CSC 411: Lecture 18: Ensemble Methods II

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Today

- Random/Decision Forest
- Mixture of Experts

What are the base classifiers?

- Popular choices of base classifier for boosting and other ensemble methods:
  - Linear classifiers
  - Decision trees

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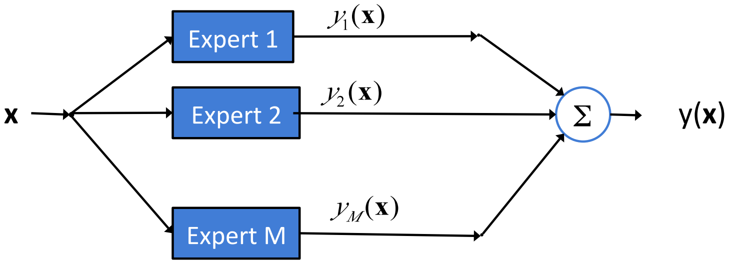
  - Aggregate the predictions of each tree to make classification decision (e.g., can choose mode vote)

Ensemble Learning: Boosting and Bagging

- Experts cooperate to predict output



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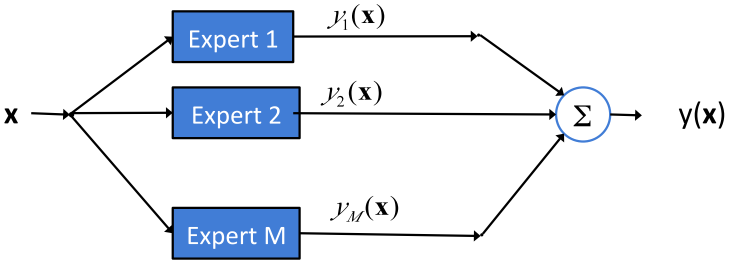
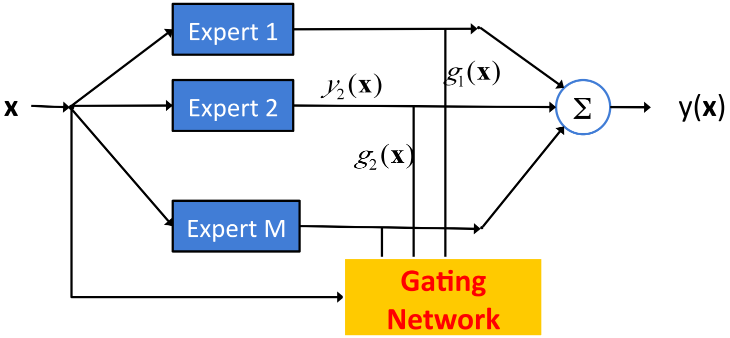


- Vote of each expert has consistent weight for each test example

$$y(\mathbf{x}) = \sum_m g_m y_m(\mathbf{x})$$

Mixture of Experts

- Weight of each expert is not constant – depends on input **x**



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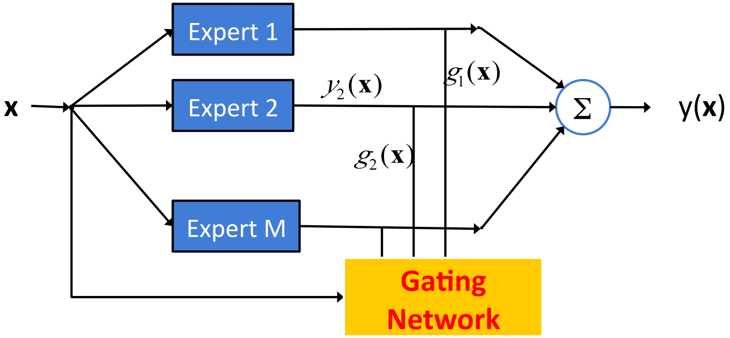

- Gating network encourages specialization (local experts) instead of cooperation

$$y(\mathbf{x}) = \sum_m g_m(\mathbf{x}) y_m(\mathbf{x})$$

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3. Allow each expert to produce distribution over outputs

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  - if its estimate for t is too low, and the average of other models is too high, then model m encouraged to lower its prediction

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- We want to estimate the parameters of the gating as well as the classifier y_m

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- For gating network, increase weight on expert when its error is less than average error of experts

$$\begin{aligned}
\frac{\partial E}{\partial y_m} &= \frac{1}{M} g_m(\mathbf{x})(t - y_m(\mathbf{x})) \\
\frac{\partial E}{\partial z_m} &= \frac{1}{M} g_m(\mathbf{x}) \left[(t - y_m(\mathbf{x}))^2 - E\right]
\end{aligned}$$

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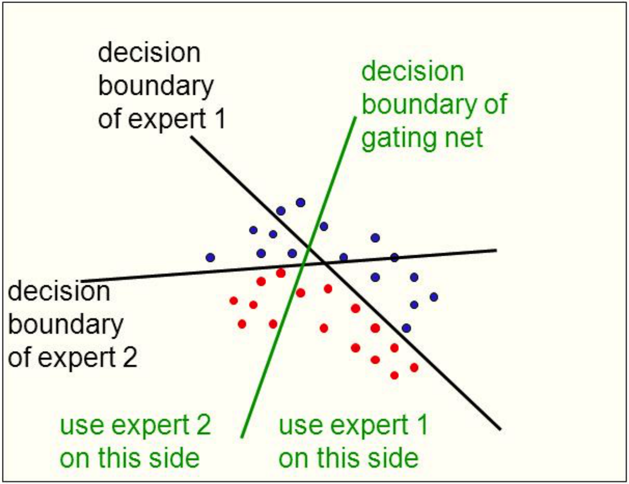
- Gradient: Error weighted by posterior probability of the expert

$$\frac{\partial E}{\partial y_m} = -2\frac{g_m(\mathbf{x}) \exp\left(-\frac{1}{2}||t - y_m(\mathbf{x})||^2\right)}{\sum_i g_i(\mathbf{x}) \exp\left(-\frac{1}{2}||t - y_i(\mathbf{x})||^2\right)}(t - y_m(x))$$

Mixture of Experts: Example



[Slide credit: G. Hinton]

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- Notes:
  - Differ in: training strategy; selection of examples; weighting of components in final classifier