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A new DEA method for supplier selection in presence of both cardinal and ordinal data

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ABSTRACT

The success of a supply chain is highly dependent on selection of best suppliers. These decisions are an important component of production and logistics management for many firms. Little attention is given in the literature to the simultaneous consideration of cardinal and ordinal data in supplier selection process. This paper proposes a new integrated data envelopment analysis (DEA) model which is able to identify most efficient supplier in presence of both cardinal and ordinal data. Then, utilizing this model, an innovative method for prioritizing suppliers by considering multiple criteria is proposed. As an advantage, our method identifies best supplier by solving only one mixed integer linear programming (MILP). Applicability of proposed method is indicated by using data set includes specifications of 18 suppliers.

1. Introduction

Competitive advantages associated with supply chain management (SCM) philosophy can be achieved by strategic collaboration with suppliers and service providers. The success of a supply chain is highly dependent on selection of good suppliers (Ng, 2008). Supplier selection involves the need to trade-off multiple criteria, as well as the presence of both quantitative and qualitative data (Wu, 2009). To manage this strategically important purchasing function effectively, appropriate method and criteria have to be chosen for the problem (Guneri, Yucel, & Ayyildiz, 2009).

Over the years, several techniques have been developed to solve the problem efficiently. Analytic hierarchy process (AHP), analytic network process (ANP), linear programming (LP), mathematical programming, multi-objective programming, data envelopment analysis (DEA), neural networks (NN), case-based reasoning (CBR) and fuzzy set theory (FST) methods have been applied in literature (Guneri et al., 2009). Using DEA, this paper proposes a model for supplier selection.

Traditionally, supplier selection models are based on cardinal data with less emphasis on ordinal data. However, with the widespread use of manufacturing philosophies such as just-in-time (JIT), emphasis has shifted to the simultaneous consideration of cardinal and ordinal data in supplier selection process (Farzipoor Saen, 2007). The main contribution of this paper is to propose a new integrated DEA model for finding most efficient supplier by considering both cardinal and ordinal data. In addition, by using this model, a method is presented for ranking suppliers with cardinal and ordinal data.

DEA is a widely recognized approach for evaluating the efficiencies of decision making units (DMUs). Because of its easy and successful application and case studies, DEA has gained too much attention and widespread use by business and academy researchers. Selection of best vendors (Liu, Ding, & Lall, 2000; Weber, Current, & Desai, 1998), evaluation of data warehouse operations (Mannino, Hong, & Choi, 2008), selection of flexible manufacturing system (Liu, 2008), assessment of bank branch performance (Camanho & Dyson, 2005), examining bank efficiency (Chen, Skully, & Brown, 2005), analyzing firm's financial statements (Edirisinghe & Zhang, 2007), measuring the efficiency of higher education institutions (Johnes, 2006), solving facility layout design (FLD) problem (Ertay, Ruan, & Tuzkaya, 2006) and measuring the efficiency of organizational investments in information technology (Shafer & Byrd, 2000) are samples of using DEA in various areas.

The organization of this paper is as follows. Section 2 reviews previous studies in supplier selection and Section 3 introduces previous related DEA models. In Section 4, a new DEA model is proposed which is able to find most efficient unit with imprecise data. Using this model, Section 5 presents a method for ranking DMUs by simultaneously considering cardinal and ordinal data. Section 6 illustrates application of proposed method. Finally, paper closes with some concluding remarks in Section 7.

2. Literature review

In previous studies, various methods have been proposed for supplier selection. For instance, Weber et al. (1998) described three approaches for selection and negotiation with vendors who were

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not selected. Furthermore, they explained how in certain situations two multi-criteria analysis tools, multi-objective programming and DEA, can be used together for this selection and negotiation process. Karpak, Kumcu, and Kasuganti (2001) presented one of the "user friendly" multiple criteria decision support systems-visual interactive goal programming (VIG). VIG facilitates the introduction of a decision support vehicle that helps improve the supplier selection decisions. Talluri and Baker (2002) presented a multi-phase mathematical programming approach for designing effective supply chain. It should be noted that their method develops and applies a combination of multi-criteria efficiency models, based on game theory concepts, and linear and integer programming methods. Kumar, Vrat, and Shankar (2004) proposed a fuzzy goal programming approach for vendor selection with multiple objectives with some fuzzy parameters. They formulated vendor selection problem as a fuzzy mixed integer goal programming vendor selection problem that includes three primary goals: minimizing the net cost, minimizing the net rejections, and minimizing the net late deliveries subject to realistic constraints regarding buyer's demand, vendors' capacity, vendors' quota flexibility, purchase value of items, budget allocation to individual vendor, etc. Liu and Hai (2005) compared the weighted sum of the selection number of rank vote, after determining the weights in a selected rank. They presented a novel weighting procedure in place of pairwise comparison of AHP for selecting suppliers. They provided a simpler method than AHP that is called voting analytic hierarchy process, but which do not lose the systematic approach of deriving the weights to be used and for scoring the performance of suppliers. Chang, Wang, and Wang (2006) proposed a fuzzy multiple attribute decision making (FMADM) method based on the fuzzy linguistic quantifier to satisfy the current product competition strategies, and also improve the effectiveness and efficiency of the entire supply chain.

Chen, Lin, and Huang (2006) developed a fuzzy decision-making approach for the supplier selection problem in supply chain system. In their method, they used linguistic values to assess the ratings and weights for criteria. These linguistic ratings can be expressed in trapezoidal or triangular fuzzy numbers. Then, a hierarchy multiple criteria decision-making (MCDM) model based on fuzzy-sets theory is proposed to deal with the supplier selection problems in the supply chain system. Finally, they showed applicability of their method in a high-technology manufacturing company. Gencer and Gurpinar (2007) used ANP and proposed a model for supplier selection. Their method include seven steps as follows: analysis of supplier selection problem, determining the goal and supplier selection criteria, determining the alternative suppliers, identification of the network structure and relationships, making the paired comparisons, building the supermatrix and finding the limiting priorities. Finally, they implemented their method in an electronic company. Xia and Wu (2007) developed an integrated approach of AHP improved by rough sets theory and multi-objective mixed integer programming to simultaneously determine the number of suppliers to employ and the order quantity allocated to these suppliers in the case of multiple sourcing, multiple products, with multiple criteria and with supplier's capacity constraints.

Önüt, Kara, and Isik (2009) proposed a supplier evaluation approach based on the analytic network process (ANP) and the technique for order performance by similarity to ideal solution (TOPSIS) methods to help a telecommunication company in the GSM sector in Turkey under the fuzzy environment. They used triangular fuzzy numbers in all pairwise comparison matrices in their method to evaluating suppliers by considering six criteria (cost, reference, quality of product, delivery time, institutionality and execution time). Demirtas and Üstün (2008) combined analytic network process (ANP) and multi-objective mixed integer linear programming (MOMILP) and proposed an approach for selecting best suppliers and defining the optimum quantities among selected suppliers to maximize the total value of purchasing and minimize the budget and defect rate. Wang, Cheng, and Huang (2009) proposed a fuzzy hierarchical TOPSIS method, which not only is well suited for evaluating fuzziness and uncertainty problems, but also can provide more objective and accurate criterion weights, while simultaneously avoiding the problem of its previous Fuzzy TOPSIS method. Ng (2008) proposed a weighted linear program for the multi-criteria supplier selection problem. Furthermore, he studied a transformation technique which enables our proposed model to be solved without an optimizer. Guneri et al. (2009) proposed an integrated fuzzy and linear programming approach for supplier selection problem. Their approach, firstly, assesses weights and ratings of supplier selection criteria with linguistic values expressed in trapezoidal fuzzy numbers. Then a hierarchy multiple model based on fuzzy set theory is expressed and fuzzy positive and negative ideal solutions are used to find each supplier's closeness coefficient. Finally, a linear programming model based on the coefficients of suppliers, buyer's budgeting, suppliers' quality and capacity constraints is developed and order quantities assigned to each supplier according to the linear programming model. Wu (2009) used grey related analysis and Dempster–Shafer theory to deal supplier selection in a fuzzy group decision making problem. It is to be noted that proposed approach uses both quantitative and qualitative data for international supplier selection.

Farzipoor Saen (2007) proposed an innovative method for selecting suppliers in conditions that both ordinal and cardinal data are present (without relying on weight assignment by decision makers). His method identifies best suppliers whose efficiency score is equal to one and is not able to find most efficient supplier. Indeed, by using his method, decision maker cannot decide which supplier is the best among other units.

Investigation of previous related works shows that identifying the best supplier whit imprecise data has gained less attention. This paper tries to fill the gap by proposing a DEA model which is able to find most efficient supplier by considering both cardinal and ordinal data. Moreover, by using this model, an innovative method for ranking suppliers is presented. In the next section, previous related DEA models are explained.

3. DEA models

Performance evaluation is an important task for a DMU to find its weaknesses so that subsequent improvements can be made. Since the pioneering work of Charnes, Cooper, and Rhodes (1978), DEA has demonstrated to be an effective technique for measuring the relative efficiency of a set of DMUs which utilize the same inputs to produce the same outputs.

Assume that there are n DMUs, (DMU_j: $j = 1, 2, \ldots, n$) which consume m inputs ($\mathbf{x}_i : i = 1, 2, \ldots, m$) to produce s outputs ($\mathbf{y}_r : r = 1, 2, \ldots, s$). The CCR input oriented (CCR-I) model evaluates the efficiency of DMU_o, DMU under consideration, by solving the following linear program:

$$
\begin{aligned}
&\max \sum_{r=1}^{s} u_r y_{rj} \\
&\text{s.t.} \\
&\sum_{i=1}^{m} w_i x_{io} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0 \quad j = 1, 2, \ldots, n \\
&w_i \geqslant \varepsilon \quad i = 1, 2, \ldots, m \\
&u_r \geqslant \varepsilon \quad r = 1, 2, \ldots, s
\end{aligned}
\tag{1}
$$

where x_{ij} and y_{rj} (all nonnegative) are the inputs and outputs of the DMU_j, w_i and u_r are the input and output weights (also referred to as multipliers). x_{io} and y_{ro} are the inputs and outputs of DMU_o. Also, ε is non-Archimedean infinitesimal value for forestalling weights to be equal to zero. The CCR-I model must be run n times, once for each unit, to get the relative efficiency of all DMUs. The envelopment in CCR is constant returns to scale meaning that a proportional increase in inputs results in a proportionate increase in outputs. Banker, Charnes, and Cooper (1984) developed the BCC model to estimate the pure technical efficiency of decision making units with reference to the efficient frontier. It also identifies whether a DMU is operating in increasing, decreasing or constant returns to scale. So CCR models are a specific type of BCC models.

New applications with more variables and more complicated models are being introduced (Emrouznejad, Tavares, & Parker, 2007). In many applications of DEA, finding the most efficient DMU is desirable. Amin and Toloo (2007) proposed an integrated model for finding most efficient DMU, as follows:

$$
\begin{aligned}
& M^* = \min M \\
& \text{s.t} \\
& M - d_j \geqslant 0 \quad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \leqslant 1 \quad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0 \quad j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} d_j = n-1 \\
& 0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0,1\} \quad j = 1,2,\ldots n \\
& w_i \geqslant \varepsilon \quad i = 1,2,\ldots,m \\
& u_r \geqslant \varepsilon \quad r = 1,2,\ldots,s
\end{aligned}
\tag{2}
$$

where d_j as a binary variable represents the deviation variable of DMU_j. β_j is considered in the Model (2) because of discrete nature of d_j and M represents maximum inefficiency which should be minimized. DMU_j is most efficient if and only if $d_j = 0$.

First constraint implies that M is equal to maximum inefficiency. Second constraint shows input-oriented nature of the Model (2). Third constraint causes efficiency of all units to be less that 1. The last one implies among all the DMUs for only most efficient unit, say DMU_p, which has $d_p^* = 0$ in any optimal solution. In addition, to determine the non-Archimedean epsilon, Amin and Toloo (2007) developed an epsilon model.

It should be noted that Model (2) is based on CCR model and identify most CCR-efficient DMU. Indeed, Model (2) is not applicable for situations in which DMUs operating in variable return to scale. To overcome this drawback, Toloo and Nalchigar (2009) proposed an integrated model which is able to find most BCC-efficient DMU. These DEA models are applicable in situations in which data of DMUs is precise. In the next section, a new DEA model is proposed which is able to find most efficient DMU while considering imprecise data.

4. Proposed model

The conventional DEA models make an assumption that input and output data are exact values on a ratio scale. Recently, Cooper, Park, and Yu (1999) addressed the problem of imprecise data in DEA, in its general form. The term "imprecise data" reflects the situation where some of the input and output data are only known to lie within bounded intervals (interval numbers) while other data are known only up to an order (Despotis & Smirlis, 2002). If imprecise data information incorporated into the original linear CCR model, the resulting DEA model is a non-linear and non-convex program, and is called imprecise DEA (IDEA). According to Cooper et al. (1999) and Kim, Park, and Park (1999) imprecise data are in following forms:

4.1. Bounded data

$$\underline{y}_{rj} \leqslant y_{rj} \leqslant \bar{y}_{rj} \text{ and } \underline{x}_{ij} \leqslant x_{ij} \leqslant \bar{x}_{ij} \text{ for } r \in \text{BO}, \quad i \in \text{BI} \tag{3}$$

where $\underline{y}_{rj}$ and $\bar{y}_{rj}$ are the lower and the upper bounds for outputs, $\underline{x}_{ij}$ and $\bar{x}_{ij}$ are the lower and the upper bounds for inputs, and BO and BI represent the associated sets containing bounded outputs and inputs, respectively.

4.2. Weak ordinal data

$$y_{rj} \leqslant y_{rk} \text{ and } x_{ij} \leqslant x_{ik} \text{ for } j \neq k, \; r \in \text{DO}, \quad i \in \text{DI}$$

or,

$$y_{r1} \leqslant y_{r2} \leqslant \cdots \leqslant y_{rk} \leqslant \cdots \leqslant y_{rn} \; (r \in \text{DO}), \tag{4}$$

$$x_{i1} \leqslant x_{i2} \leqslant \cdots \leqslant x_{ik} \leqslant \cdots \leqslant x_{in} \; (i \in \text{DI}), \tag{5}$$

where DO and DI represent the associated sets containing weak ordinal outputs and inputs, respectively.

4.3. Strong ordinal data

Strong ordinal data is subset of weak ordinal data, as follows:

$$y_{rj} < y_{rk} \text{ and } x_{ij} < x_{ik} \text{ for } j \neq k, \quad r \in \text{DO}, \quad i \in \text{DI}$$

or,

$$y_{r1} < y_{r2} < \cdots < y_{rk} < \cdots < y_{rn} \; (r \in \text{SO}), \tag{6}$$

$$x_{i1} < x_{i2} < \cdots < x_{ik} < \cdots < x_{in} \; (i \in \text{SI}), \tag{7}$$

where SO and SI represent the associated sets containing strong ordinal outputs and inputs, respectively.

4.4. Ratio bounded data

$$L_{rj} \leqslant \frac{y_{rj}}{y_{rj_o}} \leqslant U_{rj} \quad (j \neq j_o) \; (r \in \text{RO}) \tag{8}$$

$$G_{ij} \leqslant \frac{x_{rj}}{x_{ij_o}} \leqslant H_{ij} \quad (j \neq j_o) \; (i \in \text{RI}) \tag{9}$$

where L_{rj} and G_{ij} represent the lower bounds, U_{rj} and H_{ij} represent the upper bounds, and RO and RI represent the associated sets containing ratio bounded outputs and inputs, respectively.

By incorporating Eqs. (3)–(9) added to Model (2), there will be

$$
\begin{aligned}
& M^* = \min M \\
& \text{s.t} \\
& M - d_j \geqslant 0 \quad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \leqslant 1 \quad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0 \quad j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} d_j = n-1 \\
& 0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0,1\} \quad j = 1,2,\ldots,n \\
& (x_{ij}) \in \Theta_i^- \\
& (y_{rj}) \in \Theta_r^+ \\
& w_i \geqslant \varepsilon \quad i = 1,2,\ldots,m \\
& u_r \geqslant \varepsilon \quad r = 1,2,\ldots s
\end{aligned}
\tag{10}
$$

where $(x_{ij}) \in \Theta_i^-$ and $(y_{rj}) \in \Theta_r^+$ represent any or all of Eqs. (3)–(9).

Since some outputs and inputs of Model (10) are unknown decision variables, this model is nonlinear and non-convex. Hence, the optimum solution may not be a global optimum solution of the model.

To convert Model (10) into the linear program (LP), some approaches exist in literature. For instance, Despotis and Smirlis (2002) proposed a generalized model which is capable to handle both interval and ordinal data. Their approach is to transform a non-linear DEA model to a linear programming equivalent, on the basis of the original data set, by applying transformations only on the variables. In addition, Zhu (2003) proposed two different approaches in dealing with the IDEA, where some of the inputs and outputs are imprecise data in the forms of bounded data, ordinal data and ratio bounded data. One approach uses scale-transformation and variable-alternation, while the other approach converts imprecise data into exact data. In order to convert the non-linear IDEA model into a linear program, first approach of Zhu (2003) defines:

$$\begin{aligned} &X_{ij} = w_i x_{ij} \quad \forall i,j \\ &Y_{ij} = u_r y_{rj} \quad \forall r,j \end{aligned} \tag{11}$$

In this paper, by adopting Zhu's approach, Model (10) can be converted to Model (12) which is a LP.

$$\begin{aligned} &M^* = \min M \\ &\text{s.t} \\ &M - d_j \geqslant 0 \quad j = 1,2,\ldots,n \\ &\sum_{i=1}^{m} X_{ij} \leqslant 1 \quad j = 1,2,\ldots,n \\ &\sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} + d_j - \beta_j = 0 \quad j = 1,2,\ldots,n \\ &\sum_{j=1}^{n} d_j = n-1 \\ &0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0,1\} \quad j = 1,2,\ldots,n \\ &X_{ij} \in \tilde{D}_i^- \\ &Y_{rj} \in \tilde{D}_r^+ \\ &X_{ij} \geqslant \varepsilon^* \quad \forall i,j \\ &Y_{rj} \geqslant \varepsilon^* \quad \forall r,j \end{aligned} \tag{12}$$

where Θ_i^- and Θ_r^+ are replaced by $\tilde{D}_i^-$ and $\tilde{D}_r^+$ with:

- *Bounded data:* $\underline{y}_{rj} u_r \leqslant Y_{rj} \leqslant \overline{y}_{rj} u_r,\ \underline{x}_{ij} w_i \leqslant X_{ij} \leqslant \overline{x}_{ij} w_i$.
- *Ordinal data:* $Y_{rj} \leqslant Y_{rk},\ X_{ij} \leqslant X_{ik}\ \forall j \neq k$ *for some r,i.*
- *Ratio bounded data:* $L_{rj} \leqslant \frac{Y_{rj}}{Y_{rj_o}} \leqslant U_{rj}$ and $G_{ij} \leqslant \frac{X_{rj}}{X_{ij_o}} \leqslant H_{ij}\ (j \neq j_0)$.
- *Cardinal data:* $Y_{rj} = \hat{y}_{rj} u_r$ and $X_{ij} = \hat{x}_{ij}$, where $\hat{y}_{rj}$ and $\hat{x}_{ij}$ represent cardinal data.

Indeed Model (12) is extended version of Amin & Toloo's model. Hence, the following LP, which is extended version of Amin and Toloo (2007) epsilon model, is proposed to determine the non-Archimedean epsilon:

$$\begin{aligned} &\varepsilon^* = \max \varepsilon \\ &\text{s.t.} \\ &\sum_{i=1}^{m} X_{ij} \leqslant 1 \quad j = 1,2,\ldots,n \\ &\sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} \leqslant 0 \quad j = 1,2,\ldots,n \\ &X_{ij} \in \tilde{D}_i^- \\ &Y_{rj} \in \tilde{D}_r^+ \\ &X_{ij} - \varepsilon \geqslant 0 \quad \forall i,j \\ &Y_{rj} - \varepsilon \geqslant 0 \quad \forall r,j \end{aligned} \tag{13}$$

As mentioned in Section 3, the model which was proposed by Amin and Toloo (2007) is based on CCR model and evaluates DMUs in constant return to scale. Hence, Model (12) is not applicable for situations in which DMUs, in presence of cardinal and ordinal data, operating in variable return to scale. By extending previous work of Toloo and Nalchigar (2009), Appendix A proposes a model which is able to find most BCC-efficient DMU in existence of cardinal and ordinal data.

5. Ranking method

To rank DMUs in DEA, various methods have been developed by researchers. Adler, Friedman, and Stern (2002) reviewed the ranking method in DEA context and grouped them into six, to some extent related areas. The classification of methods in this paper helps to understand each area better. The readers can refer to this paper for further discussion on ranking methods.

In this section, a new method for prioritizing DMUs with cardinal and ordinal data is presented. This method is as follows:

Step 0: Let $T = \phi$ and e number of DMUs to be ranked.

Step 1: Solve following model:

$$\begin{aligned} &M^* = \min M \\ &\text{s.t.} \\ &M - d_j \geqslant 0 \quad ,j = 1,2,\ldots,n \\ &\sum_{i=1}^{m} X_{ij} \leqslant 1 \quad j = 1,2,\ldots,n \\ &\sum_{r=1}^{s} Y_{rj} - \sum_{i=1}^{m} X_{ij} + d_j - \beta_j = 0 \quad j = 1,2,\ldots,n \\ &\sum_{j=1}^{n} d_j = n-1 \\ &d_j = 1 \quad \forall j \in \mathrm{T} \\ &0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0,1\} \quad j = 1,2,\ldots,n \\ &X_{ij} \in \tilde{D}_i^- \\ &Y_{rj} \in \tilde{D}_r^+ \\ &X_{ij} \geqslant \varepsilon^* \quad \forall i,j \\ &Y_{rj} \geqslant \varepsilon^* \quad \forall r,j \end{aligned} \tag{14}$$

Suppose in optimal solution $d_p^* = 0$.

Step 2: Let $\mathrm{T} = \mathrm{T} \cup \{p\}$.

Step 3: If $|\mathrm{T}| = e$, then stop; otherwise go to Step 1.

Indeed, in Step 1 of proposed algorithm, a DMU is identified as most CCR-efficient unit in presence imprecise data. After entering this DMU to T in Step 2, in Step 3 if all DMUs are ranked, the algorithm finishes, else it goes to next iteration. By continuing the iterations to e times, decision maker is able to rank DMUs in presence of both cardinal and ordinal data. In the next section, applicability of proposed method is illustrated.

6. Illustrative example

To show applicability of proposed method, an instance of supplier selection problem is adopted from Farzipoor Saen (2007). The data utilized in this paper are obtained from previously published data set and selected from a case study that applied DEA. Inputs of suppliers are total cost of shipment (TC) which is in cardinal format and supplier reputation (SR) which is measured on an ordinal scale. In addition, bills received from the supplier without errors (NB) is considered as the bounded data output.

It is to be noted that the aim of this study is not to propose a comprehensive set of criteria for supplier selection. In other words, inputs and outputs in this study are general measures and real applications requires considering suitable criteria. Table 1 contains the supplier's attributes.

Inputs and outputs of suppliers are as follows:

$$\Theta_1^- = \{x_{11} = 253; \quad x_{12} = 268; \quad x_{13} = 259; \quad \dots \quad ; x_{118} = 216\}(\text{cardinal data})$$

$$\Theta_2^- = \{x_{218} \geqslant x_{216} \geqslant \dots \geqslant x_{217}\}(\text{ordinal data})$$

$$\Theta_1^+ = \{50 \leqslant y_{11} \leqslant 65; \quad 60 \leqslant y_{12} \leqslant 70; \quad 40 \leqslant y_{13} \leqslant 50, \quad \dots \quad , \quad 90 \leqslant y_{118} \leqslant 150\}(\text{bounded data}).$$

According to Zhu's approach,

$$\tilde{D}_1^- = \{X_{11} = 253w_1; \quad X_{12} = 268w_1; \quad X_{13} = 259w_1, \quad \dots \quad , X_{118} = 216w_1\}$$

$$\tilde{D}_2^- = \{X_{218} \geqslant X_{216} \geqslant \dots \geqslant X_{217}\}.$$

$$\tilde{D}_1^+ = \{50\mu_1 \leqslant Y_{11} \leqslant 65\mu_1; \quad 60\mu_1 \leqslant Y_{12} \leqslant 70\mu_1; \quad 40\mu_1 \leqslant Y_{13} \leqslant 50\mu_1; \quad \dots \quad ; \quad 90\mu_1 \leqslant Y_{118} \leqslant 150\mu_1\}$$

Using these equations, Farzipoor Saen (2007) evaluated 18 suppliers and identified suppliers 4, 6, 8, 9, 11, 14 and 17 as best suppliers. Indeed, his method suffers from a main shortcoming since identifies efficient suppliers and is not able to find most efficient supplier and rank them. In other words, using Farzipoor Saen's method, decision maker is not able to rank suppliers and choose the best one. The method which is proposed in this paper overcome these drawbacks, differentiates and rank efficient suppliers.

Solving Model (14) for data presented in Table 1, (with considering suitable value for epsilon, equal to 0.1972) DMU_4 is easily identified as most CCR-efficient supplier ($d_4^* = 0,\ d_{j\neq 4}^* = 1$). In second iteration of proposed method, a constraint $d_4 = 1$ is added to model. This added constraint ensure that in second iteration of algorithm, DMU_4 will not again identified as most efficient unit. By solving Model (14) in second iteration, optimal solution is ($d_{14}^* = 0,\ d_{j\neq 14}^* = 1$) which implies that DMU_{14} is second CCR-efficient supplier. By continuing this process user can rank all suppliers with imprecise data. Table 2 presents ranking of efficient suppliers by proposed method. It is notable that our method requires decision maker to solve 1 LP for identifying best supplier. However, Farzipoor Saen's method requires decision maker to solve 18 LPs to find 7 efficient suppliers.

Table 1
Data of 18 suppliers.

Supplier no. (DMU)	Inputs		Output
	TCx_{1j}	SRx_{2j} [a]	NBy_{1j}
1	253	5	[50,65]
2	268	10	[60,70]
3	259	3	[40,50]
4	180	6	[100,160]
5	257	4	[45,55]
6	248	2	[85,115]
7	272	8	[70,95]
8	330	11	[100,180]
9	327	9	[90,120]
10	330	7	[50,80]
11	321	16	[250,300]
12	329	14	[100,150]
13	281	15	[80,120]
14	309	13	[200,350]
15	291	12	[40,55]
16	334	17	[75,85]
17	249	1	[90,180]
18	216	18	[90,150]

[a] Ranking such that $18 \equiv$ highest rank, $\dots, 1 \equiv$ lowest rank($x_{218} > x_{216} > \dots > x_{217}$).

Table 2
Ranking of efficient suppliers.

Ranking	Efficient supplier no. (DMU)
1	4
2	14
3	6
4	17
5	11
6	8
7	9

7. Conclusion

Supplier selection, which is one of the most crucial components of production and logistics management, has a significant impact on various functional areas of business from procurement to production and delivery of the products to the end customer. To select the most efficient suppliers in the conditions that both ordinal and cardinal factors are present, a methodology was introduced in this study. Investigation of previous published researches indicates that SCM and the supplier (vendor) selection process have received considerable attention in literature. However, little attention is given to decisions on the appropriate selection of suppliers in existence of both cardinal and ordinal data. This paper proposed a new integrated DEA model for finding most efficient supplier with imprecise data. Using this model, decision maker is able to choose most efficient supplier by solving just one MILP. Consequently, by using this model, a method developed for ranking suppliers. Furthermore, another DEA model which is able to identify most BCC-efficient unit with imprecise data introduced in Appendix A. To provide some further insights, it is notable that the models developed in this paper are input-oriented, but can be extended to output-oriented. In addition, extending the application of proposed method to the analysis of other decision problems can be considered as a topic for future research.

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Appendix A

Toloo and Nalchigar (2009) developed Model (15) as a new integrated model for finding the most BCC-efficient DMU.

$$\begin{aligned}
& M^* = \min M \\
& s.t. \\
& M - d_j \geqslant 0 \quad j = 1,2,\dots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \leqslant 1 \quad j = 1,2,\dots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0 \quad j = 1,2,\dots,n \\
& \sum_{j=1}^{n} d_j = n - 1 \\
& 0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0,1\} \quad j = 1,2,\dots,n \\
& M, u_0 \quad free \\
& w_i \geqslant \varepsilon^* \quad i = 1,2,\dots,m \\
& u_r \geqslant \varepsilon^* \quad r = 1,2,\dots,s
\end{aligned} \tag{15}$$

Model (15) is computationally efficient and also has wider range of application than models which find most CCR-efficient DMU (Model (2)), because is capable for situation in which return to scale is variable. By incorporating Eqs. (3)–(9) to Model (15), and converting non-linear model to a LP (based on Zhu's (2003) approach), there will be:

$$
\begin{aligned}
& M^* = \min M \\
& s.t. \\
& M - d_j \geqslant 0 \quad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} X_{ij} \leqslant 1 \quad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} Y_{rj} - u_0 - \sum_{i=1}^{m} X_{ij} + d_j - \beta_j = 0 \quad j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} d_j = n-1 \\
& 0 \leqslant \beta_j \leqslant 1, d_j \in \{0,1\} \quad j = 1,2,\ldots,n \\
& X_{ij} \in \tilde{D}_i^- \\
& Y_{rj} \in \tilde{D}_r^+ \\
& X_{ij} \geqslant \varepsilon^* \quad \forall i,j \\
& Y_{rj} \geqslant \varepsilon^* \quad \forall r,j \\
& M, u_0 \quad free
\end{aligned}
\tag{16}
$$

Model (16) finds most BCC-efficient DMU in presence of both cardinal and ordinal data. Similar to Model (12), this model could be used for ranking DMUs which operate in variable return to scale.

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