CS 448/2405
Automata and Formal Languages
ASSIGNMENT # 1
DUE DATE: Friday, January 28, 2005

1. Give deterministic finite automata accepting the following languages over $\{0,1\}$. Prove that the automata that you define accepts the language in question.
   a. The set of all strings with at most one pair of consecutive 0's and at most one pair of consecutive 1's.
   b. The set of all strings with an equal number of 0's and 1's such that no prefix has two more 0's than 1's nor two more 1's than 0's.
   c. The set of all strings with at least three symbols such that the third symbol from the right end is 1.

2. Are the following languages regular? Prove or disprove your answer.
   a. The set of all strings over $\Sigma = \{a, b\}$ with an odd number of a's and an even number of b's.
   b. The complement of $\{0^n 1^n \mid n \geq 0\}$.
   c. The set of all strings of the form $0^i 1^j$ such that $gcd(i, j) = 1$.

3. Sipser, Problem 1.16
4. Sipser, Problem 1.26
5. Sipser, Problem 1.44
6. Let L be any subset of 0^*. Prove that L^* is regular.
7. This question concerns the minimization algorithm discussed in class.
   a. Prove that there exists a constant $c > 0$ such that the algorithm requires time greater than cn^2 for infinitely many DFA where n is the number of states and the input alphabet has two symbols.
   b. (Extra Credit) Give an improved algorithm for minimizing states in a DFA with smaller runtime. In particular try to get runtime $O(|\Sigma| n \log n)$ where $|\Sigma|$ is the size of the input alphabet, Σ. Hint: instead of asking for each pair of states (p, q) and each input a if $\delta(p, a)$ and $\delta(q, a)$ are distinguishable, partition the states into final and nonfinal states. Then refine the partition by considering all states whose next state under some input symbol is in one particular block of the partition. Each time a block is partitioned, refine the partition further by using the smaller subblock. Use list processing to make the algorithm as efficient as possible.