CSC 411: Lecture 19: Reinforcement Learning

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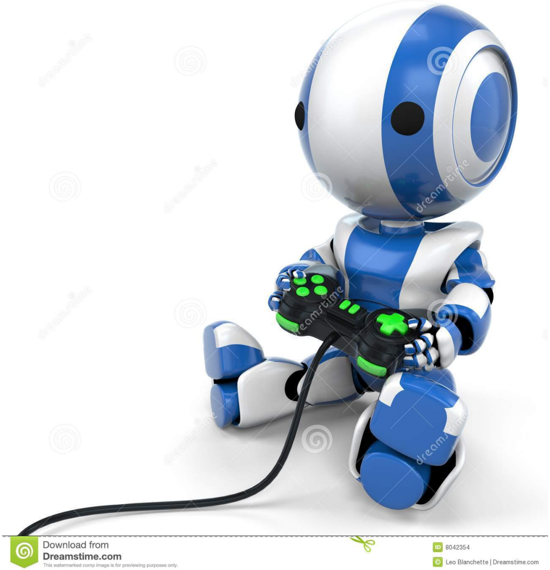
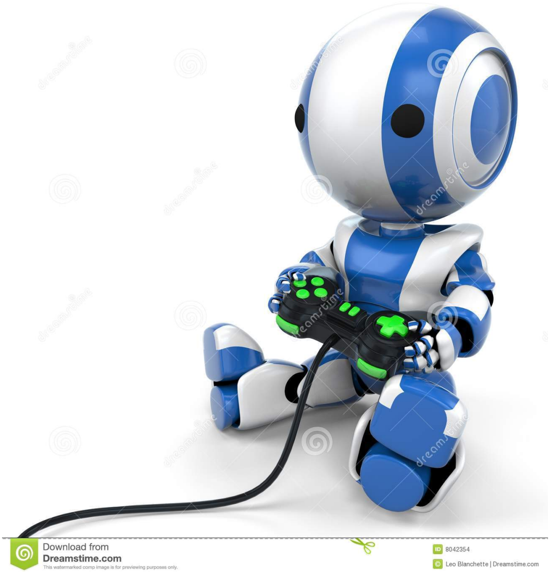
Today

- Learn to play games
- Reinforcement Learning

Playing Games: Atari



Playing Games: Super Mario



Reinforcement Learning Resources

- RL tutorial – on course website
- *Reinforcement Learning: An Introduction*, Sutton & Barto Book (1998)

Reinforcement Learning

- Learning algorithms differ in the information available to learner
  - Supervised: correct outputs
  - Unsupervised: no feedback, must construct measure of good output
  - Reinforcement learning
- More realistic learning scenario:
  - Continuous stream of input information, and actions
  - Effects of action depend on state of the world
  - Obtain reward that depends on world state and actions
    - not correct response, just some feedback

Formulating Reinforcement Learning

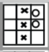
- World described by a discrete, finite set of states and actions
- At every time step t, we are in a state s_t, and we:
  - Take an action a_t (possibly null action)
  - Receive some reward r_{t+1}
  - Move into a new state s_{t+1}
- Decisions can be described by a policy
  - a selection of which action to take, based on the current state
- Aim is to maximize the total reward we receive over time
- Sometimes a future reward is discounted by γ_{k-1}, where k is the number of time-steps in the future when it is received

Tic-Tac-Toe

- Make this concrete by considering specific example
- Consider the game tic-tac-toe:
  - reward: win/lose/tie the game ($+1/-1/0$) [only at final move in given game]
  - state: positions of X's and O's on the board
  - policy: mapping from states to actions
    - based on rules of game: choice of one open position
  - value function: prediction of reward in future, based on current state
- In tic-tac-toe, since state space is tractable, can use a table to represent value function

RL & Tic-Tac-Toe

- Each board position (taking into account symmetry) has some probability

State	Probability of a win (Computer plays "o")
	0.5
	0.5
	1.0
	0.0
	0.5
etc	

- Simple learning process:
  - start with all values $= 0.5$
  - policy: choose move with highest probability of winning given current legal moves from current state
  - update entries in table based on outcome of each game
  - After many games value function will represent true probability of winning from each state
- Can try alternative policy: sometimes select moves randomly (exploration)

Acting Under Uncertainty

- The world and the actor may not be deterministic, or our model of the world may be incomplete
- We assume the Markov property: the future depends on the past only through the current state
- We describe the environment by a distribution over rewards and state transitions:

$$P(s_{t+1} = s', r_{t+1} = r' | s_t = s, a_t = a)$$

- The policy can also be non-deterministic:

$$P(a_t = a | s_t = s)$$

- Policy is not a fixed sequence of actions, but instead a conditional plan

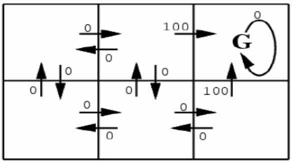
Basic Problems

- Markov Decision Problem (MDP): tuple (S, A, P, γ) where P is

$$P(s_{t+1} = s', r_{t+1} = r' | s_t = s, a_t = a)$$

- Standard MDP problems:
  1. Planning: given complete Markov decision problem as input, compute policy with optimal expected return
  2. Learning: Only have access to experience in the MDP, learn a near-optimal strategy

Example of Standard MDP Problem



$r(s, a)$ (immediate reward)

1. Planning: given complete Markov decision problem as input, compute policy with optimal expected return
2. Learning: Only have access to experience in the MDP, learn a near-optimal strategy

We will focus on learning, but discuss planning along the way

Exploration vs. Exploitation

- If we knew how the world works (embodied in P), then the policy should be deterministic
  - just select optimal action in each state
- But if we do not have complete knowledge of the world, taking what appears to be the optimal action may prevent us from finding better states/actions
- Interesting trade-off:
  - immediate reward (exploitation) vs. gaining knowledge that might enable higher future reward (exploration)

MDP Formulation

- Goal: find policy π that maximizes expected accumulated future rewards $V^{\pi}(s_t)$, obtained by following π from state s_t:

$$\begin{aligned} V^{\pi}(s_t) &= r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots \\ &= \sum_{i=0}^{\infty} \gamma^i r_{t+i} \end{aligned}$$

- Game show example:
  - assume series of questions, increasingly difficult, but increasing payoff
  - choice: accept accumulated earnings and quit; or continue and risk losing everything

What to Learn

- We might try to learn the function V (which we write as V^*)

$$V^*(s) = \max_a \left[r(s, a) + \gamma V^*(\delta(s, a))\right]$$

- We could then do a lookahead search to choose best action from any state s:

$$\pi^*(s) = arg \max_a \left[r(s, a) + \gamma V^*(\delta(s, a))\right]$$

- But there's a problem:
  - This works well if we know $\delta()$ and $r()$
  - But when we don't, we cannot choose actions this way

Q Learning

- Define a new function very similar to V^*

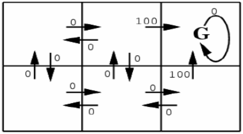
$$Q(s,a) = r(s,a) + \gamma V^*(\delta(s,a))$$

- If we learn Q, we can choose the optimal action even without knowing δ!

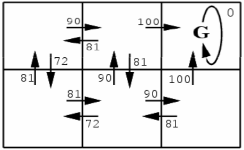
$$\begin{aligned} \pi^*(s) &= \mathit{arg}\max_a [r(s,a) + \gamma V^*(\delta(s,a))] \\ &= \mathit{arg}\max_a Q(s,a) \end{aligned}$$

- Q is then the evaluation function we will learn

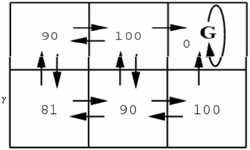
$\gamma = 0.9$



$r(s, a)$ (immediate reward) values

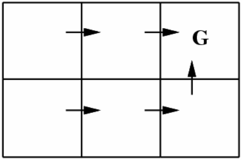


$Q(s, a)$ values



$V^*(s)$ values

$$V^*(s_5) = 0 + \gamma 100 + \gamma^2 0 + \ldots = 90$$



One optimal policy

Training Rule to Learn Q

- Q and V^* are closely related:

$$V^*(s) = \max_a Q(s, a)$$

- So we can write Q recursively:

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t)) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

- Let $\hat{Q}$ denote the learner's current approximation to Q
- Consider training rule

$$\hat{Q}(s, a) \leftarrow r(s, a) + \gamma \max_{a'} \hat{Q}(s', a')$$

where s' is state resulting from applying action a in state s

Q Learning for Deterministic World

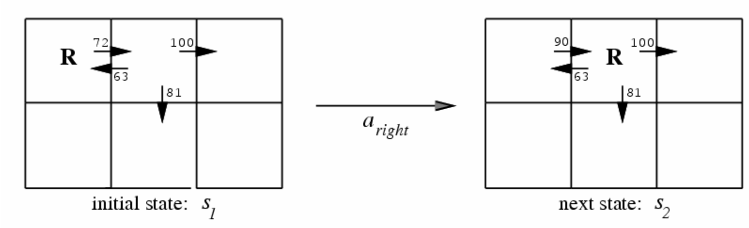
- For each s, a initialize table entry $\hat{Q}(s, a) \leftarrow 0$
- Start in some initial state s
- Do forever:
  - Select an action a and execute it
  - Receive immediate reward r
  - Observe the new state s'
  - Update the table entry for $\hat{Q}(s, a)$ using Q learning rule:

$$\hat{Q}(s, a) \leftarrow r(s, a) + \gamma \max_{a'} \hat{Q}(s', a')$$

  - $s \leftarrow s'$
- If we get to absorbing state, restart to initial state, and run thru "Do forever" loop until reach absorbing state

Updating Estimated Q

- Assume the robot is in state s_1; some of its current estimates of Q are as shown; executes rightward move



$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$
$$\leftarrow r + 0.9 \max_{a}\{63, 81, 100\} \leftarrow 90$$

- Notice that if rewards are non-negative, then $\hat{Q}$ values only increase from 0, approach true Q

Q Learning: Summary

- Training set consists of series of intervals (episodes): sequence of (state, action, reward) triples, end at absorbing state
- Each executed action a results in transition from state s_i to s_j; algorithm updates $\hat{Q}(s_i, a)$ using the learning rule
- Intuition for simple grid world, reward only upon entering goal state $\rightarrow Q$ estimates improve from goal state back
  1. All $\hat{Q}(s, a)$ start at 0
  2. First episode – only update $\hat{Q}(s, a)$ for transition leading to goal state
  3. Next episode – if go thru this next-to-last transition, will update $\hat{Q}(s, a)$ another step back
  4. Eventually propagate information from transitions with non-zero reward throughout state-action space

Q Learning: Exploration/Exploitation

- Have not specified how actions chosen (during learning)
- Can choose actions to maximize $\hat{Q}(s, a)$
- Good idea?
- Can instead employ stochastic action selection (policy):

$$P(a_i|s) = \frac{\exp(k\hat{Q}(s, a_i))}{\sum_j \exp(k\hat{Q}(s, a_j))}$$

- Can vary k during learning
  - more exploration early on, shift towards exploitation

Non-deterministic Case

- What if reward and next state are non-deterministic?
- We redefine V, Q based on probabilistic estimates, expected values of them:

$$\begin{aligned} V^{\pi}(s) &= E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots] \\ &= E[\sum_{i=0}^{\infty} \gamma^i r_{t+i}] \end{aligned}$$

and

$$\begin{aligned} Q(s,a) &= E[r(s,a) + \gamma V^*(\delta(s,a))] \\ &= E[r(s,a) + \gamma \sum_{s'} p(s'|s,a) \max_{a'} Q(s',a')] \end{aligned}$$

Nondeterministic Case: Learning Q

- Training rule does not converge (can keep changing $\hat{Q}$ even if initialized to true Q values)
- So modify training rule to change more slowly

$$\hat{Q}(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')]$$

where s' is the state land in after s, and a' indexes the actions that can be taken in state s'

$$\alpha_n = \frac{1}{1 + \text{visits}_n(s,a)}$$

where visits is the number of times action a is taken in state s