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A new integrated DEA model for finding most BCC-efficient DMU

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Abstract

In many applications of widely recognized technique, DEA, finding the most efficient DMU is desirable for decision maker. Using basic DEA models, decision maker is not able to identify most efficient DMU. Amin and Toloo [Gholam R. Amin, M. Toloo, Finding the most efficient DMUs in DEA: an improved integrated model. Comput. Ind. Eng. 52 (2007) 71–77] introduced an integrated DEA model for finding most CCR-efficient DMU. In this paper, we propose a new integrated model for determining most BCC-efficient DMU by solving only one linear programming (LP). This model is useful for situations in which return to scale is variable, so has wider range of application than other models which find most CCR-efficient DMU. The applicability of the proposed integrated model is illustrated, using a real data set of a case study, which consists of 19 facility layout alternatives.

Keywords: Integrated DEA model; Most BCC-efficient DMU; Facility layout design

1. Introduction

Data envelopment analysis (DEA) is a non-parametric linear programming based technique for measuring the relative efficiency of a set of similar units, usually referred to as decision making units (DMUs). It was introduced by Charnes et al. [1] based on Farrell's pioneering work. They generalized the single-output to single-input ratio definition of efficiency to multiple inputs and outputs. In their original DEA model, Charnes, Cooper and Rhodes (CCR model) proposed that the efficiency of a DMU can be obtained as the maximum of a ratio of weighted outputs to weighted inputs, subject to the condition that the same ratio for all DMUs must be less than or equal to one. The DEA model must be run n times, once for each unit, to get the relative efficiency of all DMUs. The CCR model evaluates both technical and scale efficiencies via the optimal value of the ratio form. The envelopment in CCR is constant returns to scale meaning that a proportional increase in

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inputs results in a proportionate increase in outputs. Banker et al. [2] developed the BCC model to estimate the pure technical efficiency of decision making units with reference to the efficient frontier. It also identifies whether a DMU is operating in increasing, decreasing or constant returns to scale. So CCR models are a specific type of BCC models.

DEA has gained too much attention by researchers because of its successful applications and case studies. Assessment of bank branch performance [3], examining bank efficiency [4], analyzing firm's financial statements [5], measuring the efficiency of higher education institutions [6], solving facility layout design (FLD) problem [7] and measuring the efficiency of organizational investments in information technology [8] are examples of using DEA in various areas.

In many applications, finding most efficient DMU is required, particularly when decision maker wants to select only one DMU among proposed DMUs. Some studies have been done in this area. For instance, Ertay et al. [7] integrated DEA and analytic hierarchy process (AHP) and presented a decision-making methodology for evaluating FLDs. In the last step of their methodology, they extended minimax DEA model to identify single most efficient DMU. Amin and Toloo [9] extended their work and proposed an integrated DEA model in order to detect the most CCR-efficient DMU. It was able to find the most CCR-efficient DMU without solving the model n times (one linear programming (LP) for each DMU) and therefore allowed the user to get faster results. It is notable that the proposed model is useful for situations that return to scale is constant. On account of the fact that of their model is based on CCR, it is not capable for situations in which DMUs are operating in increasing or decreasing returns to scale.

In this paper, we try to fill the gap by proposing a new model for finding most BCC-efficient DMU. The proposed model is capable for situations in which return to scale is variable. This model also is computationally efficient, because it detects the most BCC-efficient DMU by solving only one LP.

The rest of this paper is organized as follows: In Section 2, we describe the BCC model and the model proposed by Amin and Toloo [9] as background models. A new model for finding most BCC-efficient DMU is proposed in Section 3. In Section 4 applicability of proposed model is illustrated. The paper is closed with some concluding remark in Section 5.

2. Background models

DEA is commonly used to evaluate the relative efficiency of a number of DMUs. The basic DEA model in Charnes et al. [1], called the CCR model, has lead to several extensions, most notably the BCC model of Banker et al. [2]. Assume that there are n DMUs, (DMU_j: $j = 1,2, \ldots ,n$) which consume m inputs ($\mathbf{x}_i$: $i = 1,2,\ldots,m$) to produce s outputs ($\mathbf{y}_r$: $r = 1,2, \ldots ,s$). The BCC input oriented (BCC-I) model evaluates the efficiency of DMU_o, DMU under consideration, by solving the following linear program:

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{s} u_r y_{rj} - u_0 \\
\text{s.t.} \quad & \sum_{i=1}^{m} w_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0, \quad j = 1,2,\ldots,n \\
& u_0, \quad \text{free} \\
& w_i \geqslant \varepsilon, \quad i = 1,2,\ldots,m \\
& u_r \geqslant \varepsilon, \quad r = 1,2,\ldots,s
\end{aligned}
\tag{1}
$$

where x_{ij} and y_{rj} (all nonnegative) are the inputs and outputs of the jth DMU, w_i and u_r are the input and output weights (also referred to as multipliers). x_{io} and y_{ro} are the inputs and outputs of DMU_o. Also, ε is non-Archimedean infinitesimal value for forestalling weights to be equal to zero. On account of the fact that basic DEA models identify more than one DMU as efficient units, finding the most efficient DMU is an issue.

Amin and Toloo [9] proposed an integrated model for finding most CCR-efficient DMU, as follows:

$$\begin{aligned}
M^* = \min \quad & M \\
\text{s.t.} \quad & M - d_j \geqslant 0, \quad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0, \quad j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} d_j = n-1 \\
& 0 \leqslant \beta_j \leqslant 1, d_j \in \{0,1\}, \quad j = 1,2,\ldots,n \\
& w_i \geqslant \varepsilon, \quad i = 1,2,\ldots,m \\
& u_r \geqslant \varepsilon, \quad r = 1,2,\ldots,s
\end{aligned} \tag{2}$$

where d_j as a binary variable represents the deviation variable of DMU_j. DMU_j is most CCR-efficient if and only if $d_j = 0$. The constraint $\sum_{j=1}^{n} d_j = n-1$ forces among all the DMUs for only single most CCR-efficient unit.

This model proposed to overcome drawbacks of its previous model which was proposed by Ertay et al. [7]. Ertay et al. [7] model was a parametric extended minimax DEA model to identify single most CCR-efficient DMU. Ertay et al. [7] model requires to be solved n times for n DMUs. In addition this model uses trial and error method in objective function. Through an example, Amin and Toloo [9] showed that their DEA trial and error approach may not converge to a single CCR-efficient DMU. So, Ertay et al. [7] minimax DEA procedure does not terminate to a single CCR-efficient alternative in all situations. Consequently, Amin and Toloo [9] proposed non-parametric integrated Model (2) which finds most CCR-efficient DMU without solving the LP n times.

3. Proposed model

The model proposed by Amin and Toloo [9] is based on CCR model and is not useful for situations in which DMUs operating in variable return to scale. In this paper we propose a new integrated model which is useful for these situations. The model proposes as:

$$\begin{aligned}
M^* = \min \quad & M \\
\text{s.t.} \quad & M - d_j \geqslant 0, \quad j = 1,2,\ldots,n \\
& \sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1,2,\ldots,n \\
& \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} + d_j - \beta_j = 0, \quad j = 1,2,\ldots,n \\
& \sum_{j=1}^{n} d_j = n-1 \\
& 0 \leqslant \beta_j \leqslant 1, \quad d_j \in \{0,1\}, \quad j = 1,2,\ldots,n \\
& M, u_0, \quad \text{free} \\
& w_i \geqslant \varepsilon^*, \quad i = 1,2,\ldots,m \\
& u_r \geqslant \varepsilon^*, \quad r = 1,2,\ldots,s
\end{aligned} \tag{3}$$

DMU_j is most BCC-efficient if and only if $d_j = 0$. Similar to Amin and Toloo [9], the main idea of Model (3) is trying to find only one most efficient DMU, but in situations in which return to scale is variable. Adding free variable u_o, enhance the capability of model to situations in which DMUs act in variable return to scale, meaning that most BCC-efficient can be found using Model (3). The constraint $\sum_{j=1}^{n} d_j = n - 1$ forces among all the DMUs for only single most BCC-efficient unit.

Indeed, Model (3) is an extended version of Amin and Toloo [9] model. Hence, the following LP, which is extended version of Amin and Toloo [9] epsilon model, is proposed to determine the non-Archimedean epsilon:

$$
\begin{aligned}
\varepsilon^* = \max \quad & \varepsilon \\
\text{s.t.} \quad & \sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1, 2, \ldots, n && (4.1) \\
& \sum_{r=1}^{s} u_r y_{rj} - u_0 - \sum_{i=1}^{m} w_i x_{ij} \leqslant 0, \quad j = 1, 2, \ldots, n && (4.2) \\
& w_i - \varepsilon \geqslant 0, \quad i = 1, 2, \ldots, m && (4.3) \\
& u_r - \varepsilon \geqslant 0, \quad r = 1, 2, \ldots, s && (4.4)
\end{aligned}
\tag{4}
$$

Theorem 1. *In Model (4), (4.2) and (4.4) are redundant.*

Proof. The dual of Model (4) is as follows:

$$
\begin{aligned}
\min \quad & \sum_{j=1}^{n} \delta_j \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\delta_j - \sum_{j=1}^{n} x_{ij}\beta_j - \gamma_i = 0, \quad i = 1, \ldots, m \\
& \sum_{j=1}^{n} y_{rj}\beta_j - \eta_r = 0, \quad r = 1, \ldots, s \\
& \sum_{j=1}^{n} \beta_j = 0 \\
& \sum_{i=1}^{m} \gamma_i + \sum_{r=1}^{s} \eta_r = 1 \\
& \delta_j \geqslant 0, \quad \beta_j \geqslant 0, \quad j = 1, 2, \ldots, n \\
& \gamma_i \geqslant 0, \quad \eta_r \geqslant 0, \quad i = 1, 2, \ldots, n, \quad r = 1, \ldots, s
\end{aligned}
\tag{5}
$$

Considering the structure of Model (5), $\forall j,r$: $\beta_j = 0$, $\eta_r = 0$. Because,

$$
\sum_{j=1}^{n} \beta_j = 0 \;\&\; \forall j : \beta_j \geqslant 0 \Rightarrow \forall j : \beta_j = 0 \Rightarrow \forall r : \eta_r = 0
$$

So, the dual of Model (4) is simplified as follows:

$$
\begin{aligned}
\delta^* = \min \quad & \sum_{j=1}^{n} \delta_j \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij}\delta_j - \gamma_i = 0, \quad i = 1, \ldots, m \\
& \sum_{i=1}^{m} \gamma_i = 1 \\
& \delta_j \geqslant 0, \quad j = 1, 2, \ldots, n \\
& \gamma_i \geqslant 0, \quad i = 1, 2, \ldots, n
\end{aligned}
\tag{6}
$$

But the dual of Model (6) is Model (7):

$$\begin{aligned} \varepsilon^* = \max \quad & \varepsilon \\ \text{s.t.} \quad & \sum_{i=1}^{m} w_i x_{ij} \leqslant 1, \quad j = 1, 2, \ldots, n \\ & w_i - \varepsilon \geqslant 0, \quad i = 1, 2, \ldots, m \end{aligned} \tag{7}$$

The dual of the dual is the primal, hence the proof is completed. □

Lemma 1. *Model (7) is feasible.*

Proof. Let $\varepsilon = 0$, $\forall i$: $w_i = 0$. Clearly (ε, $\mathbf{w}$) is a feasible solution of Model (7). □

Lemma 2. *Model (6) is feasible.*

Proof. Let

$$\forall j : \delta_j = \frac{1}{n \sum_{i=1}^{m} x_{ij}} \quad \text{and} \quad \forall i : \gamma_i = \frac{1}{n} \times \sum_{j=1}^{n} \left(\frac{x_{ij}}{\sum_{k=1}^{m} x_{kj}} \right)$$

Clearly (δ, γ) is a feasible solution of Model (6). □

Lemma 3. $0 < \varepsilon^* < +\infty$

Proof. On the contrary, assume that $\varepsilon^* = 0$. Hence, $\delta^* = \sum_{j=1}^{n} \delta_j^* = 0$. On the other hand, $\forall j : \delta_j^* \geqslant 0$, implies that $\forall j : \delta_j^* = 0 \Rightarrow \gamma_j^* = 0 \Rightarrow \sum_{j=1}^{n} \gamma_j^* = 0$ which contradicts to the last constraint of Model (6), so $\varepsilon^* = \delta^* > 0$.

According to Lemma 1 and Lemma 2, Model (7) and Model (6) are feasible, respectively. Therefore, $\varepsilon^* = \delta^* < +\infty$. □

Lemma 4. *Let (ε^*, $\mathbf{w}^*$) be an optimal solution of Model (7) and $J = \{j : \sum_{j=1}^{n} w_i^* x_{ij} = 1\}$, then $|J| > 0$.*

Proof. On the contrary, assume that $|J| = 0$. The complementary slackness conditions imply that $\delta^* = 0$, which contradicts to Lemma 3. This implies that $|J| > 0$. □

Theorem 2. *Model (3) is feasible.*

Proof. Suppose that (ε^*, $\mathbf{w}^*$, $\mathbf{u}^*$, u_0) is an optimal solution of Model (4) and $\mathbf{w}^* \mathbf{x}_p = 1$ (according to Lemma 4 such index exists, ties are broken arbitrary). Let $M = 1$, $\mathbf{w} = \mathbf{w}^*$, $\mathbf{u} = \mathbf{u}^*$, $u_0 = u_0^*$, $d_p = 0$, $\forall j \neq p$: $d_j = 1$, $\forall j : \beta_j = \mathbf{u}^* \mathbf{y}_j - u_0^* - \mathbf{w}^* \mathbf{x}_j - d_j$, $\mathbf{d} = (d_1, \ldots, d_n)$, $\beta = (\beta_1, \ldots, \beta_n)$.

Clearly (M, $\mathbf{w}$, $\mathbf{u}$, u_0, $\mathbf{d}$, β) is a feasible solution of Model (3). □

Lemma 5. $M^* = 1$.

Proof. Model (3) implies that $M^* = \min\{\max\{d_j : j = 1, 2, \ldots n\}\}$, $d_j \in \{0, 1\}$, $\sum_{j=1}^{n} d_j = n - 1$. This completes the proof. □

4. Illustrative example

To operate production and service systems efficiently, systems should not only have to be operated with optimal planning and operational policies, but also be well designed. FLD has a very important effect on the performance of a manufacturing system. The concept of FLD is usually considered as a multi objective problem. For this reason, a layout generation and its evaluation are often challenging and time consuming due to their inherent multiple objectives in nature and their data collection process. In addition, an effective facility layout evaluation procedure necessitates the consideration of qualitative criteria, e.g., flexibility in volume and variety and quality related to the product and production, as well as quantitative criteria such as material handling cost, adjacency score, shape ratio, and material handling vehicle utilization in the decision process. The criteria that are to be minimized are viewed as inputs whereas the criteria to be maximized are considered as outputs ([7]). We are to select the most BCC-efficient FLD.

As Ertay et al. [7], we use a real data set of 19 FLDs which consume 2 inputs, cost and adjacency score, to produce four outputs, shape ration, flexibility, quality and hand-carry utility. This data are presented in Table 1.

Table 1
Inputs and outputs of DEA

DMU	DEA Inputs		DEA Outputs			
	Cost	Adjacency	Shape ratio	Flexibility	Quality	Hand-carry utility
1	20309.56	6405.00	0.4697	0.0113	0.0410	30.89
2	20411.22	5393.00	0.4380	0.0337	0.0484	31.34
3	20280.28	5294.00	0.4392	0.0308	0.0653	30.26
4	20053.20	4450.00	0.3776	0.0245	0.0638	28.03
5	19998.75	4370.00	0.3526	0.0856	0.0484	25.43
6	20193.68	4393.00	0.3674	0.0717	0.0361	29.11
7	19779.73	2862.00	0.2854	0.0245	0.0846	25.29
8	19831.00	5473.00	0.4398	0.0113	0.0125	24.80
9	19608.43	5161.00	0.2868	0.0674	0.0724	24.45
10	20038.10	6078.00	0.6624	0.0856	0.0653	26.45
11	20330.68	4516.00	0.3437	0.0856	0.0638	29.46
12	20155.09	3702.00	0.3526	0.0856	0.0846	28.07
13	19641.86	5726.00	0.2690	0.0337	0.0361	24.58
14	20575.67	4639.00	0.3441	0.0856	0.0638	32.20
15	20687.50	5646.00	0.4326	0.0337	0.0452	33.21
16	20779.75	5507.00	0.3312	0.0856	0.0653	33.60
17	19853.38	3912.00	0.2847	0.0245	0.0638	31.29
18	19853.38	5974.00	0.4398	0.0337	0.0179	25.12
19	20355.00	17402.00	0.4421	0.0856	0.0217	30.02

Table 2
BCC-I efficiency scores of DMUs

Rank	DMU	BCC-I efficiency score
1	19	1
1	1	1
1	17	1
1	16	1
1	15	1
1	5	1
1	14	1
1	7	1
1	12	1
1	9	1
1	10	1
12	13	0.998526
13	11	0.998326
14	3	0.998281
15	8	0.997871
16	18	0.996468
17	6	0.996457
18	2	0.995887
19	4	0.989942

Table 3
Comparison results of proposed model with Amin and Toloo's model

Variable	Model (3)	Amin and Toloo's Model
d_j^*	$d_{14}^* = 0,\ d_{j\neq 14}^* = 1$	$d_{16}^* = 0,\ d_{j\neq 16}^* = 1$
w_1^*	0.000025	0.000026
w_2^*	0.000028	0.328456
u_1^*	0.556423	0.000026
u_2^*	0.860412	0.020036
u_3^*	0.000025	0.000027
u_4^*	0.026153	0.000026
u_{o1}^*	0.458908	–

Using DEA-Solver, the basic BCC-I model is solved and results are presented in Table 2. Solving 19 LP models, there are 11 BCC-efficient DMUs. Obviously, in this situation decision maker is not able to select most BCC-efficient FLD. In this situation, Model (3) is applicable.

Solving Model (7) for the real data regarding to 19 FLDs shown in Table 1, optimal value is: $\varepsilon^* = 0.000025$. There also exists a polynomial time algorithm, *Epsilon algorithm*, which introduced by Amin and Toloo [10]. Applying this algorithm, resulted in same value as received from solving Model (7).

Using this value and solving Model (3), DMU_{14} is identified as most BCC-efficient DMU. It should be noted that Model (3) for this FLD problem includes 83 constraints. To provide some further insights, a comparison of results from Model (3) and Amin and Toloo's model is presented in Table 3.

As Table 3 indicates DMU_{14} is most BCC-efficient unit whereas DMU_{16} is most CCR-efficient unit.

5. Conclusion

In this paper, we proposed a new integrated model for finding the most BCC-efficient DMU. Using the proposed model, decision maker is able to find most BCC-efficient DMU by solving only one LP, rather than n LPs, so can get faster results. The proposed model is computationally efficient and also has wider range of application than models which find most CCR-efficient DMU, because is capable for situation in which return to scale is variable. It also has another merit in comparison to basic DEA models. This model compares all DMUs by a single formulation and common set of weights. It should be noted that depend on number of DMUs the proposed model could be large because it includes $4n + m + s + 1$ constraints, respect to BCC model which contains $n + m + s + 1$ constraint. It is worthwhile mentioning here that the model developed in this paper is input-oriented, but can be extended to output-oriented.

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