CS 448/2405
Automata and Formal Languages
ASSIGNMENT # 2
DUE DATE: Friday, February 25, 2005

1. Give context free grammars generating the following languages.

   a. The set of all strings over $\{0, 1\}$ with exactly twice as many 0's as 1's.

   b. The set of all strings over $\{a, b, c\}$ of the form $a^i b^j c^k$ such that either $i \neq j$ or $j \neq k$.

   c. The set of all strings over $\{a, b\}$ not of the form ww for some string $w \in \{a, b\}^*$.

2. Let G be the grammar

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

   Prove that the language generated by the above grammer consists of strings x such that each prefix of x has at least as many a's as b's.

3. Are the following languages context free? Prove or disprove your answer.

   a. $\{a^i b^j \mid i \neq j \text{ and} i \neq 3j\}$

   b. $\{a^i b^j \mid j = i^2\}$

   c. $\{a^i \mid i \text{ is a prime}\}$

4. Prove that if L is a context-free language over a one-symbol alphabet, then L is regular.

5. Prove that the class of context-free languages are not closed under complementation.

6. Sipser, problem 2.17.