

Photo: Killarney Provincial Park, Sept 2020

## Climate Change: What we know and how we know it

### Week 3: The Forecast Factory

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Today we're going to take a break from talking about climate change, and talk about the weather. I like to joke that everyone is happy talking about the weather, but nobody really wants to talk about climate.

We'll explore the difference more fully next week, but for now, just remember: climate is what you expect, weather is what you get. Or you could say the climate tells you what to buy for your wardrobe, the weather tells you what to wear each day.

Forecasting the weather is complicated and we'll need some heavy math, so let's get that out of the way first...

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - w \frac{\partial u}{\partial z} - f u - \frac{1}{p} \frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - w \frac{\partial v}{\partial z} - f v - \frac{1}{p} \frac{\partial p}{\partial y}$$

$$\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial x} - v \frac{\partial w}{\partial y} - w \frac{\partial w}{\partial z} - g - \frac{1}{p} \frac{\partial p}{\partial z}$$

$$\frac{\partial \rho}{\partial t} = -u \frac{\partial \rho}{\partial x} - v \frac{\partial \rho}{\partial y} - w \frac{\partial \rho}{\partial z} - \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\partial p}{\partial t} = -u \frac{\partial p}{\partial x} - v \frac{\partial p}{\partial y} - w \frac{\partial p}{\partial z} - \frac{C_p}{C_v} p \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} - w \frac{\partial T}{\partial z} - \frac{RT}{C_v} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)$$

Reproduced from Amy Dahan Dalmedico, "History and Epistemology of Models: Meteorology (1946–1963) as a Case Study," *Archive for the History of Exact Sciences* 55 (2001): 398.

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These are known as the “primitive equations” for forecasting the weather. Some of the mathematicians in the audience might recognize the form of these equations – it’s a set of partial differential equations for calculating the rate of change of key meteorological variables.

For everyone else, don’t worry about what the equations say. The key idea is that each equation tells you how to calculate the next value for each of a number of key weather variables. The first three equations deal with wind speed, and really just capture Newton’s laws of motion. The other three equations deal with air pressure, water vapour, and temperature.

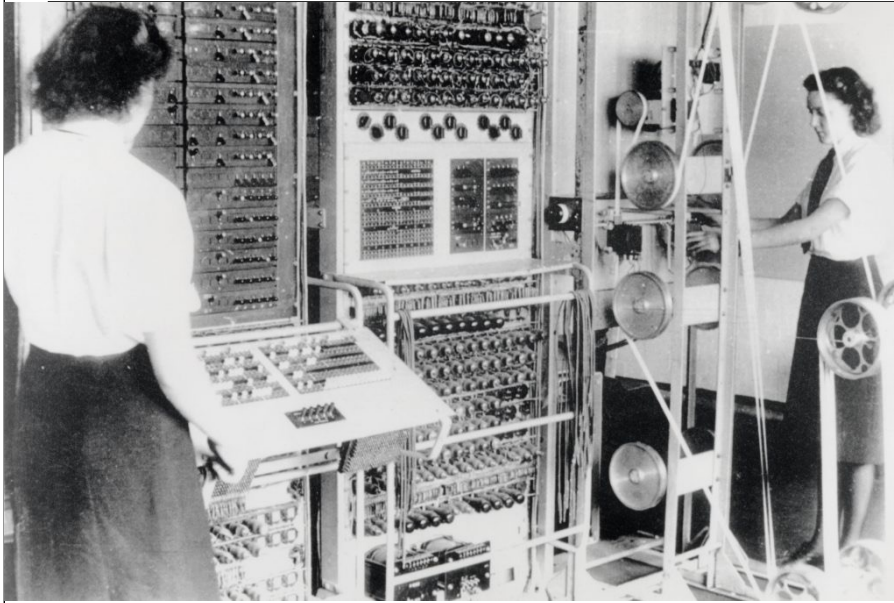
For more details, see:

Amy Dahan Dalmedico, “History and Epistemology of Models: Meteorology (1946–1963) as a Case Study,” *Archive for the History of Exact Sciences* 55 (2001) 398.

<https://link.springer.com/article/10.1007/s004070000032>

But that’s enough about the math...

## Colossus at Bletchley Park, UK



Elsie Booker (R) and Dorothy du Boisson programming the Colossus machine at Bletchley Park in 1945. (Photo UK National Archives)

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Numerical weather forecasting was one of the first big uses of electronic computers, because you need a lot of computing power to solve those equations.

So let's have a quick look at the history of the electronic computer.

Colossus was the first computer to be fully automatic, fully electronic, and programmable. Created towards the end of the Second World War, in January 1944.

It was built by Tommy Flowers, in secret, at Bletchley Park in the UK, to crack the Lorenz Code used by German high command.

UK had already cracked the Enigma code. The Lorenz was even harder.

Here it is shown operated by Wrens, Elsie Booker (R) and Dorothy du Boisson. Before these electronic computers, the word 'computer' referred to a human, or rather a job carried out by a person who did calculations. Most computers were women.

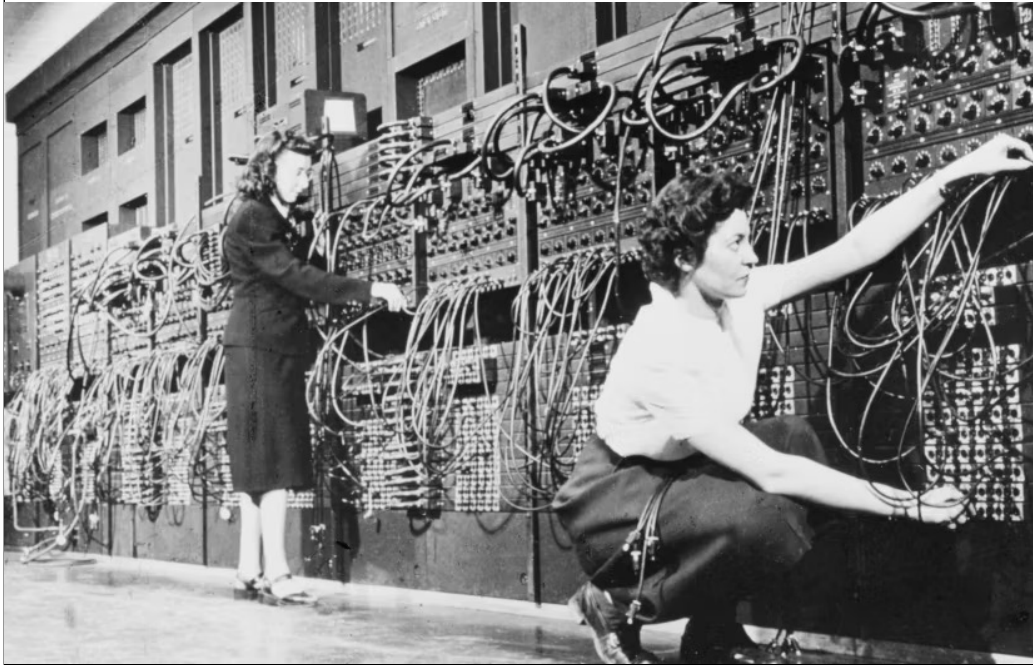
This computer was kept secret for many decades after the war, so is missing from many histories of computing.

But it was designed for only one thing (code-breaking), so isn't really a general-purpose computer.

For more details see:

<https://www.bletchleypark.org.uk/our-story/75-years-since-colossus-arrived-at-bletchley/>

## ENIAC at UPenn, US



So if Colossus wasn't a general purpose computer, that means the first computer to meet all four conditions (fully automatic, electronic, programmable, and general purpose) was ENIAC.

This machine became operational in December 1945, and was unveiled to the public in February 1946.

Here shown operated by Marlyn Wescoff (L) and Ruth Lichterman. A team of 5 women altogether were employed to operate it.

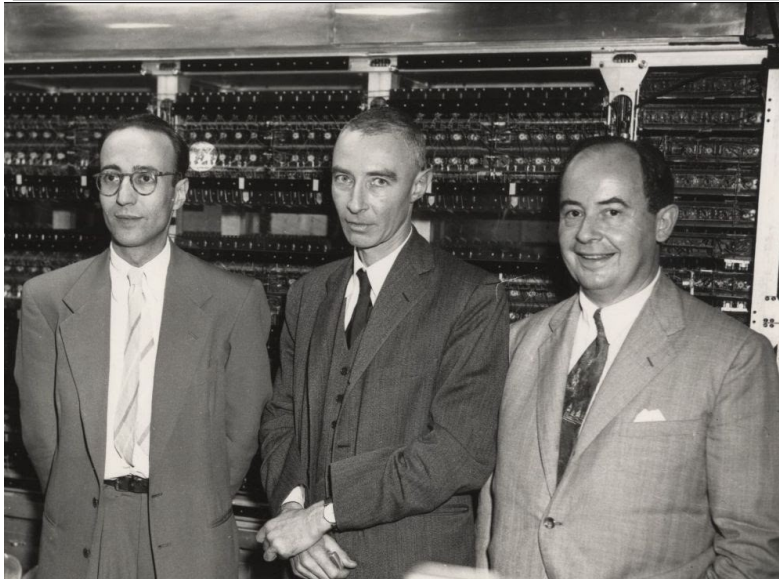
Designed by John Mauchly and J. Presper Eckert at the University of Pennsylvania, sponsored by the US Army, who wanted it to do ballistics calculations.

See for example:

<https://www.aps.org/apsnews/2022/11/eniac-first-top-secret-program>



## A chance meeting



Herman Goldstein, J. Robert Oppenheimer, and John von Neumann in 1952 at the Institute for Advanced Study at Princeton University. This photo is from the dedication of the new IAS computer, fondly known as "Johnniac". (Photo: IAS)

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Two key characters for today's lecture were John von Neumann and Herman Goldstein. The guy in the middle, Robert Oppenheimer has a whole movie about him, but he isn't really relevant to this lecture, except that he was the director of the Institute for Advanced Study at Princeton after the war, and he's in this photo because they're celebrating the launch of the IAS's new computer.

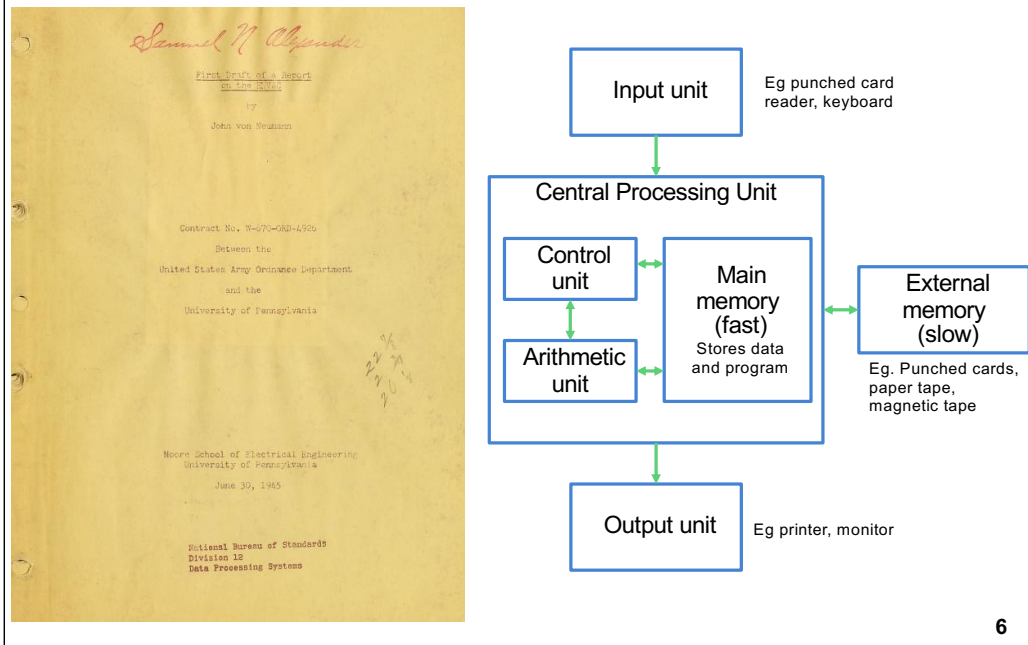
Von Neumann was one of the leading mathematicians of his generation. Towards the end of the war, he was working as a consultant at Los Alamos on the mathematics of shockwaves, needed for analysis of nuclear bombs. In his quest to find a suitable machine to run the computations, he was one of the only Los Alamos scientists allowed to freely come and go.

In the summer of 1944, at a railway station in Aberdeen, Maryland (site of a major US Army facility, the Aberdeen Proving Ground), Goldstein recognized von Neuman, and struck up a conversation, telling him all about ENIAC.

Von Neuman immediately arranged a trip to UPenn to visit the machine while it was still under construction. Intended to calculate ballistics trajectories, von Neuman instead pulled strings to use it for a thermodynamics analysis needed for the hydrogen bomb project. The program was encoded onto a million punched cards and shipped to Philadelphia. It took 6 weeks to run, and completed successfully.

After the war, von Neuman moved to the the IAS at Princeton, and recruited Goldstein to manage the development of their own computer.

## The von Neuman architecture



Among computer scientists, Von Neuman is generally known as the father of the modern computer, and the design of today's computers is still called the von Neumann architecture, as they still use the overall design von Neumann described in this report, which he wrote by hand, and sent to Goldstein, who had it typed up and circulated it widely.

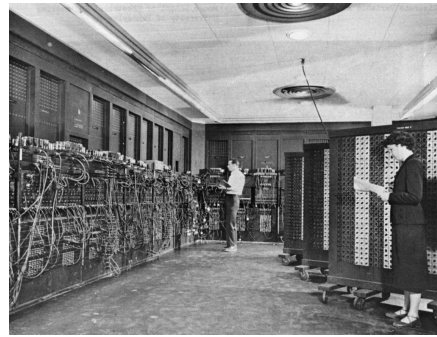
He was describing the design for EDVAC, which was the successor to ENIAC, and von Neuman was working closely with the team designing it.

This report also became the subject of a lawsuit, as it was cited the following year as a public disclosure of the design, when Mauchly and Eckhert tried to patent their design. It caused a huge rift, especially as only von Neuman's name appears as author. von Neuman eventually won the ensuing lawsuit. He said he felt that the design should be shared openly, rather than patented for profit.

## First Numerical Simulation of Weather

Late 1940s: John von Neumann & Jule Charney develop a weather forecasting program for the 1st electronic computer, ENIAC:

Funded by the US Army for use in weather control, geo-engineering, etc.



See: Lynch, P. (2008). The ENIAC Forecasts: A Recreation. Bulletin of the American Meteorological Society, 1–11

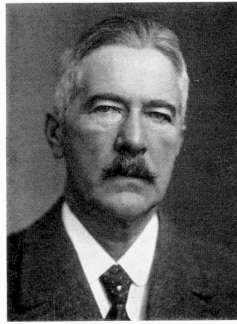
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Von Neumann was fascinated by the early electronic computers, as he realized they would enable him to do calculations that had been impossible previously, especially some of the numerical methods he had worked on for things like fluid motion.

He persuaded the US Army that these new computers could be used to forecast the weather, and that weather forecasting would give the army a critical battlefield advantage.

He hired a promising young meteorologist, Jule Charney to design the weather forecasting program. We already met Charney back in week 1, towards the end his career, when he was asked to respond to president Jimmy Carter's questions about climate change. Here he is at the start of his career, developing the first weather program.

## Stockholm Cosmic Physics Society, 1893



ARVID HÖGBOM

Arvid Högbom  
Geologist  
Where does carbon  
dioxide come from?  
Where does it go?



Nils Ekholm.  
Fotograf.

Nils Ekholm  
Meteorologist  
& Explorer  
What caused the  
ice ages?



Svante Arrhenius

Svante Arrhenius  
Chemist  
How much would  
temperatures drop if  
there was less  
carbon dioxide?



Vilhelm Bjerknes  
Physicist  
Can weather be  
predicted using  
mathematical  
principles?

Image source: Easterbrook, S. M. (2023) Computing the Climate. Cambridge University Press.

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But how did Charney write a weather forecasting program for ENIAC?

To answer that question we have to go back to the four scientists we met last last week, all members of the Stockholm Cosmic Physics Society.

To write his weather forecasting program, Charney was building on the work of the fourth member of this group – the Norwegian scientist Vilhelm Bjerknes.

Bjerknes originally trained as a physicist, and was following in his father's footsteps, doing important work on magnetic fields. In the process, he came up with the key questions for forecasting movement of the atmosphere.

## Basic physical equations

1904: Vilhelm Bjerknes identified the “primitive equations”

These capture the flow of mass and energy in the atmosphere;

Sets out a manifesto for practical forecasting



Zonal (East-West) Wind:

$$\frac{\partial u}{\partial t} = \eta v - \frac{\partial \Phi}{\partial x} - c_p \theta \frac{\partial \pi}{\partial x} - z \frac{\partial u}{\partial \sigma} - \frac{\partial (u^2 + v^2)}{\partial x}$$

Meridional (North-South) Wind:

$$\frac{\partial v}{\partial t} = -\eta u - \frac{\partial \Phi}{\partial y} - c_p \theta \frac{\partial \pi}{\partial y} - z \frac{\partial v}{\partial \sigma} - \frac{\partial (u^2 + v^2)}{\partial y}$$

Temperature:

$$\frac{\delta T}{\delta t} = \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z}$$

Precipitable Water:

$$\frac{\delta W}{\delta t} = u \frac{\partial W}{\partial x} + v \frac{\partial W}{\partial y} + w \frac{\partial W}{\partial z}$$

Air pressure:

$$\frac{\partial}{\partial t} \frac{\partial p}{\partial \sigma} = u \frac{\partial}{\partial x} x \frac{\partial p}{\partial \sigma} + v \frac{\partial}{\partial y} y \frac{\partial p}{\partial \sigma} + w \frac{\partial}{\partial z} z \frac{\partial p}{\partial \sigma}$$

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Bjerknes thought that weather ought to be predictable in the same way the movements of the planets are: If you know the current state of the atmosphere, and you have a set of equations that express how forces such as gravity and the planet’s rotation act upon the atmosphere, you should be able to calculate how the weather will change over time. His “manifesto” for weather prediction called for a two step: process: (1) a diagnostic step to assess the current state of the atmosphere and (2) a prognostic step to calculate the next state of the atmosphere.

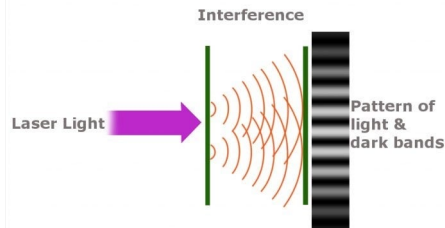
Bjerknes thought that there was already enough data being collected that step (1) was feasible. For step 2, he worked out a set of equations, based on Newton’s laws of motion, but realized they would be too complicated to use for practical forecasting. He worked out seven equations, one for each of the seven key variables he had identified: wind speed in three directions (N-S, E-W, and vertical), temperature, air pressure, water content, and a final equation for conservation of mass.

The five equations on this slide are a slightly simplified set (!).



## An aside: “Luminiferous Aether”

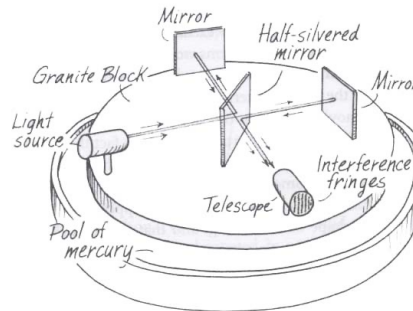
Light sometimes behaves like a wave:



But how do light waves travel through the vacuum of space?

19th Century scientists believed an invisible substance called the luminiferous aether carried the waves

The Michelson-Morley experiment disproved this:



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Originally, Bjerknes wasn't interested in weather forecasting at all. His early work was on magnetic fields, and he spent ten years writing up his father's work as a book on magnetism. At the time, the mechanisms by which magnetism worked were mysterious, and generally explained by assuming an invisible substance known at the “Luminiferous Aether”. In the middle of the nineteenth century, James Clerk Maxwell had showed that light and magnetism are essentially the same thing, both travelling through space as a series of waves. The classic experiment to show that light acts as a wave is to pass light through two narrow slits. After passing through the slits the waves spread out, like ripples on a pond, and interfere with one another, so that the image projected looks like bands – light where two waves are in sync, and dark where they cancel each other. But waves must be carried by something, so the Aether was assumed to be a fluid that could not be seen or measured, the only way of explaining how light can travel through the vacuum of space, and how magnets apply a force to things they are not touching.

But the Michelson-Morley experiment proved it doesn't exist. The idea is that planet Earth must be moving through this mysterious Aether. But this should be detectable: if you split a beam of light, and send it in two different directions, you should be able to align things so that one of beam is heading into the Aether (which will compress the waves), and the other away from it, which will stretch them. But nothing happens! Then along came Einstein and explains it all with relativity and curved space-time, which we won't get into today, but it meant that Bjerknes's book was irrelevant almost as soon as it was published.

## Andrée's Arctic Ballooning Expedition



Photo of the crashed balloon taken in 1897 by Nils Strindberg. The film from his camera was recovered in 1930, when the crash site was finally discovered. (Photo: public domain)

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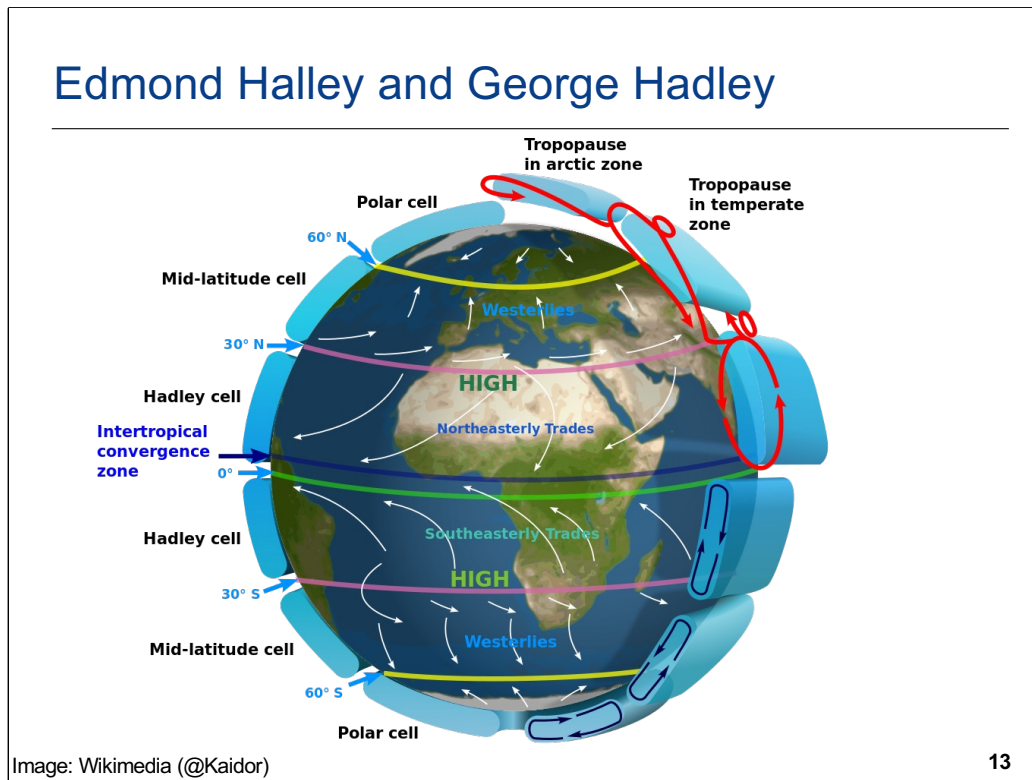
Despite the lack of interest from physics, Bjerknes's colleagues in Stockholm were enthused by his work on fluid flow, as they saw the potential for applications to understanding and predicting the winds.

Then a tragedy struck – a major ballooning expedition set off to fly over the North Pole, with several explorers from Stockholm on board. It was sponsored by the King of Sweden, and got a huge amount of publicity. The explorer Nils Eckholm from the Stockholm Cosmic Physics Society was supposed to be on it, but he backed out at the last minute as he mistrusted the design of the balloon.

The balloon took off from Svalbard in July 1897. The expedition was supposed to last 30 days, but the balloon crashed onto the pack ice in the arctic ocean after only two days. The three men in the balloon, Salomon August Andrée, Knut Freankel, and Nils Strindberg survived the crash, and trekked across the ice for 30 days. But with poor equipment and faulty navigation, they never made it back to safety.

For many years, nobody knew what had happened, and the fate of the balloon expedition remained a huge mystery in Sweden until the final camp was discovered in 1930.

In the days after they lost contact, there was a lot of talk of mounting a rescue, but they would need to predict which way the winds would have taken it. They needed Bjerknes's theorems. His colleagues started a fund to pay for the work, named it after Bjerknes, and gradually drew him into the project.



Bjerknes's equations derive from Newton's laws of motion.

The first person to apply Newton's laws to study the movement of the atmosphere was Edmond Halley (of Halley's comet fame).

In 1686, he published maps of the trade winds.

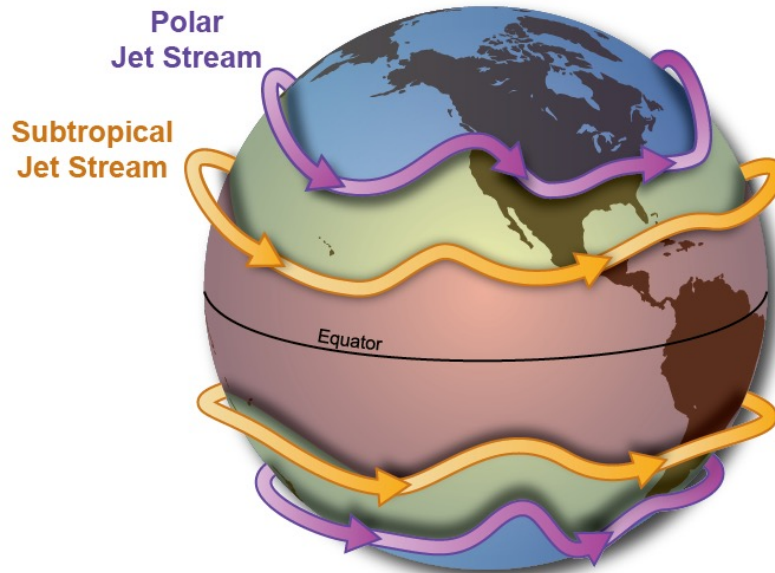
Hot air rises in the tropics over the equator, drawing in cooler air from the poleward regions to replace it. By the time this air reaches the northern edge of the subtropics, it has cooled, and so it sinks again, creating vertical cycles of air (the red arrows on this diagram).

Halley thought the characteristic pattern of trade winds (NE to SW) in the northern tropics was due to the sun progressing east to west each day, so the rising air is always further east than the cooler air that replaces it.

But in 1735 meteorologist George Hadley came up with a better explanation, so the patterns are named after him.

But it wasn't until Bjerknes that anyone worked out the equations needed to predict these patterns.

## Rossby Waves in the Jet Stream

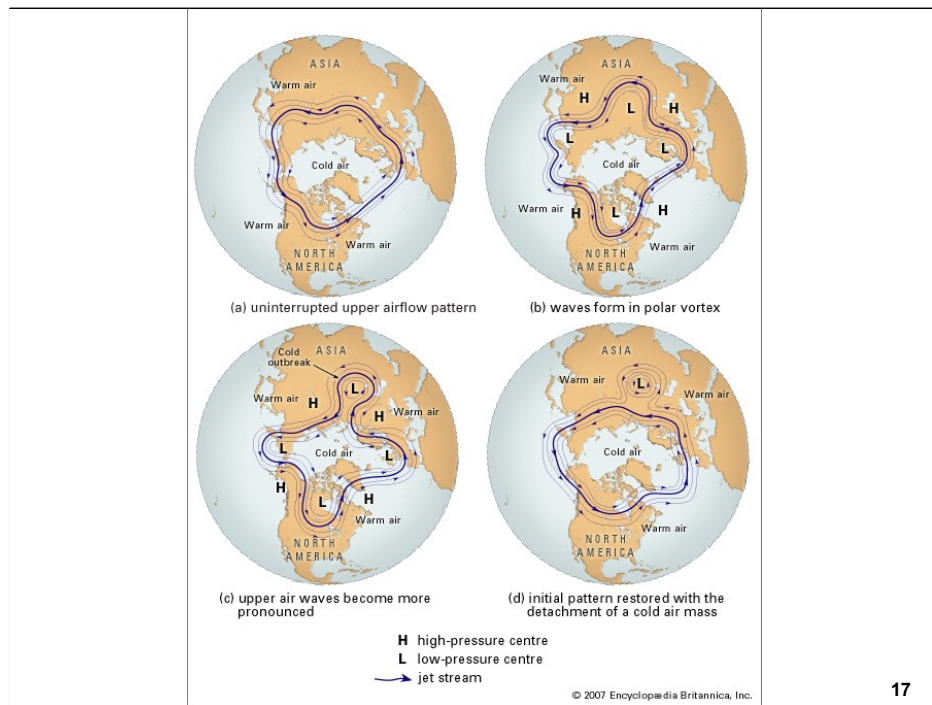


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Because of the Coriolis force, the air moving away from the equator in the Hadley cells is deflected towards the east. When it gets far enough from the equator, it ends up moving due east. And this pattern persists all the way around the planet.

So the Hadley cells cause a characteristic pattern of jet streams in the upper atmosphere – there are two main jet streams in each hemisphere, one over the subtropics, and one closer to the polar regions.

These streams of fast moving air circle the planet, but meander a little as they do. The resulting waves are known as Rossby waves, after the meteorologist who first discovered them.



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The jet stream also acts as a barrier that circles the planet, keeping the cold air over the poles contained. But the meanders caused by Rossby waves occasionally form complete loops, which then break off and take a parcel of freezing cold air with them. In the winter, when we get a “polar vortex” in Toronto, that’s what’s happening.

If you can predict the path of the Rossby waves, you have a headstart on predicting changing weather patterns in the regions close to the Jetstream – which includes most of North America and Europe – as the jet streams are the main driver that moves weather systems around the planet.



## Towards Numerical Forecasts

1910s: Lewis Fry Richardson performs the first numerical weather forecast, imagines a giant computer to do this regularly;

First plan for massively parallel computation



Image Source: Lynch, P. (2008). The origins of computer weather prediction and climate modeling.

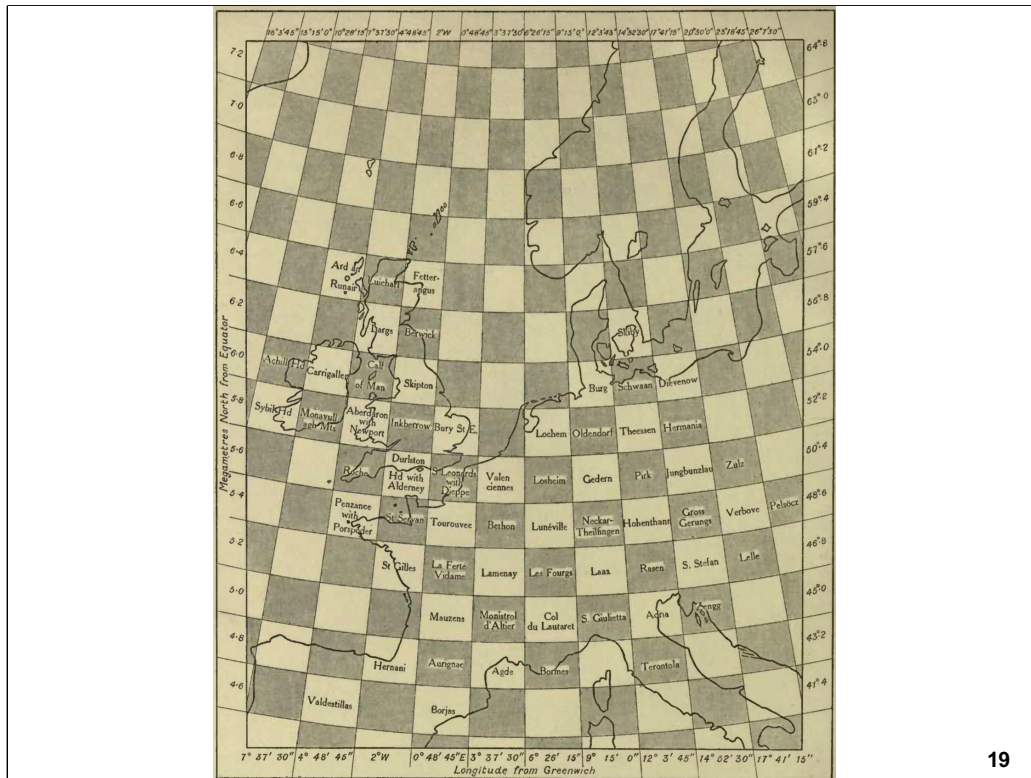
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Bjerknes had worked out the equations, but didn't have a computer to run them on – he thought using them to calculate the weather would be impossible. However, in the early 20<sup>th</sup> Century the English meteorologist Lewis Fry Richardson made an attempt. In the middle of his work as an ambulance driver on the front line in France during WW1, Richardson computed the first ever numerical forecast, calculating the change in wind and air pressure at two locations in France, using the starting conditions that were observed for 20 May 1910, and using the equations to calculate the weather conditions for 24 hours later. It took him about 2 years (!) to do the calculations, working roughly 10 hours per week in his spare time (mostly in a tent with a gaslamp).

His forecast was spectacularly wrong, but largely because of inaccuracies in the initial conditions. His notes were rediscovered (under a coal heap!) after the war, and he published them as a book.

He estimated that 32 people working together could have done the calculation fast enough to keep up with the real weather.

Furthermore, that was for just one grid point. He imagined dividing up the entire globe into 200km squares, needing about 2000 columns of air, and assigning 32 “computers” (people!!) to each, for a total of 64,000 people. He imagined them on balconies in a huge hollow sphere, with a conductor in the middle shining beams of light on those computers who were falling behind in their task.



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Here's how Richardson's method worked:

Measure the current air pressure at the centre of each shaded cell, and measure the current wind speed at the centre of each unshaded cell.

You can then predict the next measurement for pressure in the shaded cells (say an hour later) by calculating how the winds in the neighbouring cells are pushing air in or out.

Likewise, you can predict the next measurement for wind speed in each unshaded cells by the difference in air pressure in the cells each side of it.

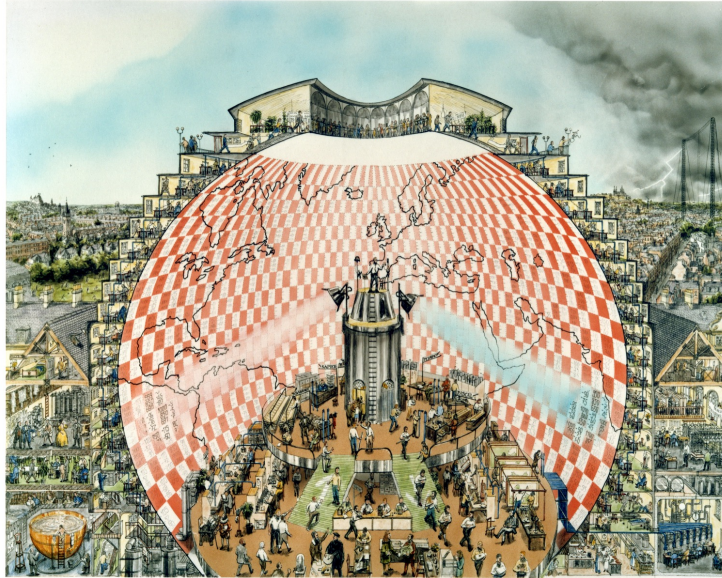
You also need a bit more information for each cell: e.g. temperature and humidity, as these also affect air pressure. But the key idea is that the next value for air pressure is calculated from differences in the wind speeds, and the next value for winds speeds is calculated from differences in air pressure.

This way you can keep going, step by step, calculating the next value (air pressure or wind speed) for each cell.

Problem: you don't know what's going on beyond the edge of your map, so it's like you lose all the edge cells at each step. Your map gets smaller and smaller as you predict further into the future.

This image is from inside the front cover of his 1922 book.

## Richardson's Forecast Factory



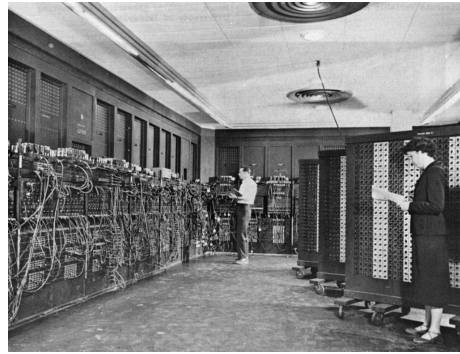
Here's another artist's rendering of Richardson's "forecast factory". You can see the conductor standing on the podium in the middle, shining beams of light to tell the "computers" (people) to speed up or slow down.

Incidentally, this very similar to how we do weather forecasts today, except the computers are electronic processors, and we use tens of thousands of them working in parallel, each doing a small part of the calculations.

## First Numerical Simulation of Weather

**Late 1940s:** John Von Neumann & Jule Charney develop a weather forecasting program for the 1st electronic computer, ENIAC:

Funded by the US Army for use in weather control, geo-engineering, etc.



See: Lynch, P. (2008). The ENIAC Forecasts: A Recreation. Bulletin of the American Meteorological Society, 1–11

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So, back the 1940s, and Charney and von Neumann's first weather forecasting program.

They used Bjerknes's equations, and Richardson's method. They realized the reason Richardson's one attempt to do a weather forecast (by hand) was wrong was because of problems with his data for the starting point of the forecast. If the measurements used to start the forecast were affected by local gusts of wind, they won't really represent the average wind speed over the whole of the grid square. So you need several measurements in each grid cell, and you need to smooth them out to remove any local gusts or anomalies.





Before testing their program on ENIAC, they did a smaller test to see if the program would work.

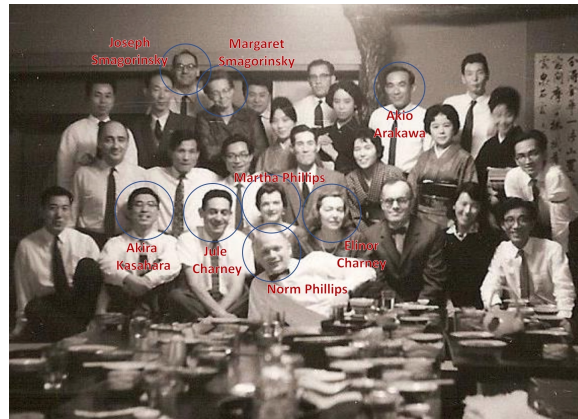
Instead of using grid cells over the whole planet, they just looked at a single latitude band, at the 45th parallel. That's just slightly north of Toronto, and it passes through southern Europe and central Asia. They chose 36 points around this latitude band, approximately 800km apart. And they only looked at East-West winds (ignoring North-South winds), so they could simplify the problem so that it could be calculated without an electronic computer.



## Hidden Figures



IAS team members in 1952. From left: Jule Charney, Norm Phillips, G Lewis, Norma Gilbarg, George Platzman (Photo: Joseph Smagorinsky, IAS)



Dinner party at the first International Numerical Weather Prediction conference in Tokyo, November 1960 (photo: Akira Kasahara)

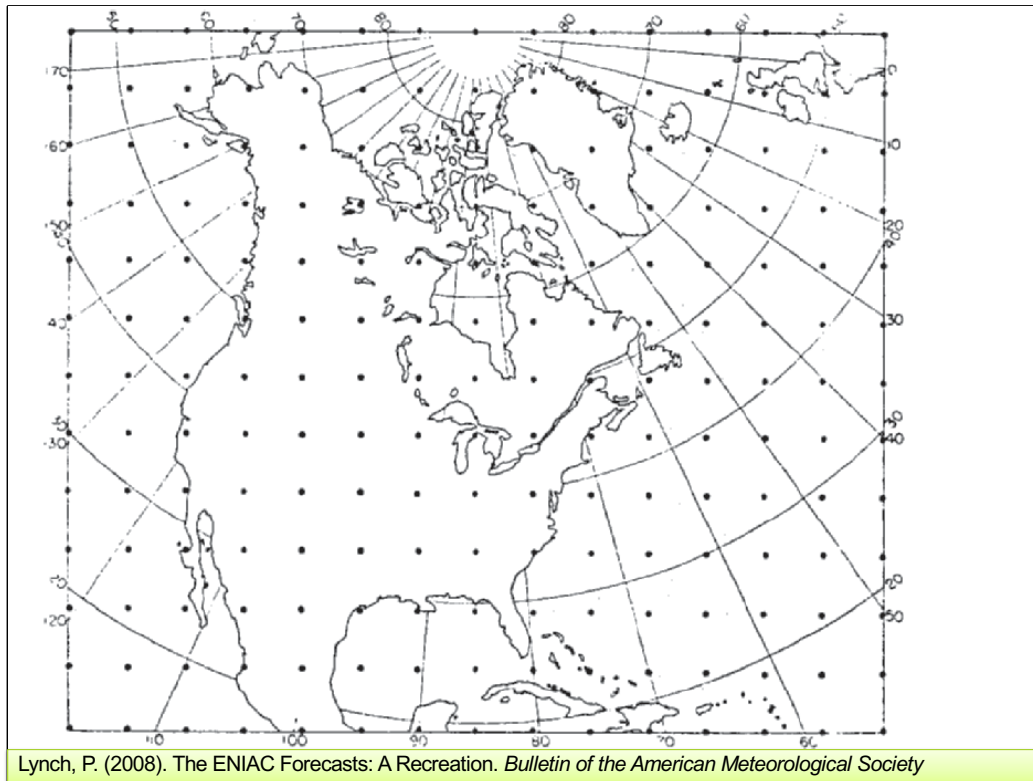
Missing from photos: Ellen-Kristine Eliassen (née Nome)

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The task was taken on by three women meteorologists: Norma Gilbarg, Margaret Smagorinsky and Ellen-Kristine Eliassen.

That means the first ever successful numerical weather forecast wasn't calculated by a machine – it was a 24 hour weather forecast for the 45<sup>th</sup> parallel, calculated by three women.

Just like the movie *Hidden Figures*, which tells the story of the women who did the calculations for NASA's Apollo missions, the work of these three women has been largely ignored – they weren't even mentioned in the published papers reporting this work.



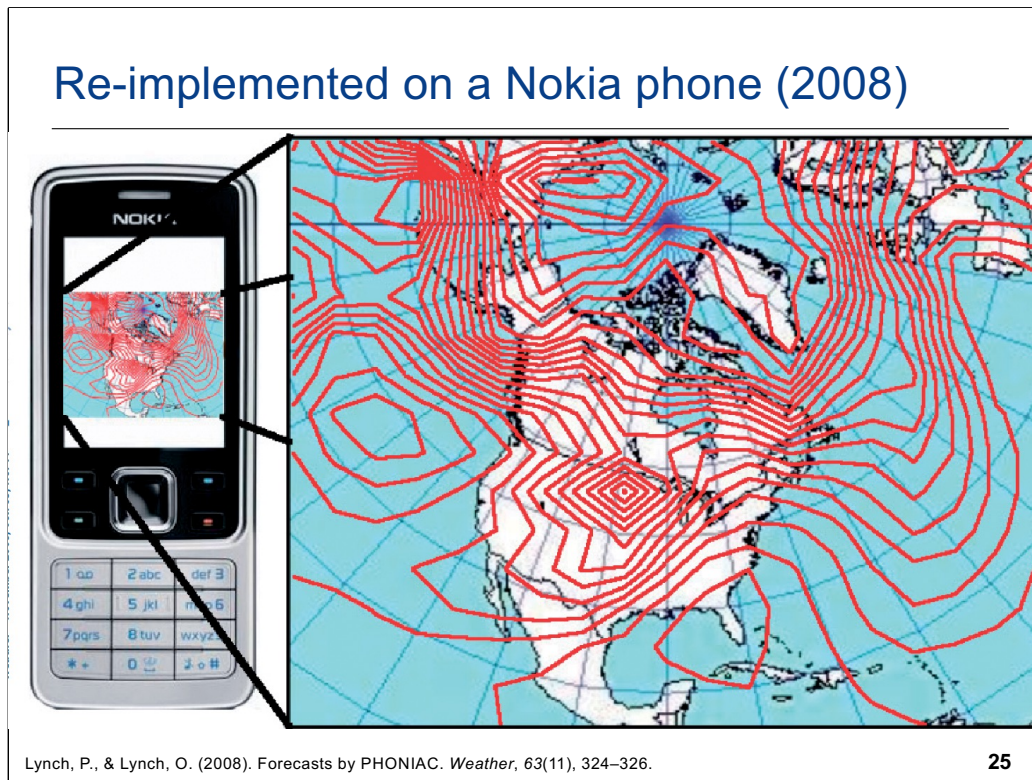
Charney and von Neumann were delighted with how well that worked, so they made plans to do a more complex forecast on ENIAC. They chose a grid covering just North America, with grid points still about 800km apart.

The program only calculated one single weather variable at each grid point, representing air pressure about halfway up the atmosphere.

Meteorologists call this the 500millibar Geopotential Height. Technically, that means the height at which the air is at 500 millibars of pressure. As air pressure at sea level is approximately 1000millibars, and air pressure gets steadily less as you go up, 500mb is the point where half of the mass of the atmosphere is above you, and half is below you.

One way to think about this is as if the bottom half of the atmosphere is made of a fluid substance that's different from air. Geopotential height then expresses how far off the ground the surface of this substance is at each of the grid points. Areas of high pressure (representing good weather) will be like peaks in this surface, and areas of low pressure (representing bad weather) will be like troughs.

Lynch, P. (2008). The ENIAC Forecasts: A Recreation. *Bulletin of the American Meteorological Society*, (January), 1–11.



It took more than 24 hours on ENIAC to calculate a 24 hour forecast for all the grid points in this region. In other words, ENIAC was slower than the real weather.

But that didn't bother Charney and von Neumann: they knew all they needed was faster computer, and they could calculate the forecasts faster than the real weather.

Computers have gotten steadily faster since those days. A few years ago, someone adapted their program to run on a Nokia cellphone, and it took less than a second to run.

### (What a forecast factory actually looks like)



The Yellowstone supercomputer at the NCAR Wyoming Supercomputing Center, Cheyenne

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Today, we do weather forecasts on massive supercomputers that look like this. They're a bit like Richardson's idea for a forecast factory, in that they contain tens of thousands of processors, running in parallel, each doing one small piece of the calculation.

And today's weather forecast programs are a lot more complex: they calculate all the weather variables at many different heights in the atmosphere, using a grid with much smaller spacing than the grid that Charney and von Neumann used.

But at heart, the method is the same: the same set of equations, and Richardson's "finite differences" method to calculate how each variable changes at the next time-step (typically each hour, or every three hours)