CS 448/2405
Automata and Formal Languages
ASSIGNMENT # 2
DUE DATE: Monday, March 6

1. Give PDA's for the following languages.

   a. The set of all strings over $\{0, 1\}$ with exactly twice as many 0's as 1's.

   b. The set of all strings over $\{a, b, c\}$ of the form $a^i b^j c^k$ such that either $i \neq j$ or $j \neq k$.

2. Give context-free grammars generating the following languages.

   a. The set of all strings over $\{a, b\}$ not of the form ww for some string $w \in \{a, b\}^*$.

   b. The complement of the language $\{a^n b^n \mid n \geq 0\}$

3. Are the following languages context free? Prove or disprove your answer.

   a. $\{a^i b^j \mid j = i^2\}$

   b. $\{a^i \mid i \text{ is a prime}\}$

   c. $\{w\#x \mid w \text{ is a substring of } x, \text{ where } w, x \in \{a, b\}^*\}$

4. Prove that if L is a context-free language over a one-symbol alphabet, then L is regular.

5. Prove that the class of context-free languages are not closed under complementation.

6. (Hard) Let $D = \{xy \mid x, y \in \{0, 1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}$. Show that D is a context-free language.

7. (Hard) Let $PRIME$ be the set of strings over $\{0, 1\}$ that are prime numbers in binary notation. Prove that $PRIME$ is not a context-free language.