CS 448/2405
Automata and Formal Languages
ASSIGNMENT # 3
DUE DATE: Monday, April 3, 2006

The due date for the extra credit problem is Monday April 10, 2006. All other problems are due on April 3, 2006.

1. Which of the following sets are countable and which are uncountable? Justify your answer. (If countable, provide a one-to-one mapping from the set to the natural numbers, and if uncountable, prove it using diagonalization.)
   (a) The set of square matrices over $\{0, 1\}$.
   (b) The set of all functions from $\{0, 1\}^*$ to $\{0, 1\}^*$.

2. Prove that A is Turing-recognizable (r.e.) if and only if A is mapping reducible to A_{TM}.

3. Prove that A is decidable (recursive) if and only if A is mapping reducible to the language 0^*1^*.

4. Prove that a language L over $\{0, 1\}$ is recursive if and only if there is a program (algorithm) that enumerates exactly the strings in L in lexicographic order. (By this I mean that the program outputs an infinite sequence of 0's and 1's, where the ith bit is 1 if and only if the lexicographically ith string is in the language, and otherwise the ith bit is 0.)

5. Classify each of the following languages as either: (i) recursive, (ii) recursively enumerable but not recursive, or (iii) not recursively enumerable. Prove your answers. You may not appeal to Rice's theorem.
   a. Given the encoding of a Turing Machine M, is there a string of every even length for which M halts?
   b. Given the encoding of a Turing Machine, M, is the size of the language accepted by M divisible by 3?
   c. Given the encoding of a Turing Machine M, and input w, does M on input w ever move its tape head to the left at least two times consecutively?
   d. Given the encoding of a Turing Machine M, a state q and a string w, does M ever reach state q when run on input w?
   e. Given a CFL L (described by a context free grammar), is the complement of L also a context free language?

6. (Extra Credit) Show that for a context free language L, it is undecidable whether $L = L^R$.