

# CSC2720H: Systems Thinking for Global Problems

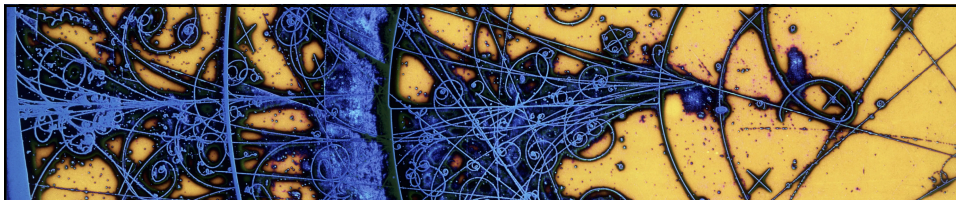
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<http://www.cs.toronto.edu/~sme/SystemsThinking>



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## Week 8

- Group Presentations!
- Review of Chaos and Complexity Theory
- Self-Organized Criticality
  - Population Models
  - Self-organised criticality
- Leverage Points



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## Complex Systems Theory - Key concepts

### → Non-linear Dynamical Systems

↳ Change in inputs/parameters lead to disproportionate change in outputs

### → Sensitivity to Initial Conditions

↳ The “butterfly effect”

↳ Small differences in initial conditions tend to grow exponentially

### → Attractors (Simple and Strange)

↳ Chaotic systems are not random

↳ Can predict the shape of the attractor, but not specific future states

### → Self-similarity and Fractals

### → Criticality and Tipping Points



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## The Harder a System is Pushed the Faster the Change Comes



### Feigenbaum Ratio

Named for mathematician Mitchell J. Feigenbaum

The Feigenbaum ratio is a bifurcation fractal produced by a period-doubling cascade, such as in the logistic bifurcation diagram.

The period doubling comes at a constant rate given by the delta constant: 4.66921166091029 etc.

This constant arises in any dynamical system that approaches chaotic behavior via period-doubling bifurcations:

fluid-flow turbulence,  
electronic oscillators, chemical  
reactions, the Mandelbrot set.



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## Tipping points and Instabilities

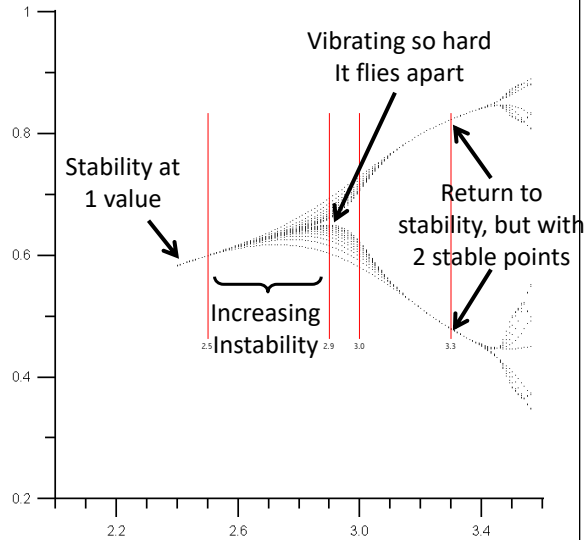
Logistic equations, plotted to show only X values from 20-25.

As a bifurcation approaches the system takes progressively longer to attenuate.

For  $r=3.3$  the system attenuates to one value after 5 iterations.

For  $r=3.0$  the system is still attenuating at the decimal places 4 - 6 after 1 million iterations.

The bifurcation occurs because the instability grows until it flies apart.



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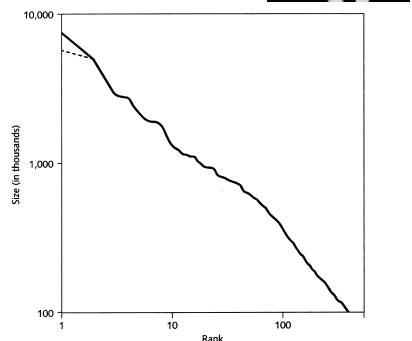
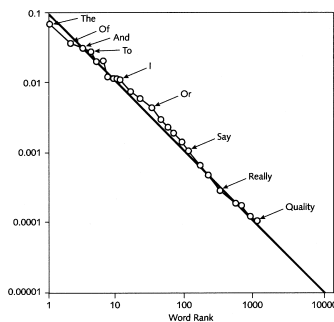
## Zipf's Law

Formulated by George Kingsley Zipf who observed some striking patterns in human systems.

Examples:

Word usage in the English (and other) languages,

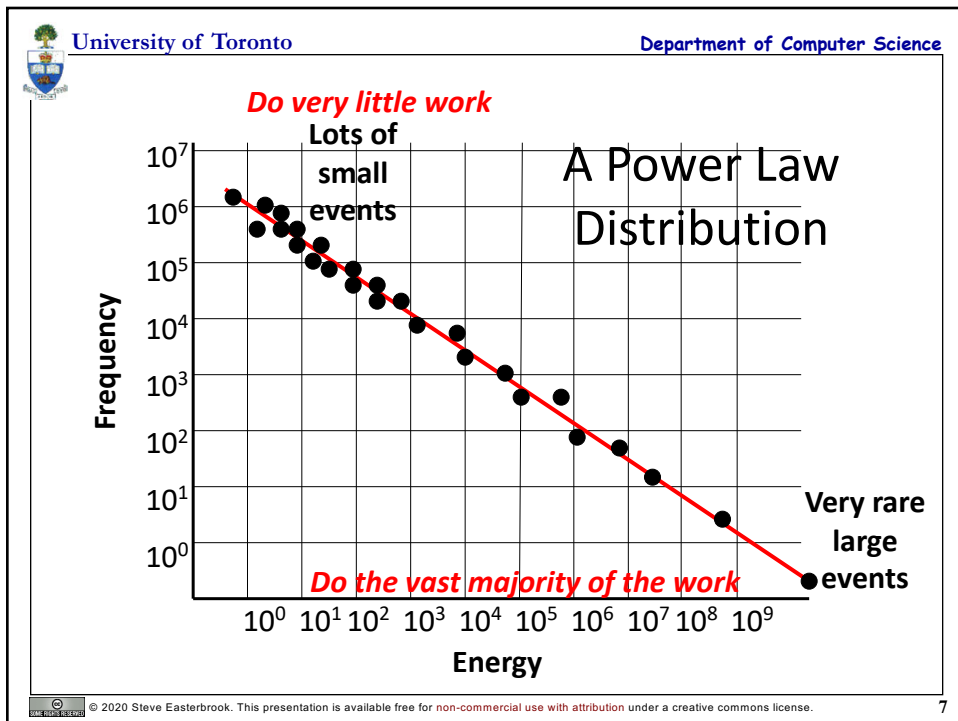
Distribution of city population sizes.



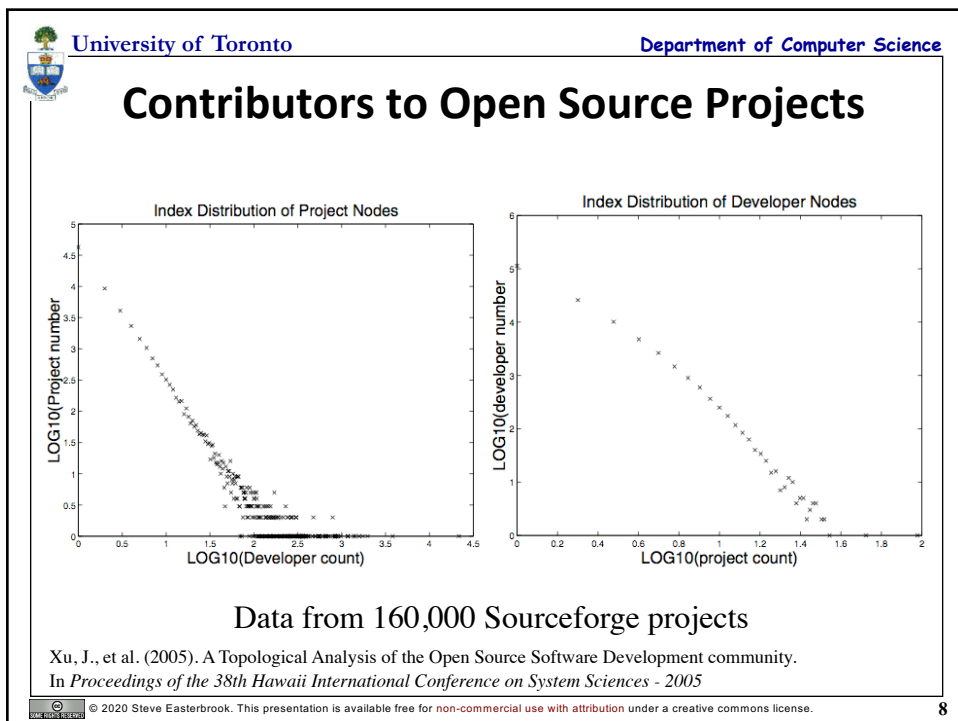
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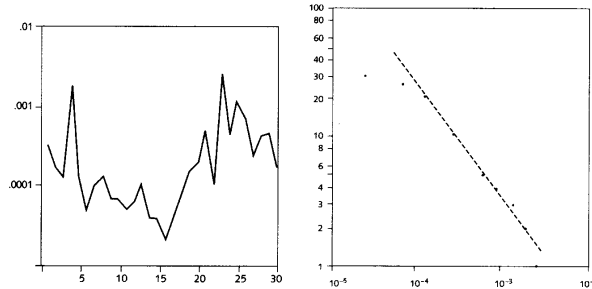
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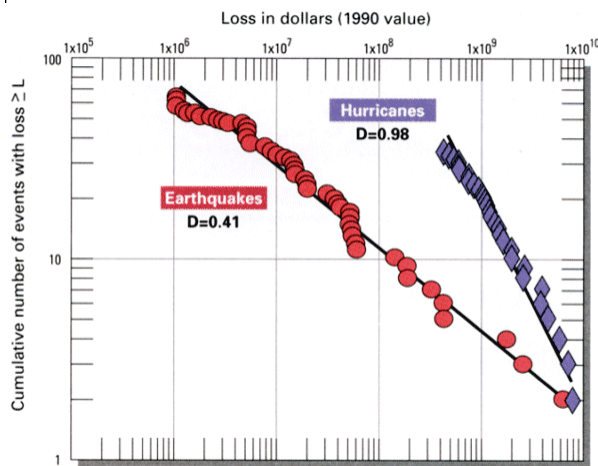
## Power Law Relationships – Cotton Prices



Mandelbrot's (1963) analysis of monthly variations in cotton prices during a 30 month period. The left plot shows the month by month changes. Note how they vary; lots of small changes, and fewer large changes. The right logarithmic graph shows the same data is a power-law distribution, indicating the cotton commodities market is at the critical level (SOC). Other commodities follow a similar pattern.



## Power Law Relationships – Natural Disasters



Plot of cumulative frequency of dollar loss due to earthquakes and hurricanes in the U.S. between 1900 and 1989. Data presented in this manner reveal linear trends which provide the basis for forecasting the probability of future dollar loss.



## Self-Organized Criticality

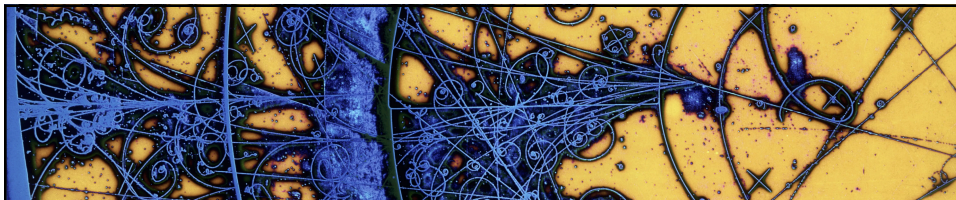


Simple model of how  
avalanches work:  
Bak, Tang, and  
Wiesenfeld's Sandpile  
model

<https://www.youtube.com/watch?v=h737qbQRPME&feature=youtu.be&t=4m>

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Game

## Triangles



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