CS137 Discrete Mathematics and its Applications 2

Coursework 3

Due by Monday 25 February 2013 at 12noon

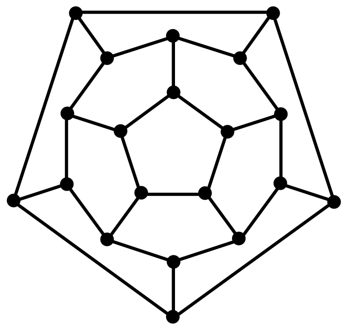
Submit with an appropriate coversheet to a collection box in CS0.06

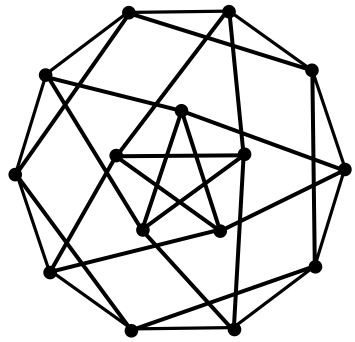
Attempt to solve **ALL SEVEN** of the following problems.
Submit a solution to **THREE** of the seven problems, **ONE** from **EACH GROUP**.

Group 1

1. (a) Draw all pairwise non-isomorphic connected graphs with 4 edges.

   (b) A cycle is *Hamiltonian* if it contains all vertices of the graph. Find an Euler circuit and a Hamiltonian cycle in the following graphs. If it does not exist, explain why.





2. A walk with endpoints u, v is a uv-walk. A path with endpoints u, v is a uv-path. Let G be a graph. Prove that

   (a) G contains a uv-walk if and only if G contains a uv-path.

   (b) G contains a closed walk of odd length if and only if G contains a cycle of odd length.

Group 2

1. Reconstruct the trees from their Prüfer codes: $(3, 3, 5, 5, 6, 6)$, $(1, 5, 1, 5, 9, 8, 2)$, $(1, 5, 2, 2, 1, 5, 5)$

   Let i be a positive integer. What tree has the Prüfer code

   (a) $(i, i, \ldots, i)$?

   (b) $(i-2, i-3, \ldots, 1)$?

2. Let T be a tree with n vertices, k leaves, and no vertex of degree 2.

   (a) Prove that $k \geq (n+2)/2$.

   (b) What does T look like if $k = (n+2)/2$?

3. Let G be a graph with n vertices, m edges, and k connected components. Prove that

$$n - k \leq m \leq \binom{n-k+1}{2}$$

Group 3

1. Let G be a graph m edges and chromatic number $\chi(G) = k$. Prove that $m \geq \binom{k}{2}$.

2. Let G be a graph with $n \geq 11$ vertices. Prove that G and $\overline{G}$ cannot be both planar. (Hint: use Euler's formula)