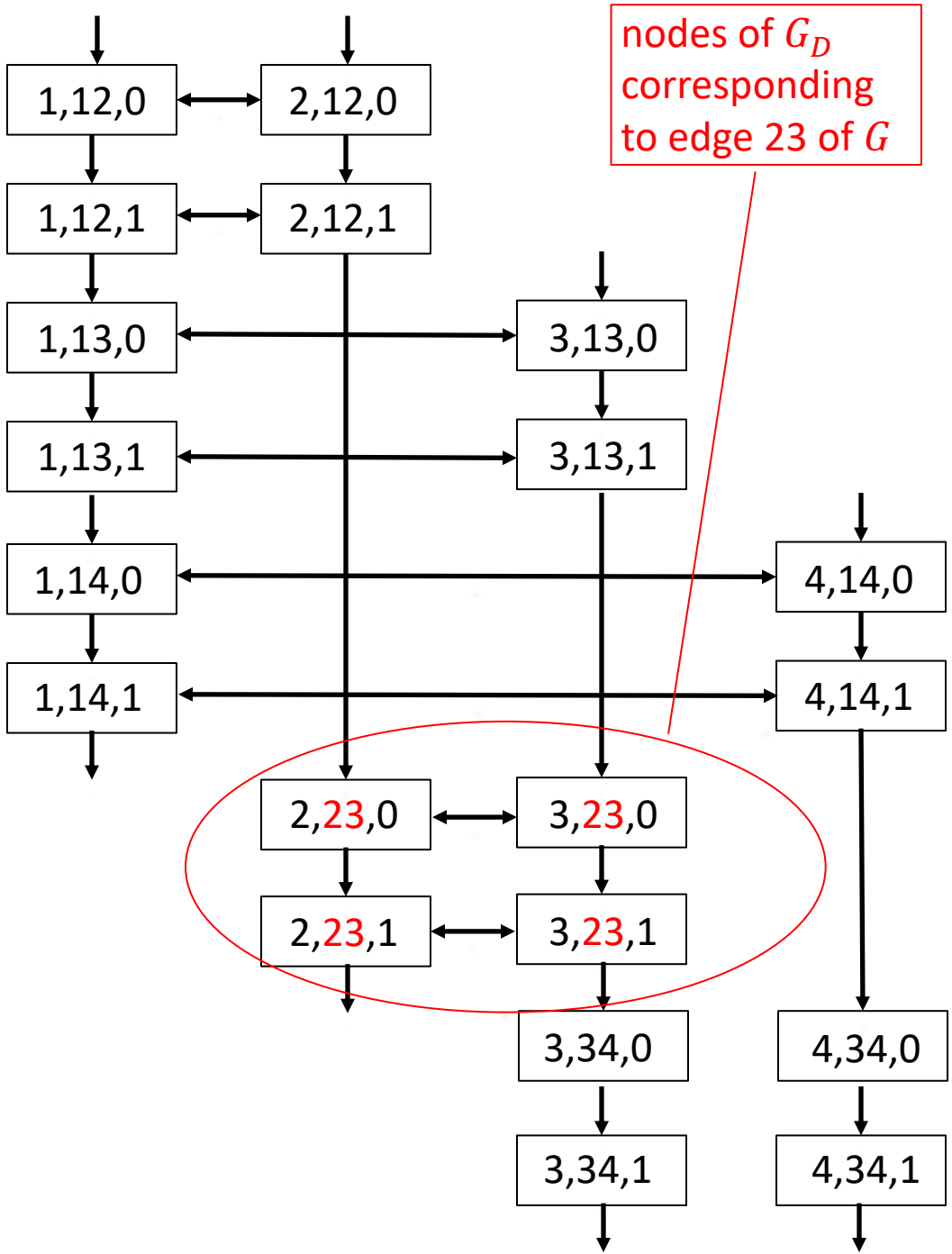
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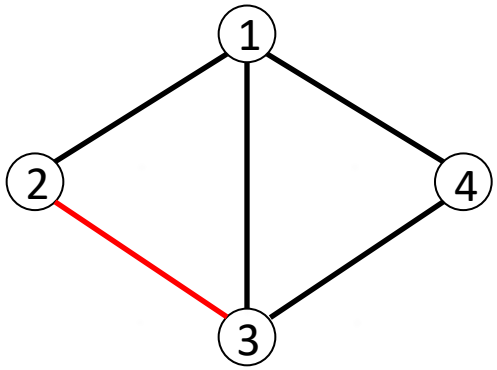
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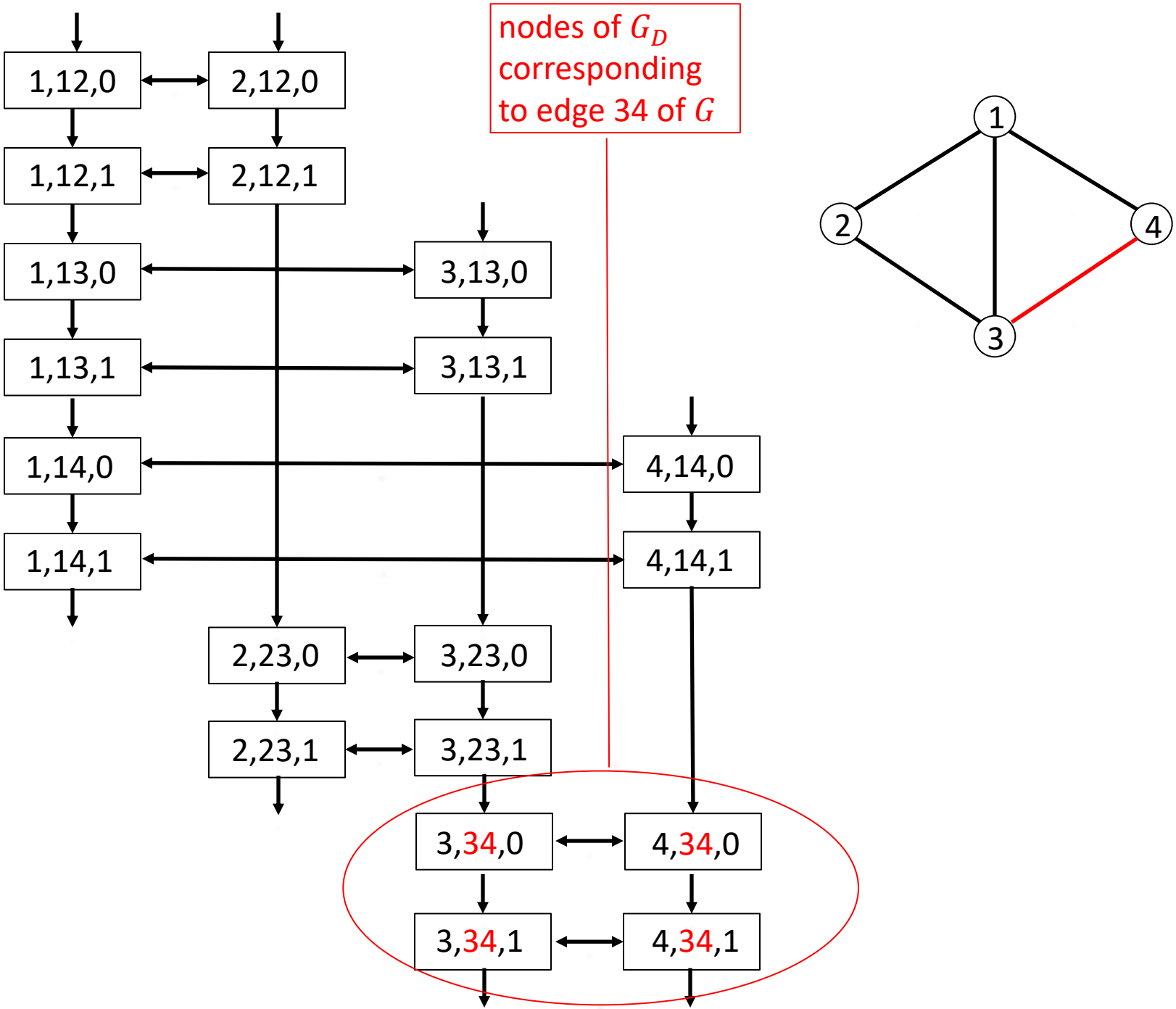
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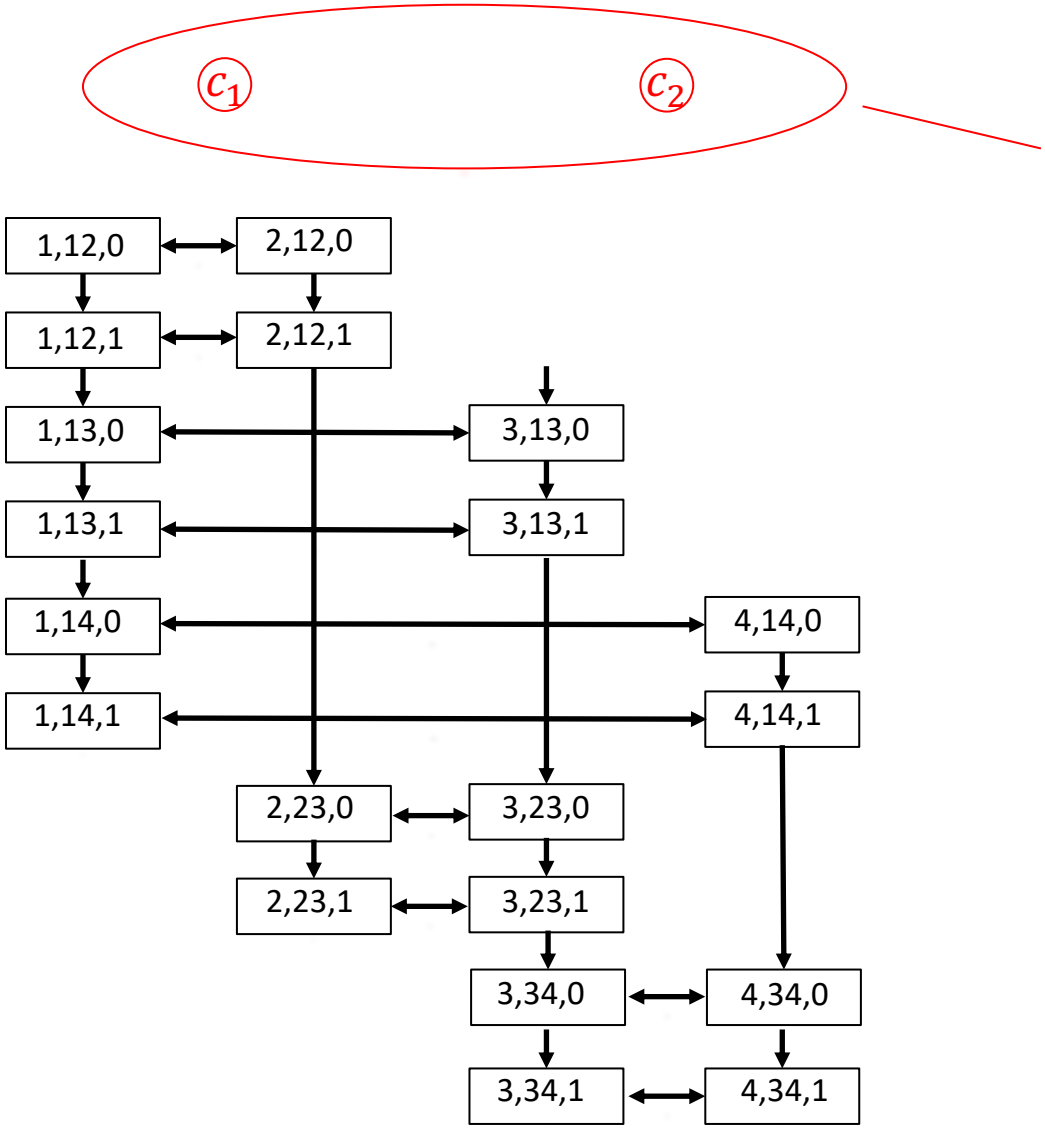

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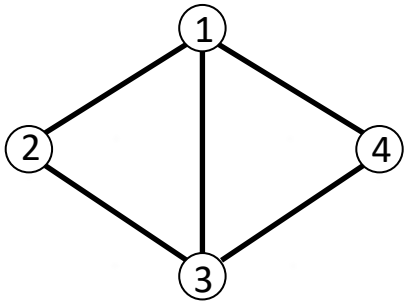

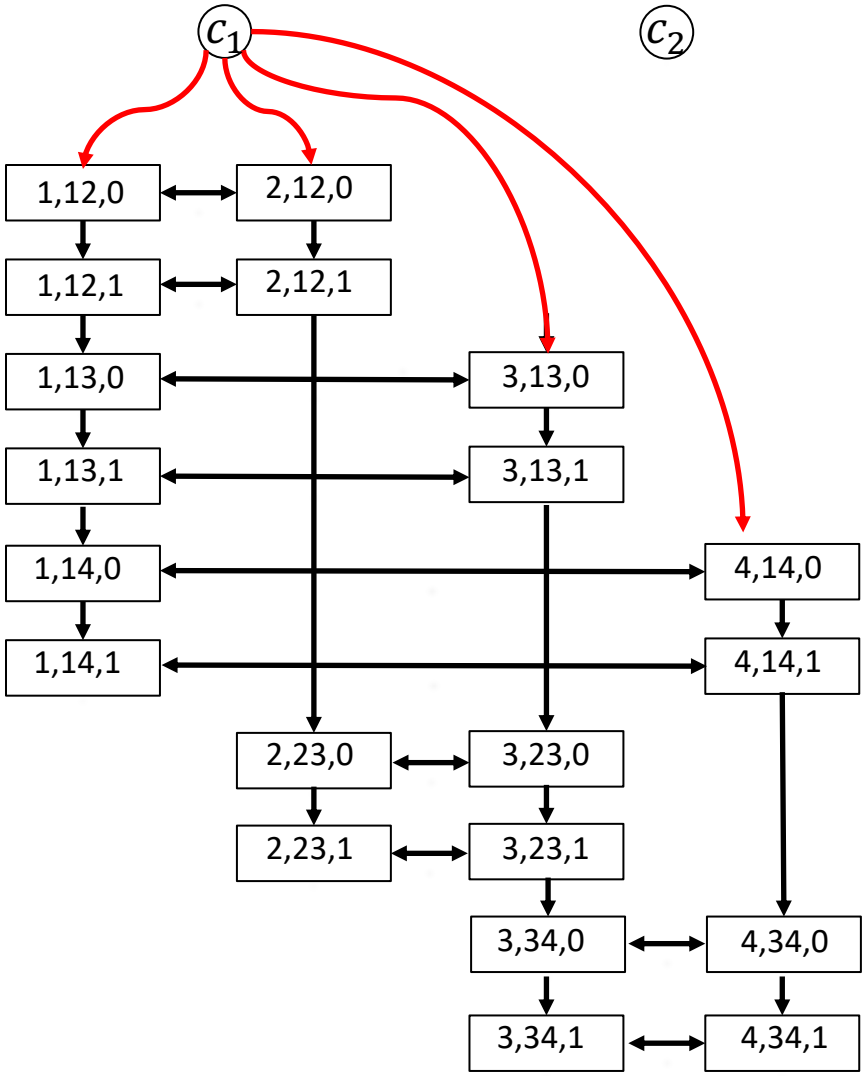
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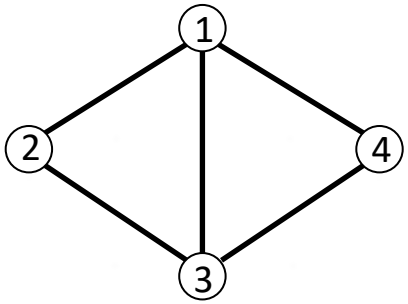
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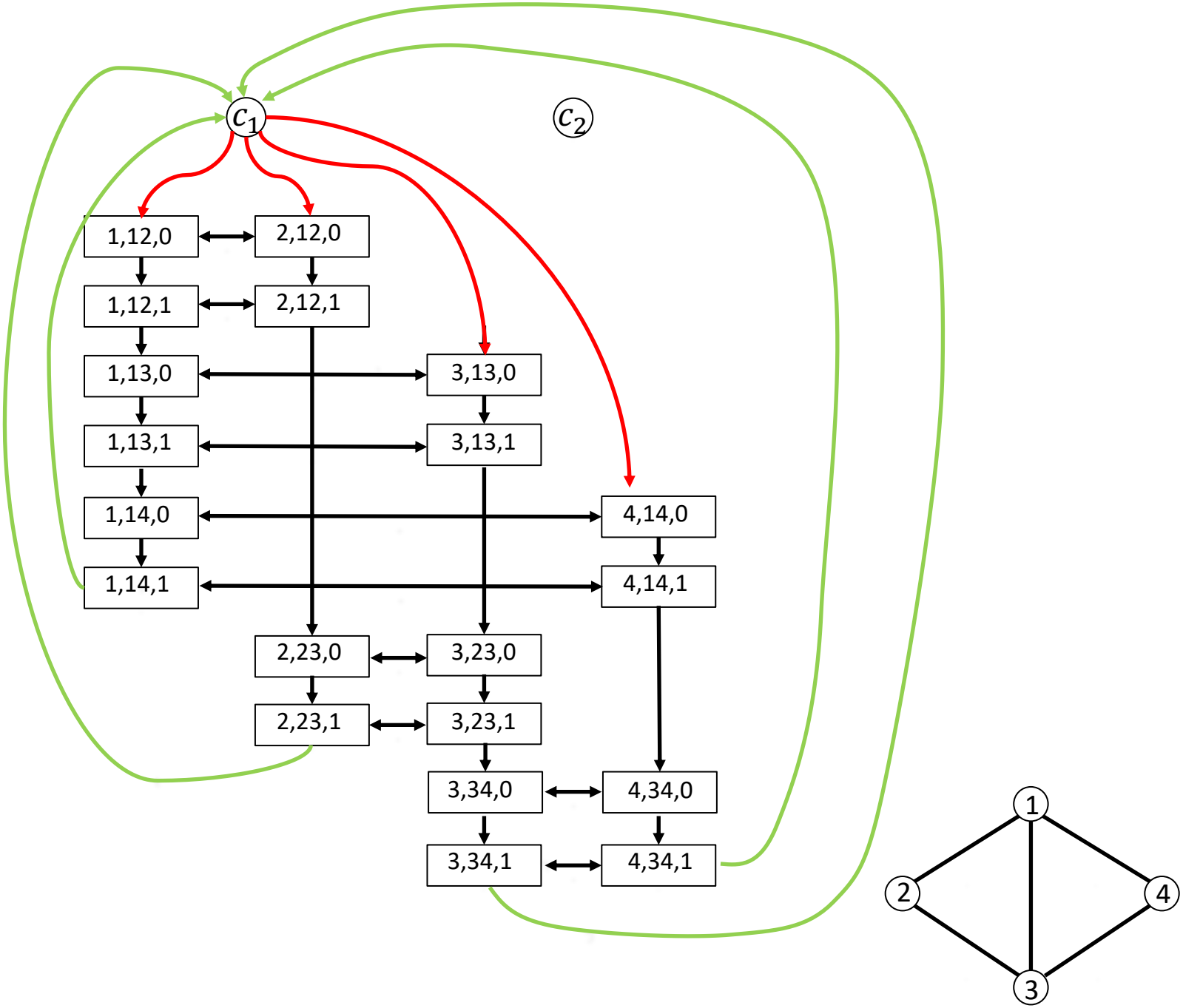
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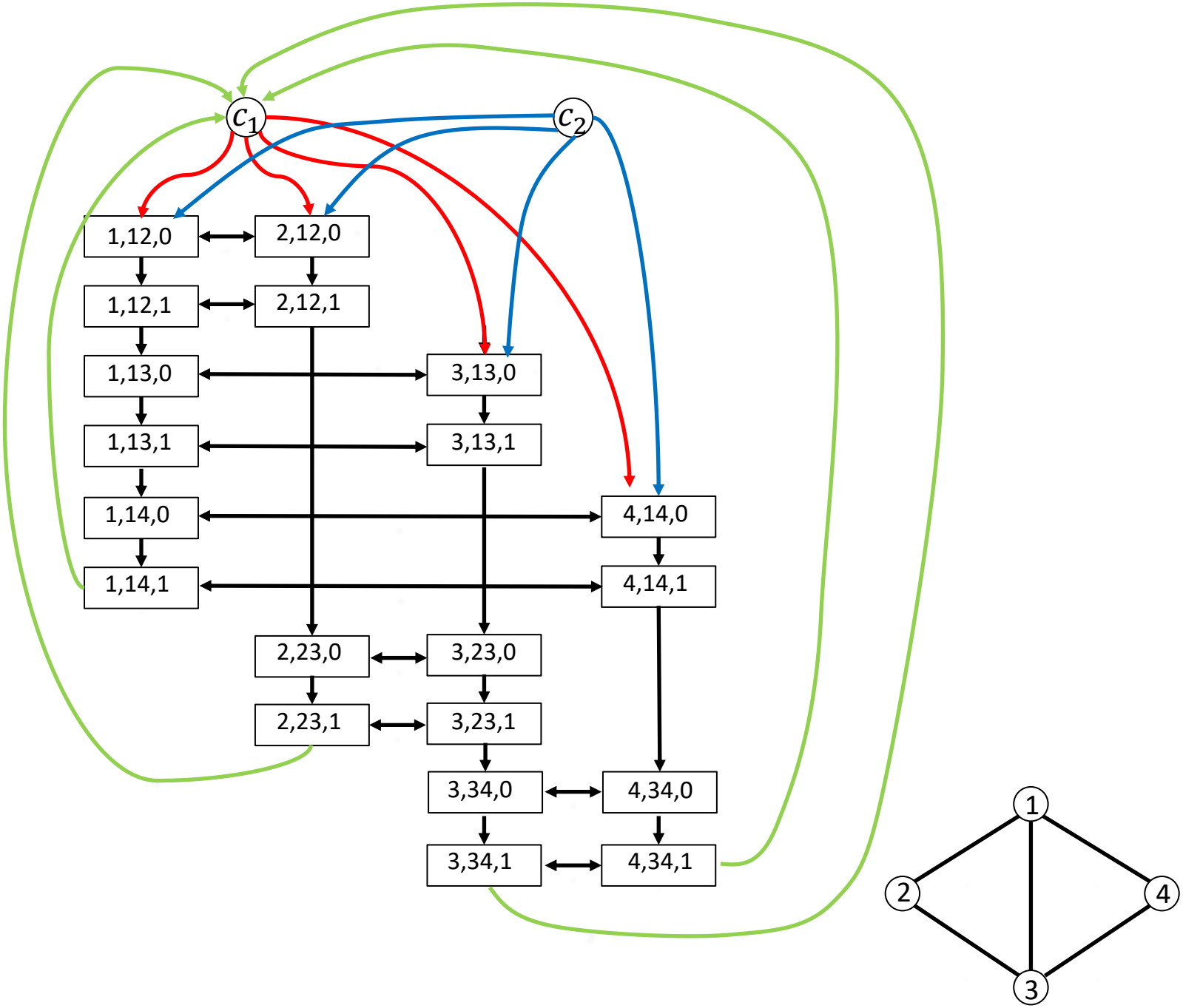
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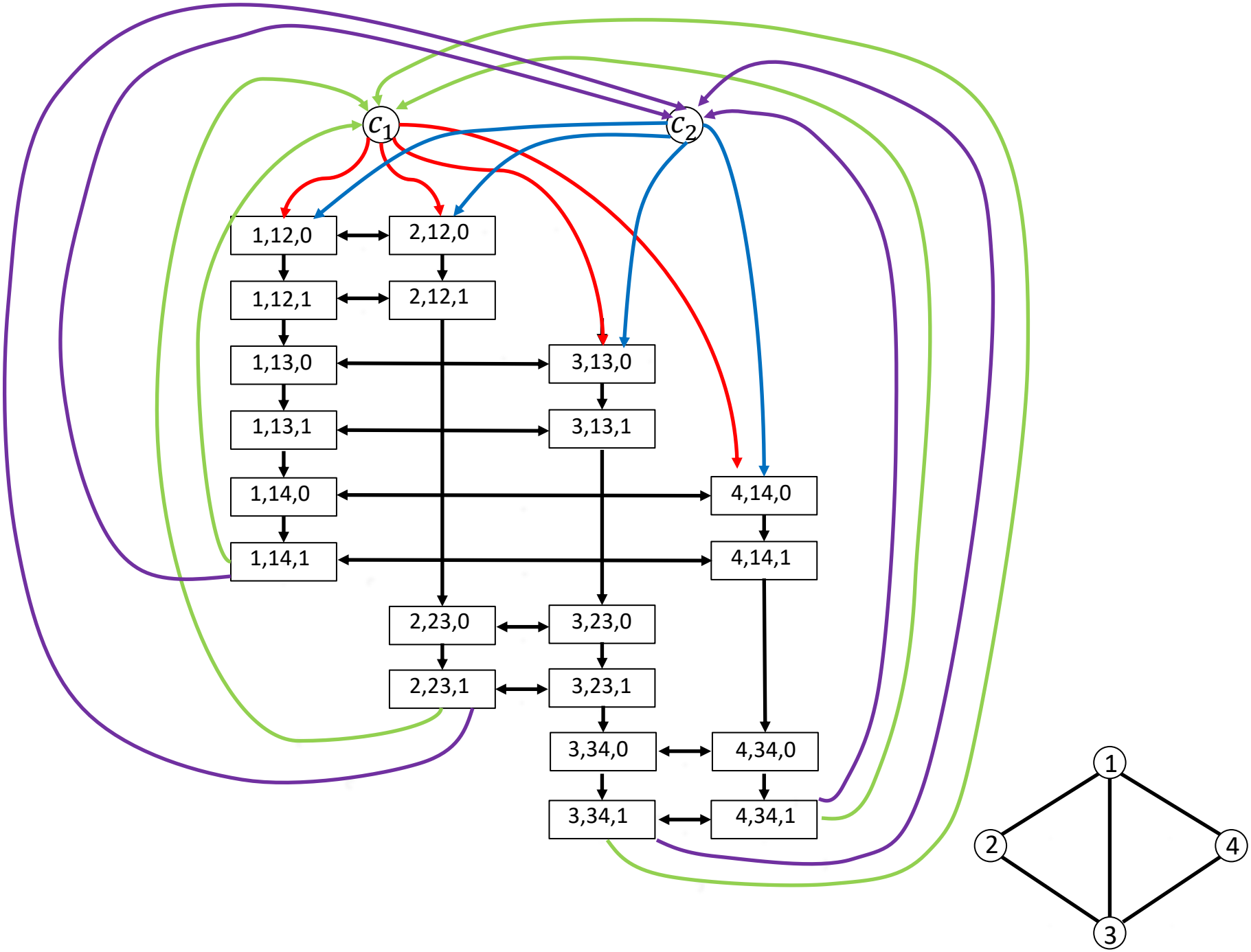
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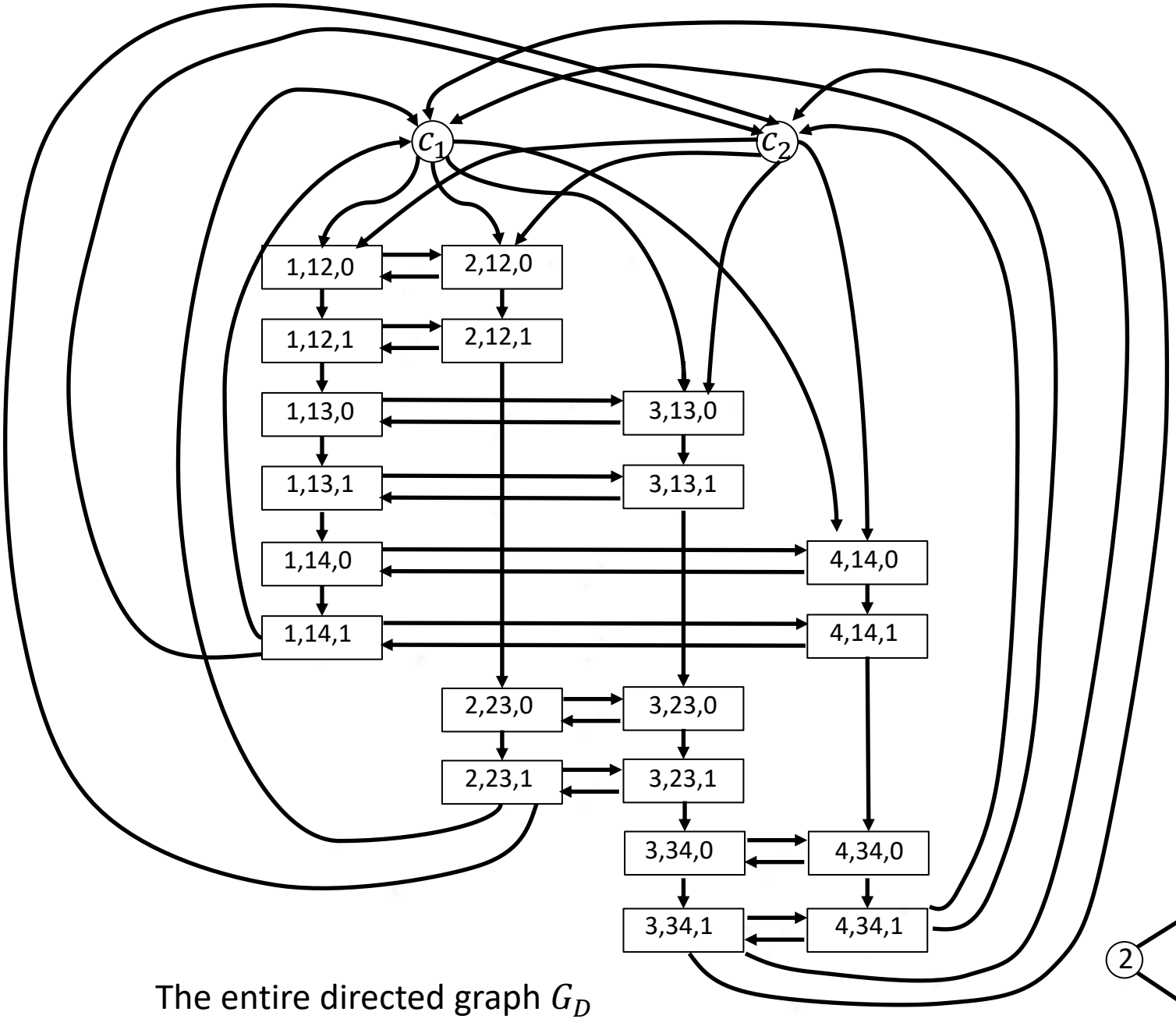
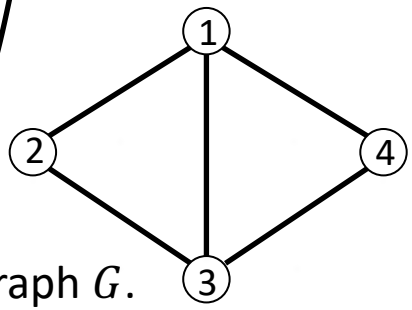
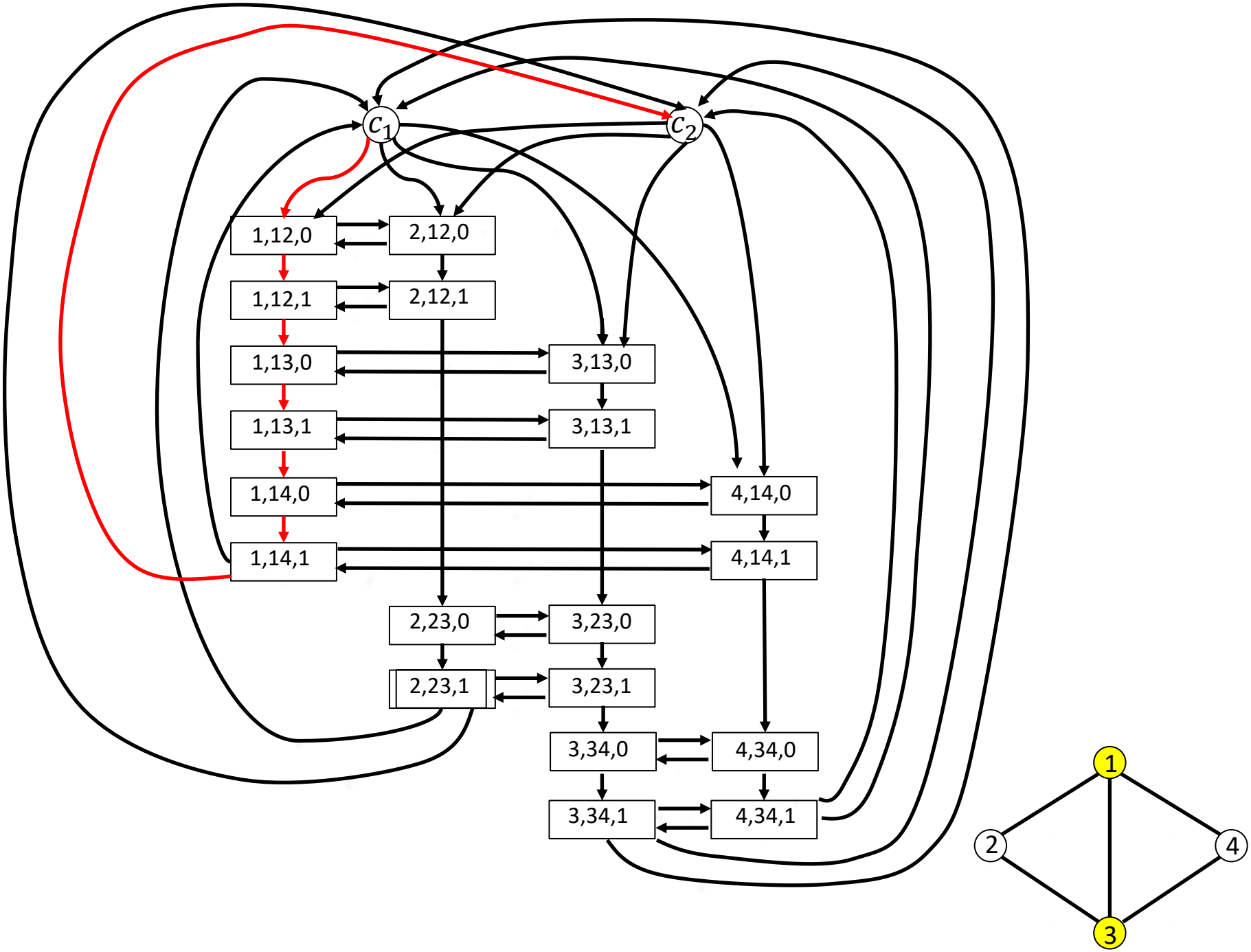


The entire directed graph G_D constructed from G.



The undirected graph G.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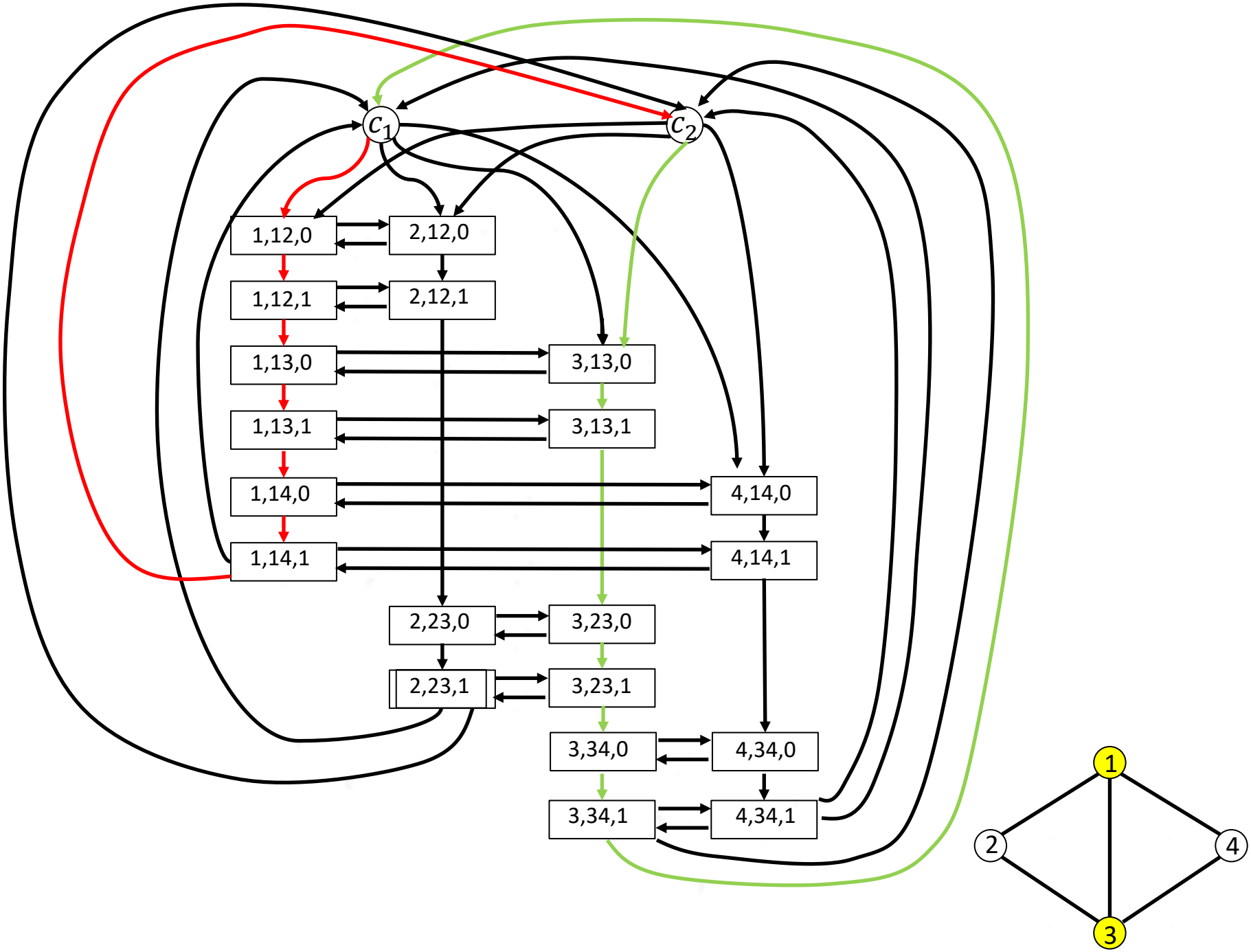
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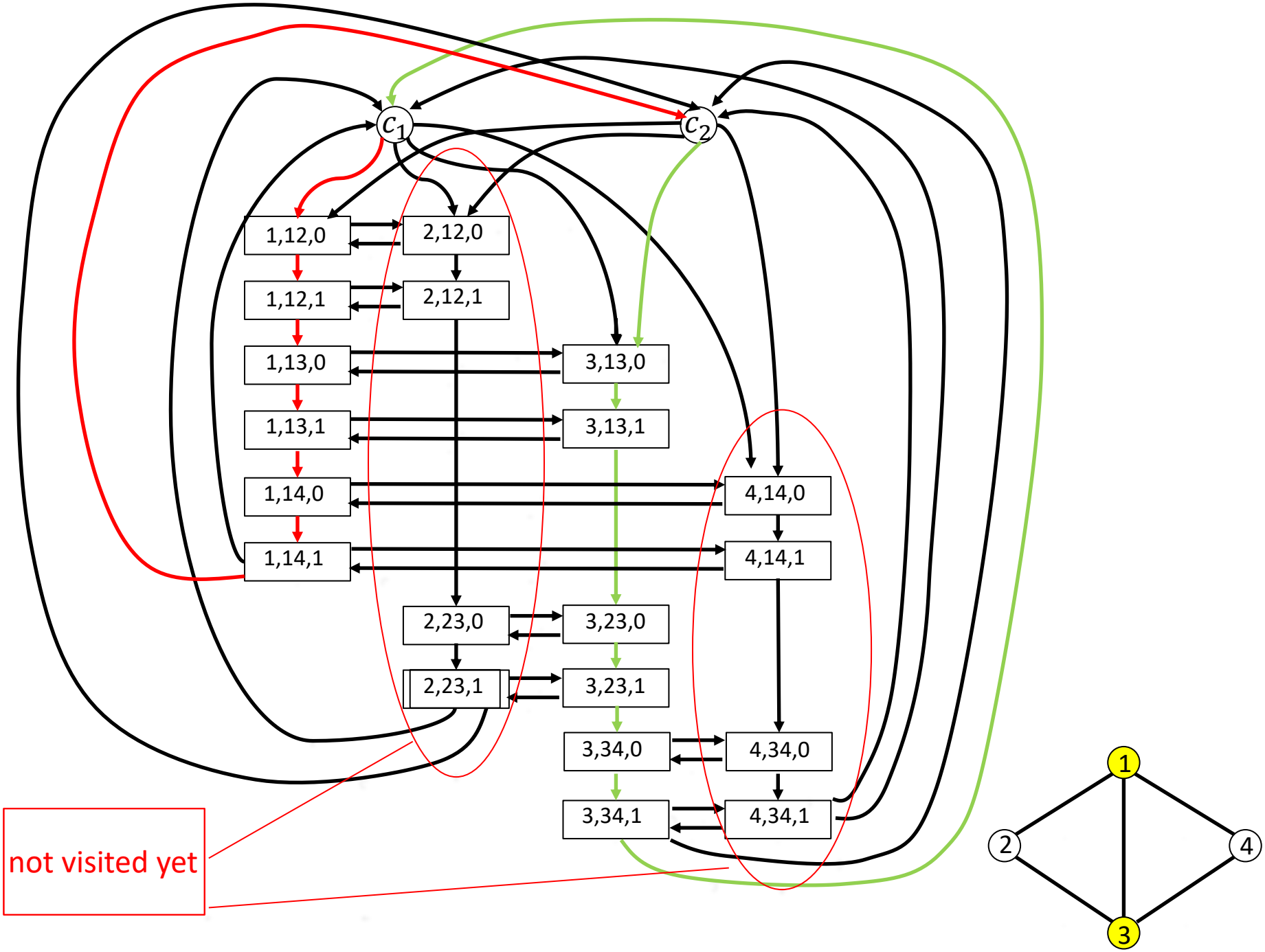
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3
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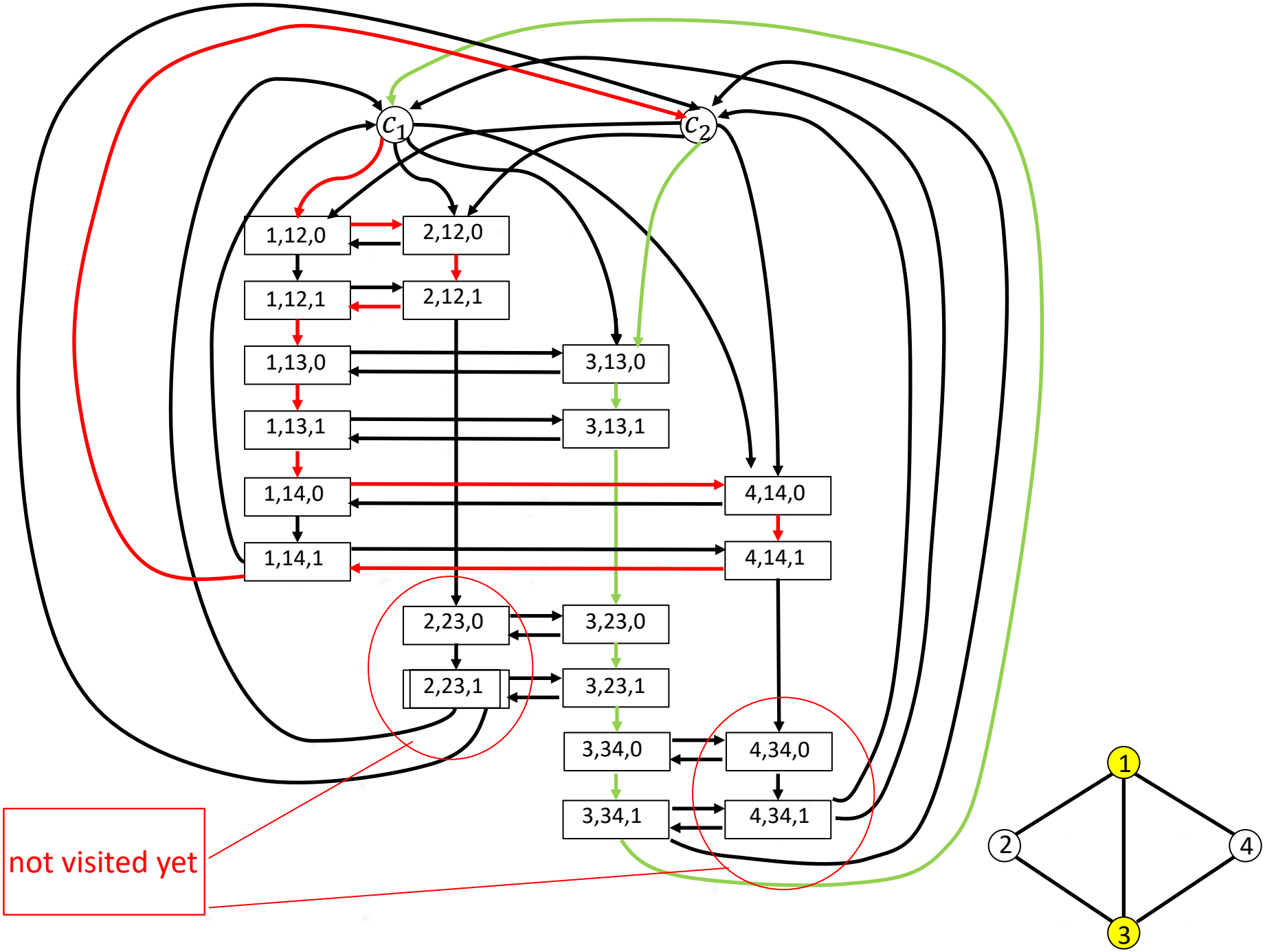
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3
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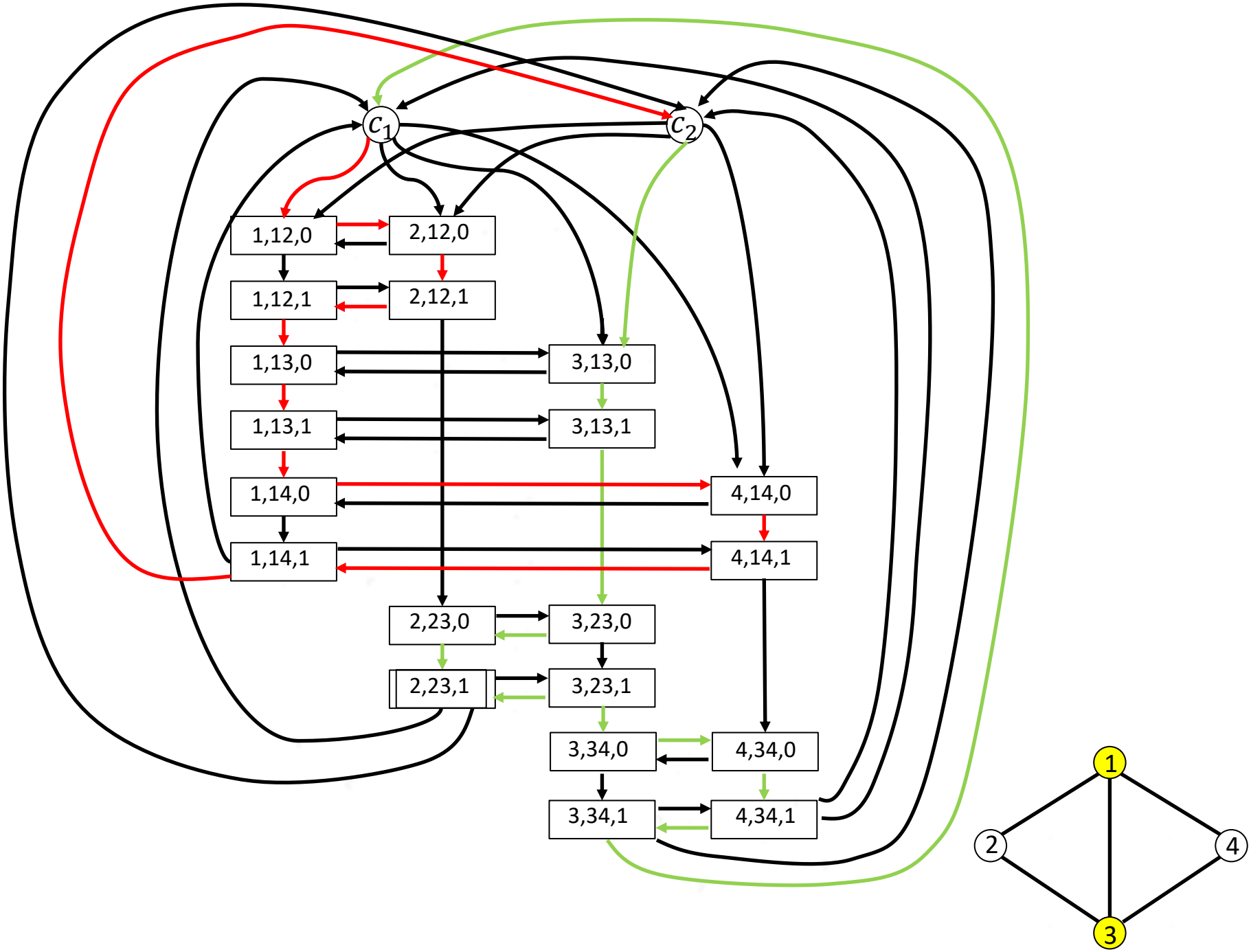
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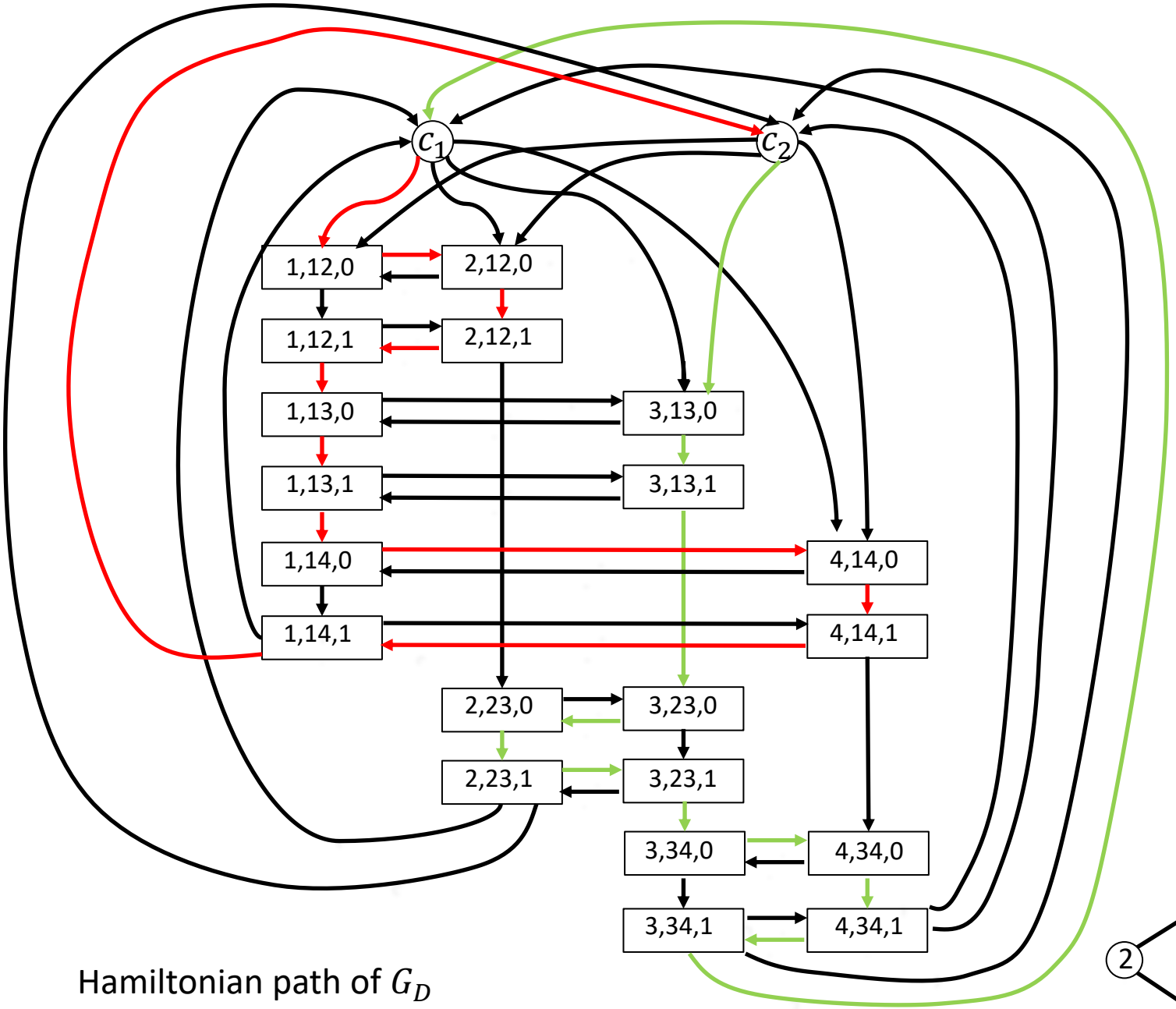

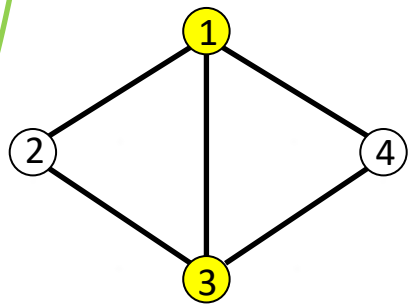

Hamiltonian path of G_D corresponding to vertex cover {1,3} of G.