Deep Generative Models

Shenlong Wang

Overview

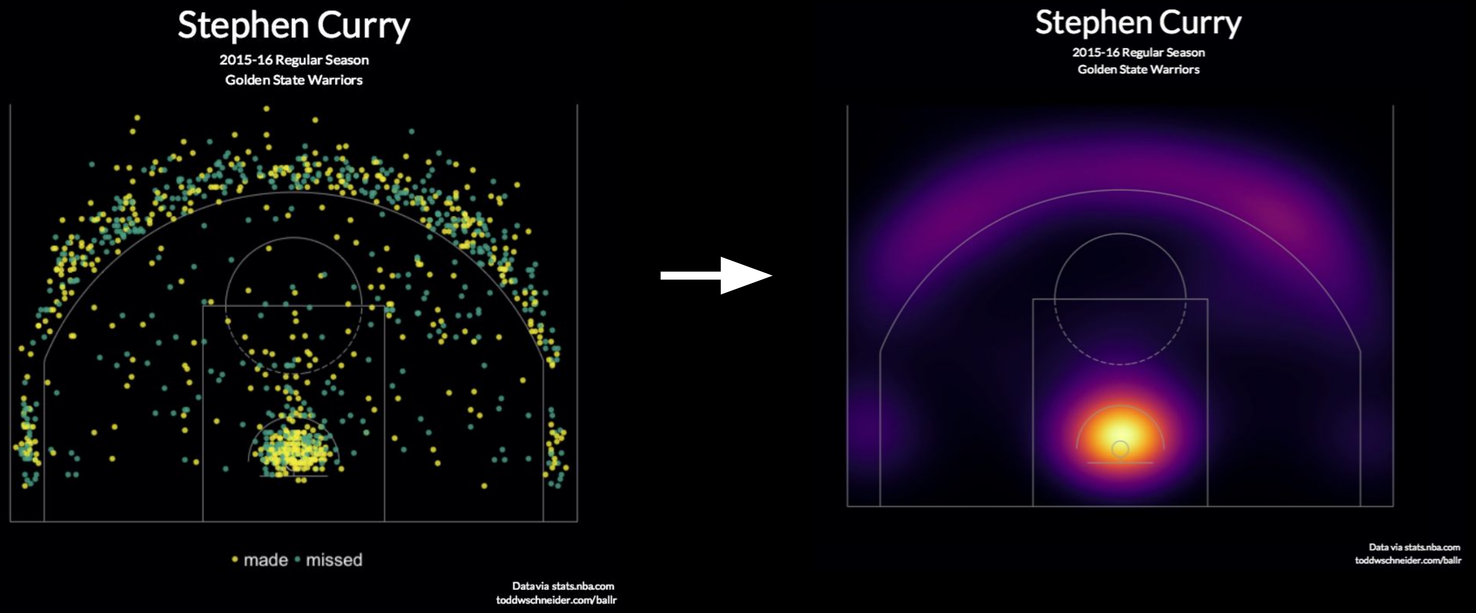
- Why unsupervised learning?
- Old-school unsupervised learning
  - PCA, Auto-encoder, KDE, GMM
- Deep generative models
  - VAEs, GANs

Unsupervised Learning

- No labels are provided during training
- General objective: inferring a function to describe hidden structure from unlabeled data
    - Density estimation (continuous probability)
    - Clustering (discrete labels)
    - Feature learning / representation learning (continuous vectors)
    - Dimension reduction (lower-dimensional representation)
    - etc.

Why Unsupervised Learning?

- Density estimation: estimate the probability density function p(x) of a random variable x, given a bunch of observations {X1, X2, ...}



2D density estimation of Stephen Curry's shooting position

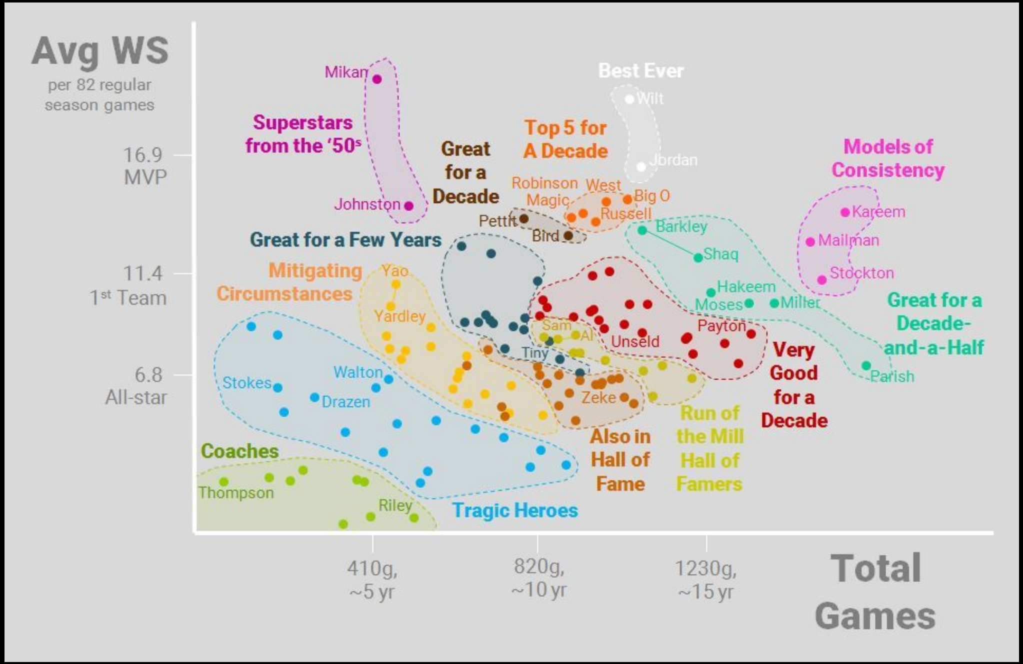
Credit: BallR

Why Unsupervised Learning?

- Density estimation: estimate the probability density function p(x) of a random variable x, given a bunch of observations {X1, X2, ...}

Why Unsupervised Learning?

- Clustering: grouping a set of input {X1, X2, ...} in such a way that objects in the same group (called a cluster) are more similar

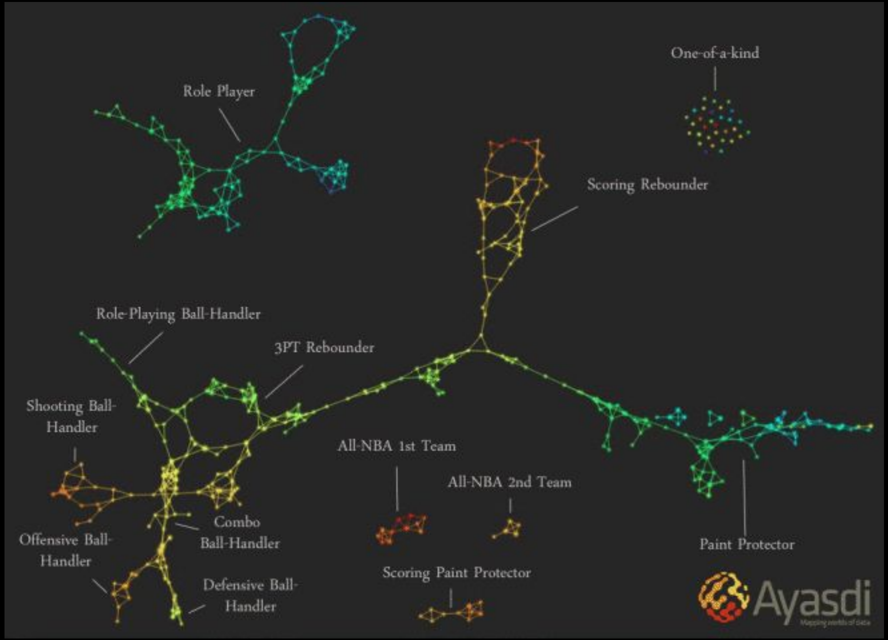


Clustering analysis of Hall-of-fame players in NBA

Credit: BallR

Why Unsupervised Learning?

- Feature learning: a transformation of raw data input to a representation that can be effectively exploited in machine learning tasks

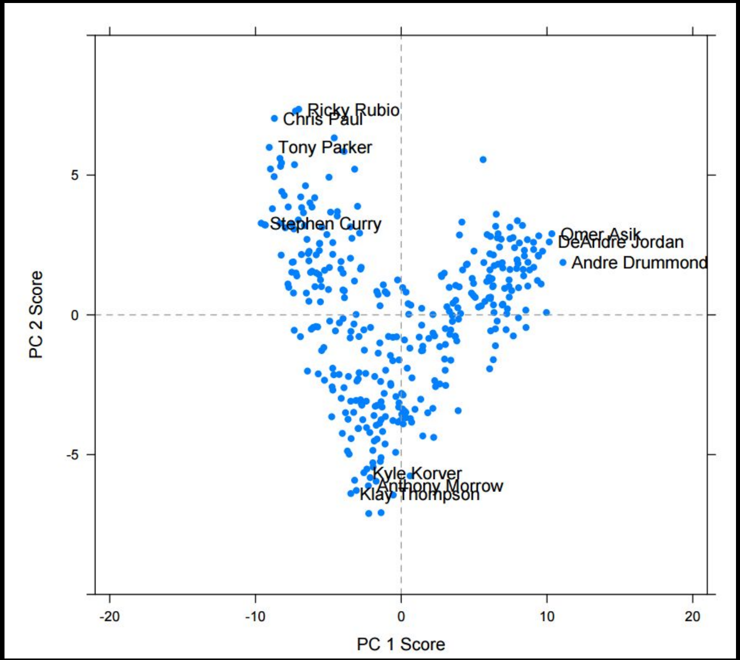


2D topological visualization given the input how similar players are with regard to points, rebounds, assists, steals, rebounds, blocks, turnovers and fouls

Credit: Ayasdi

Why Unsupervised Learning?

- Dimension reduction: reducing the number of random variables under consideration, via obtaining a set of principal variables



Principle component analysis over players trajectory data

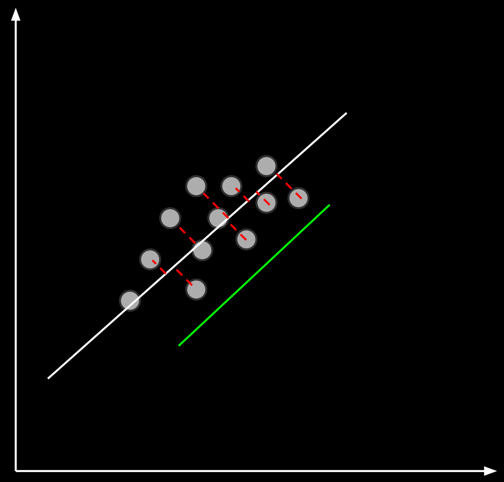
Credit: Bruce, Arxiv 2016

Principle Component Analysis (PCA)

An algorithm that conducts dimension reduction

Intuition:

- Finds the lower-dimension projection that minimizes reconstruction error
- Keep the most information (maximize variance)



See more details in Raquel's CSC411 slides:
http://www.cs.toronto.edu/~urtasun/courses/CSC411_Fall16/14_pca.pdf

Principle Component Analysis (PCA)

An algorithm that conducts dimension reduction

Intuition:

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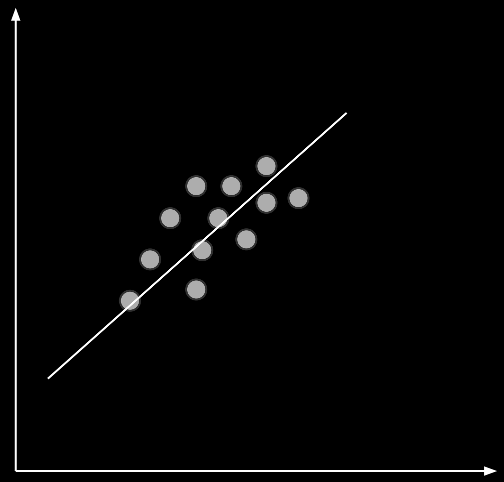
Algorithm:

- Conduct eigen decomposition
- Find K-largest eigenvectors
- Linear projection with the matrix composed of K eigenvectors



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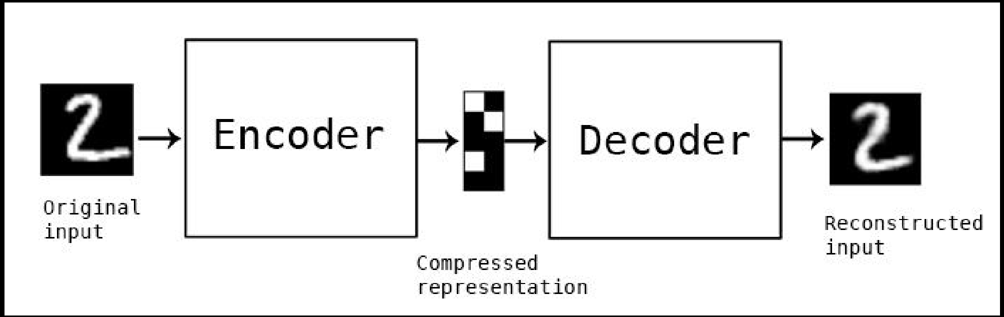
Auto-encoder

A neural network that the output is the input itself.

Intuition:

- A good representation should keep the information well (reconstruction error)
- Deep + nonlinearity might help enhance the representation power

$$\min_{\mathbf{w}_1, \mathbf{w}_2} \|\mathbf{x}_i - g(f(\mathbf{x}_i; \mathbf{w}_1); \mathbf{w}_2)\|_2^2$$



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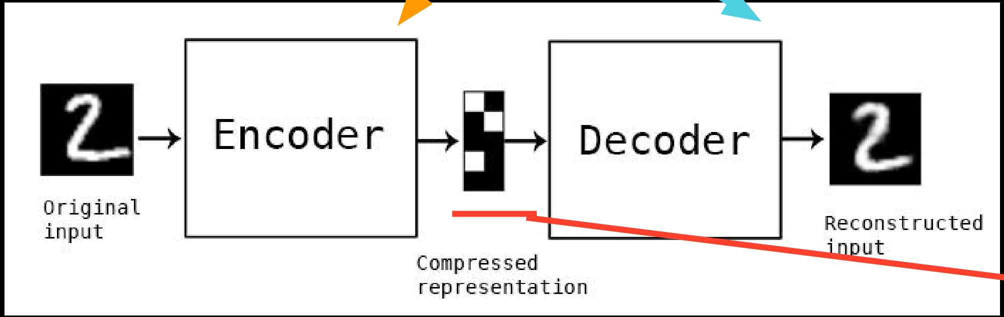


Learnt representation

Credit: LeCun

Auto-encoder

A neural network that the output is the input itself.

10-dimensional Auto-encoder feature embedding based on players shooting tendency

Credit: Wang et al. 2016 Sloan Sports Conference

Kernel Density Estimation (KDE)

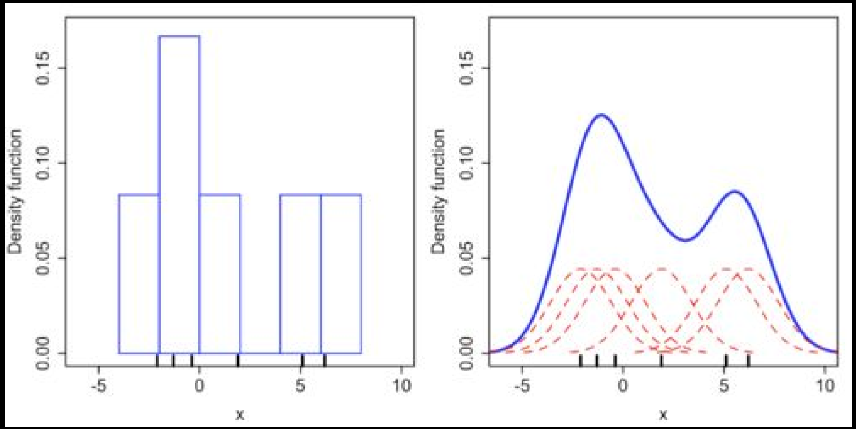
A nonparametric way to estimate the probability density function of a random variable

Intuition:

- Point with more neighbouring samples have higher density
- Smoothed histogram, centered at data point

$$f(\mathbf{x}) = \frac{1}{N}\sum_i \frac{1}{h_i} \underline{K(\frac{\mathbf{x}-\mathbf{x}_i}{h_i})}$$

Kernel function, measures the similarity



Credit: Wikipedia

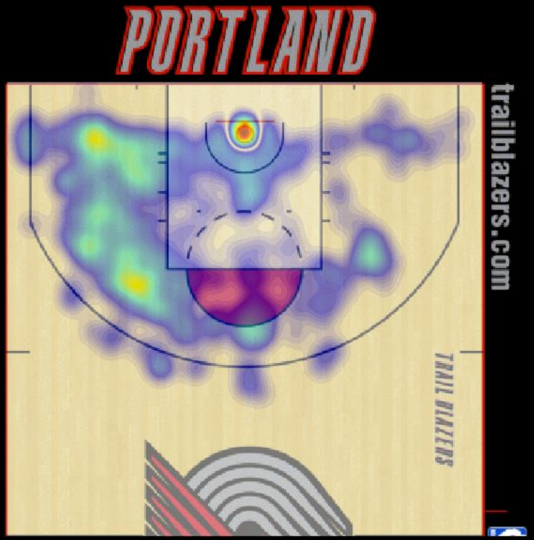
Kernel Density Estimation (KDE)

A nonparametric way to estimate the probability density function of a random variable

Applications:

- Visualization
- Sampling

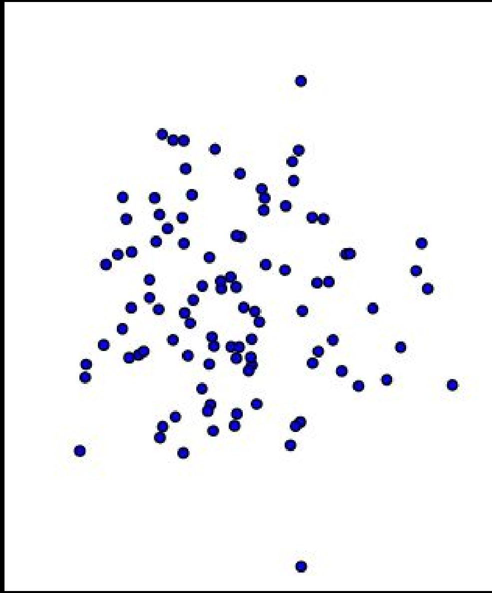
$$f(\mathbf{x}) = \frac{1}{N}\sum_i \frac{1}{h_i} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h_i}\right)$$



Shooting heat map of Lamarcus Aldridge 2015-2016. Credit: Squared Statistics

Generative models

Task: generate new samples follows the same probabilistic distribution of a given a training dataset



Generative models

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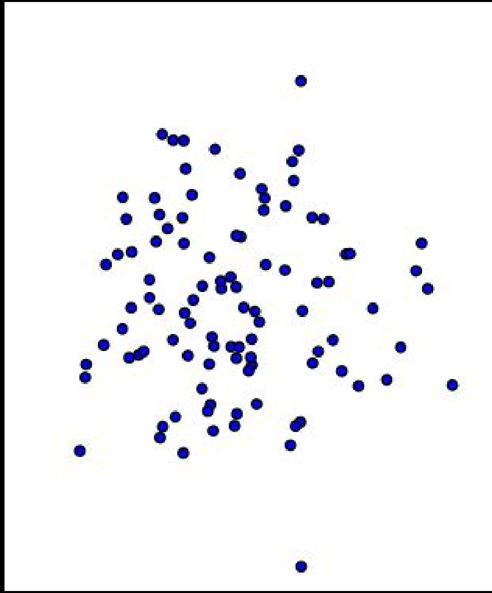
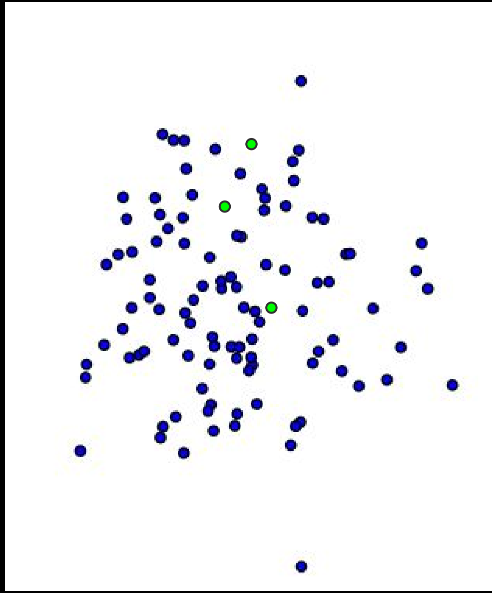


$$Z \sim \mathcal{N}(0, 1)$$



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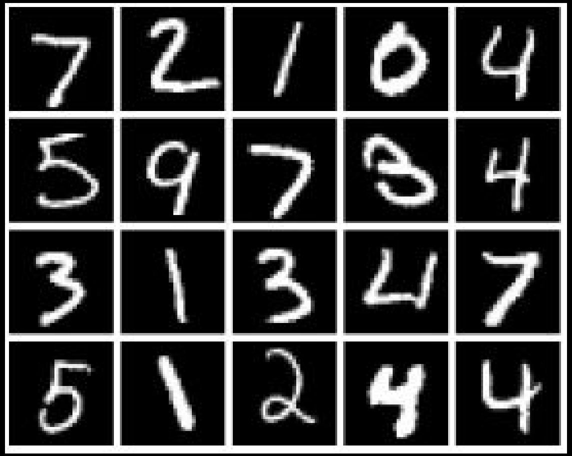
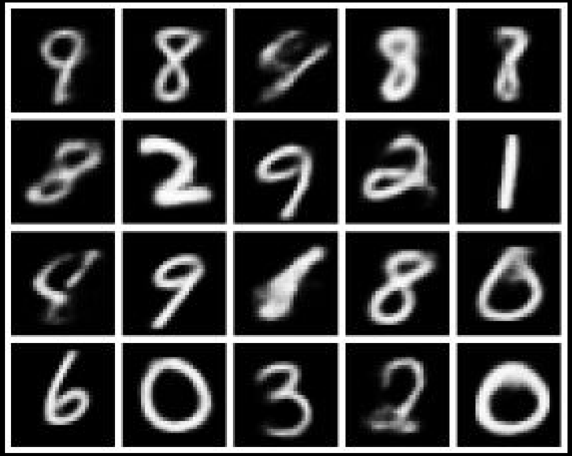


Training samples

$$p(x) = ?$$



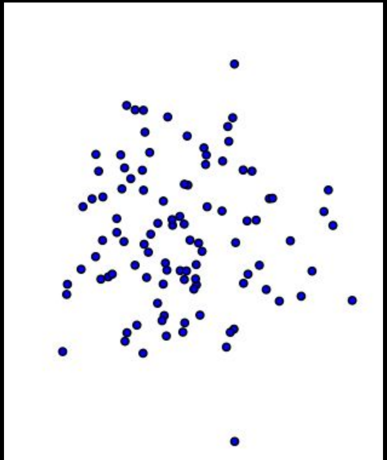
Generated samples

Credit: Kingma

Note: sometimes it’s fine if we cannot estimate the explicit form of p(x), since it might be over complicated

Variational Auto-encoder (VAE)

Intuition: given a bunch of random variables that can be sampled easily, we can generate random samples following other distributions, through a complicated non-linear mapping x = f(z)



$$f(z) = z/10 + z/\|z\|$$

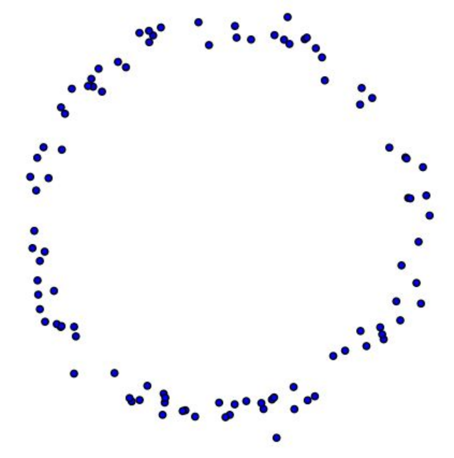
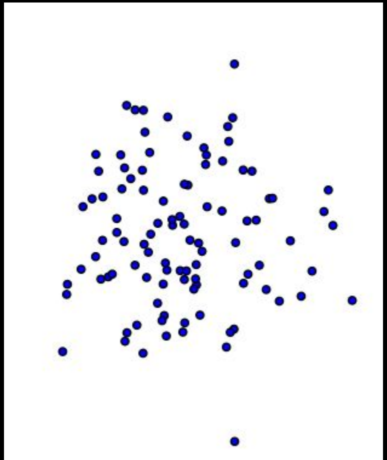


Image Credit: Doersch 2016

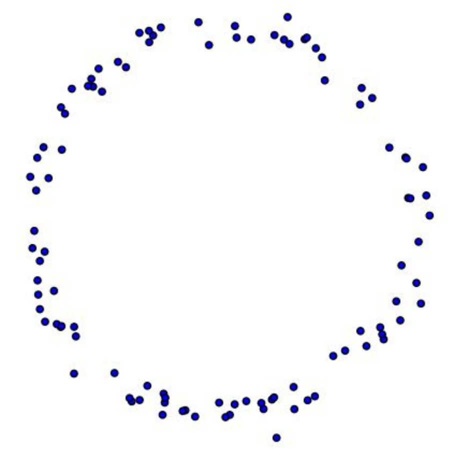
Variational Auto-encoder (VAE)

Intuition: given a bunch of random variables that can be sampled easily, we can generate some new random samples through a complicated non-linear mapping x = f(z)



$Z \sim \mathcal{N}(0,1)$

$X \sim \mathcal{N}(f(z;\theta), \sigma^2 I)$



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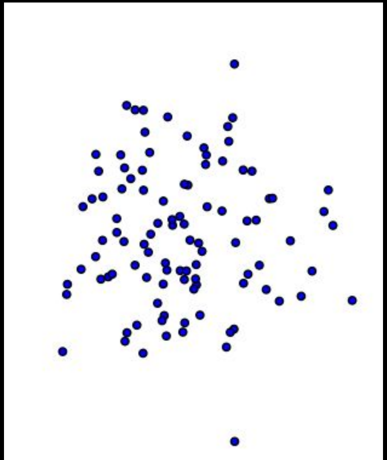

$Z \sim \mathcal{N}(0, 1)$

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Gaussian

NN

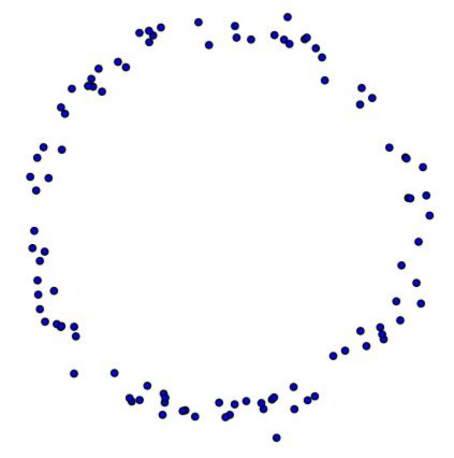
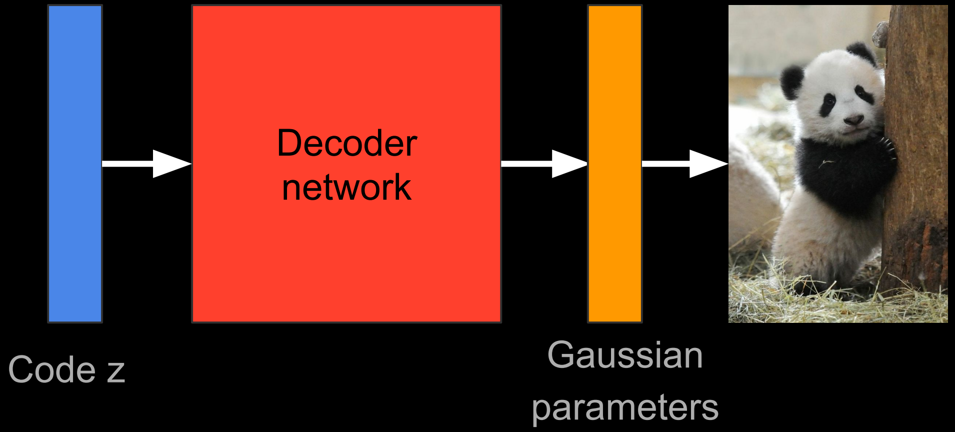


Image Credit: Doersch 2016

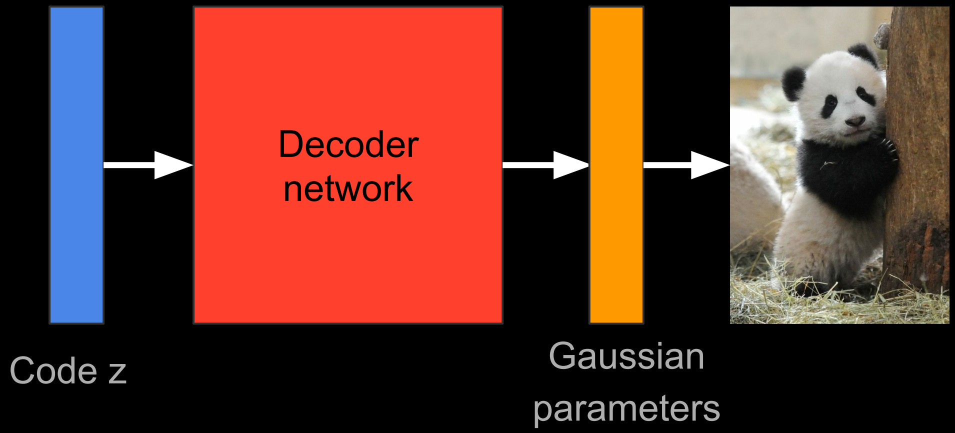
Variational Auto-encoder (VAE)

You can consider it as a decoder!



Variational Auto-encoder (VAE)

How do we learn the parameters of decoder network?



Variational Auto-encoder (VAE)

Review: Marginalization

$$p(\mathbf{x}) = \int p_\theta(\mathbf{x}|\mathbf{z})p_\theta(\mathbf{z})d\mathbf{z}$$

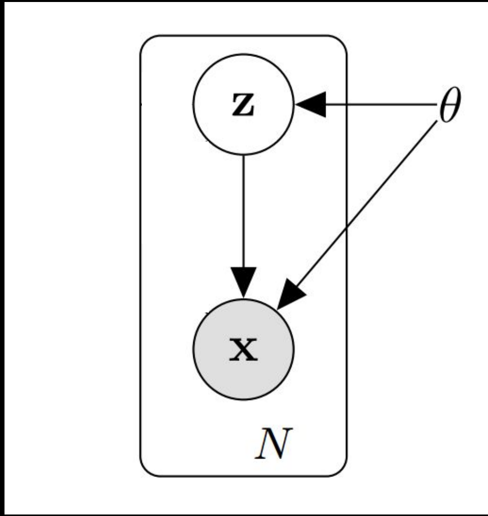


Image Credit: Doersch 2016

Variational Auto-encoder (VAE)

Review: Marginalization

$$p(\mathbf{x}) = \int p_\theta(\mathbf{x}|\mathbf{z})p_\theta(\mathbf{z})d\mathbf{z}$$

[0, 0.2, -0.5]

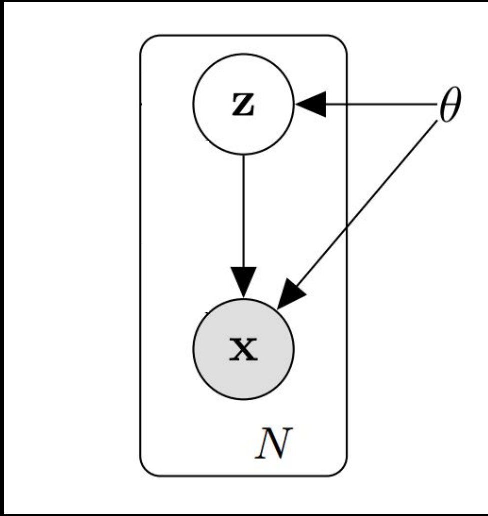


Image Credit: Doersch 2016

Variational Auto-encoder (VAE)

Learning objective: maximize the log-probability

$$\max_{\theta} \sum_{i} \log p_{\theta}(\mathbf{x}_i)$$

Training images should have high probability

$$p_{\theta}(\mathbf{x}) = \int p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\theta}(\mathbf{z}) d\mathbf{z}$$

Image Credit: Doersch 2016

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Integration over a neural network. Difficult!

Image Credit: Doersch 2016

Variational Auto-encoder (VAE)

Learning objective: maximize the log-probability

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Integration over a neural network. Difficult!

Quiz: Why not do this?

$$\log p_\theta(\mathbf{x}) \approx \log \frac{1}{N} \sum_j p_\theta(\mathbf{x}|\mathbf{z}_j)$$

Image Credit: Doersch 2016

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$$p_{\theta}(\mathbf{x}) = \int p_{\theta}(\mathbf{x}|\mathbf{z}) p_{\theta}(\mathbf{z}) d\mathbf{z}$$

many sampled z will have a close-to-zero p(x|z)

Quiz: Why not do this?

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Image Credit: Doersch 2016

Variational Auto-encoder (VAE)

Learning objective: maximize variational lower-bound

$$\log p_\theta(\mathbf{x}_i) \geq \underbrace{\mathbb{E}_{q(\mathbf{z})}[\log p_\theta(\mathbf{x}_i|\mathbf{z})] - KL[q(\mathbf{z})||p_\theta(\mathbf{z})]}$$

Variational lower-bound

Proposal distribution

Quiz: How to choose a good proposal distribution?

Variational Auto-encoder (VAE)

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Variational lower-bound

Proposal distribution

Quiz: How to choose a good proposal distribution?

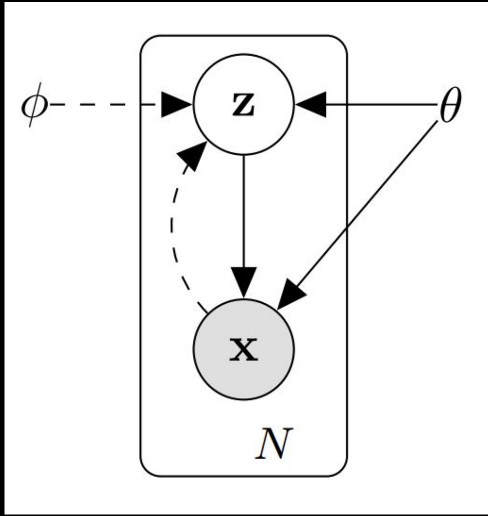
- Easy to sample
- Differentiable wrt parameters
- Given a training sample X, the sampled z is likely to have a non-zero p(x|z)

Variational Auto-encoder (VAE)

Learning objective: maximize variational lower-bound

$$\log p_\theta(\mathbf{x}_i) \geq \mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x}_i)}[\log p_\theta(\mathbf{x}_i|\mathbf{z})] - KL[q_\phi(\mathbf{z}|\mathbf{x}_i)||p_\theta(\mathbf{z})]$$

Answer: Another neural network + Gaussian to approximate the posterior!



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Reconstruction error:

- Training samples have higher probability

Prior:

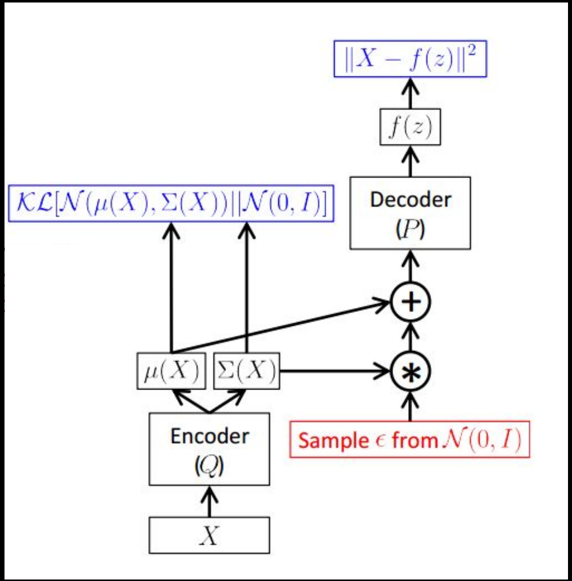
- Proposal distribution should be like Gaussian

Variational Auto-encoder (VAE)

Learning objective: maximize variational lower-bound

$$\log p_\theta(\mathbf{x}_i) \geq \underline{\mathbb{E}_{q_\phi(\mathbf{z}|\mathbf{x}_i)}[\log p_\theta(\mathbf{x}_i|\mathbf{z})]} - \underline{KL[q_\phi(\mathbf{z}|\mathbf{x}_i)||p_\theta(\mathbf{z})]}$$

- KL-Divergence: closed-form and differentiable if both are Gaussians
- Reconstruction error: approximate by just sampling one z



Computation graph
Credit: Doersch

Variational Auto-encoder (VAE)

Why it is the variational lower-bound?

$$\log p_\theta(\mathbf{x}) = \log \int p_\theta(\mathbf{x}|\mathbf{z}) p_\theta(\mathbf{z}) d\mathbf{z}$$

$$\log p_\theta(\mathbf{x}) = \log \int p_\theta(\mathbf{x}|\mathbf{z}) \frac{p_\theta(\mathbf{z})}{q(\mathbf{z})} q(\mathbf{z}) d\mathbf{z}$$

Jenson inequality

$$\log \int p(\mathbf{x}) g(\mathbf{x}) d\mathbf{x} \geq \int p(\mathbf{x}) \log g(\mathbf{x}) d\mathbf{x}$$

$$\log p_\theta(\mathbf{x}) \geq \int q(\mathbf{z}) \log \left(p_\theta(\mathbf{x}|\mathbf{z}) \frac{p_\theta(\mathbf{z})}{q(\mathbf{z})} \right) d\mathbf{z}$$

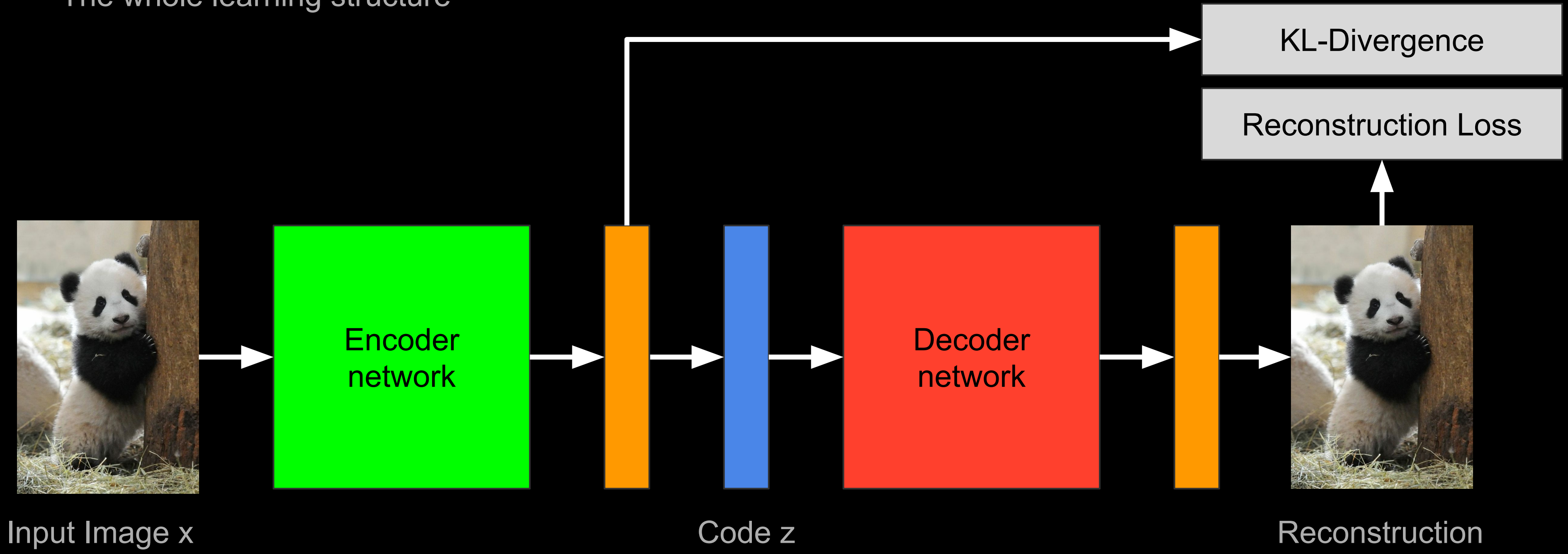
$$\log p_\theta(\mathbf{x}) \geq \int q(\mathbf{z}) \log p_\theta(\mathbf{x}|\mathbf{z}) d\mathbf{z} - \int q(\mathbf{z}) \log \frac{p_\theta(\mathbf{z})}{q(\mathbf{z})} d\mathbf{z}$$

$$\log p_\theta(\mathbf{x}) \geq \mathbb{E}_{q(\mathbf{z})}[\log p_\theta(\mathbf{x}|\mathbf{z})] - KL[q(\mathbf{z})||p_\theta(\mathbf{z})]$$

Kingma et al. 2014

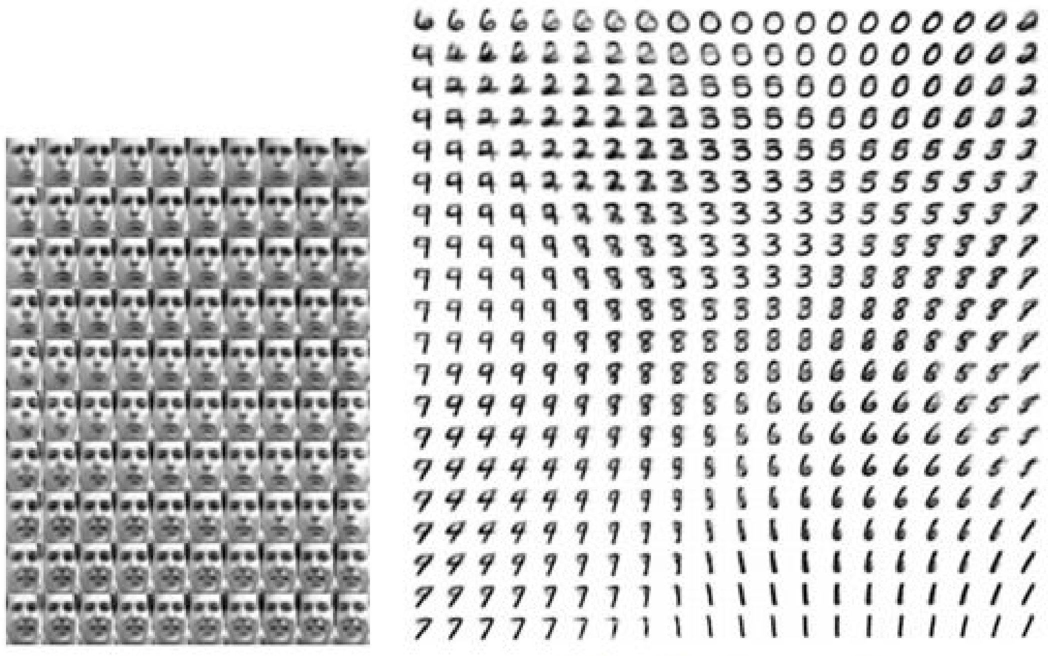
Variational Auto-encoder (VAE)

The whole learning structure



Variational Auto-encoder (VAE)

Results



(a) Learned Frey Face manifold (b) Learned MNIST manifold

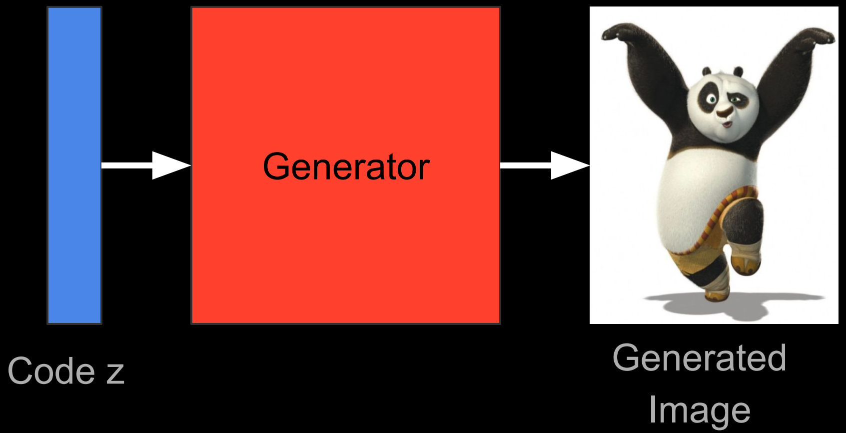
Kingma et al. 2014

Variational Auto-encoder (VAE)

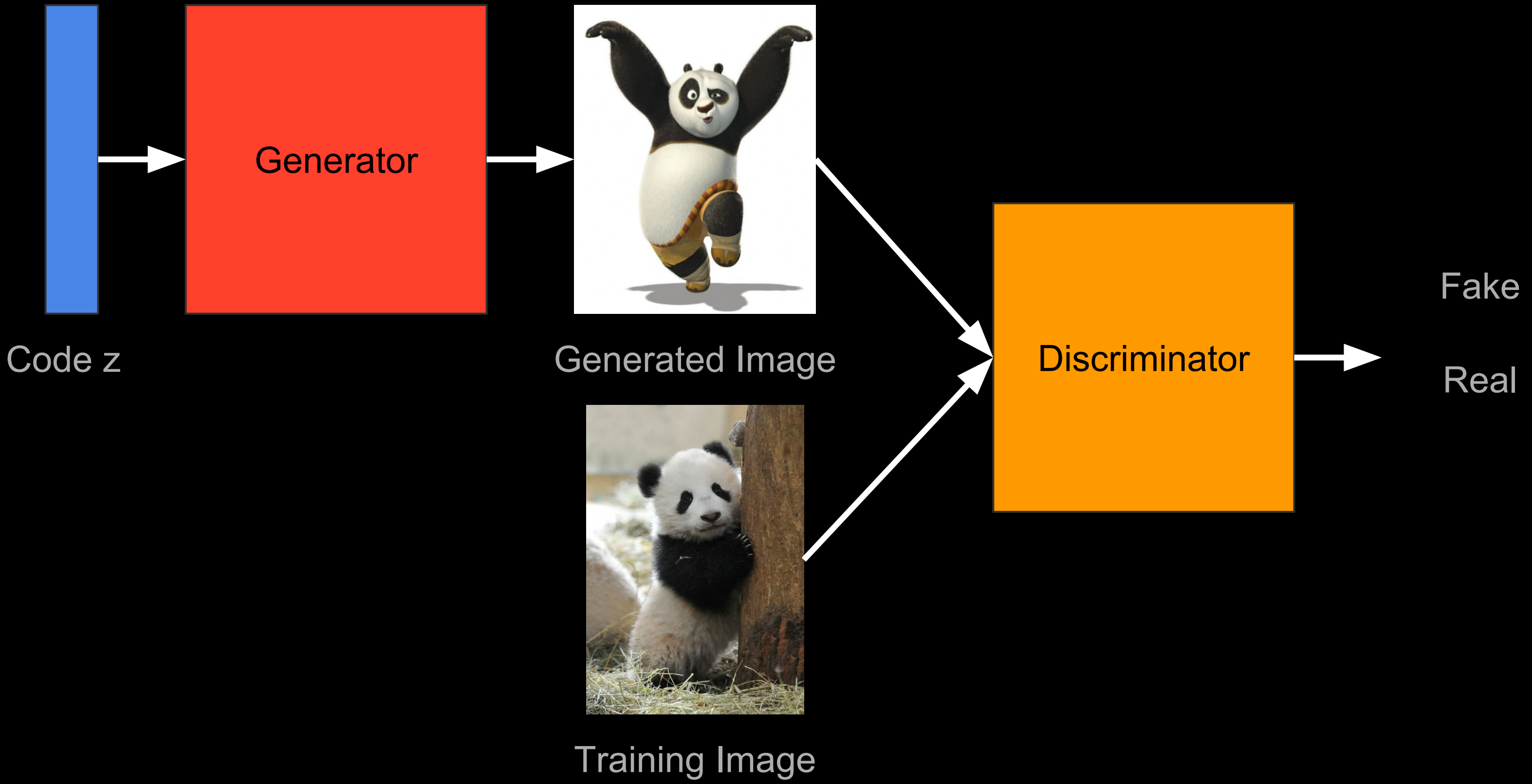
VAE Demo

Kingma et al. 2014

Generative Adversarial Network (GAN)



Generative Adversarial Network (GAN)



Generative Adversarial Network (GAN)

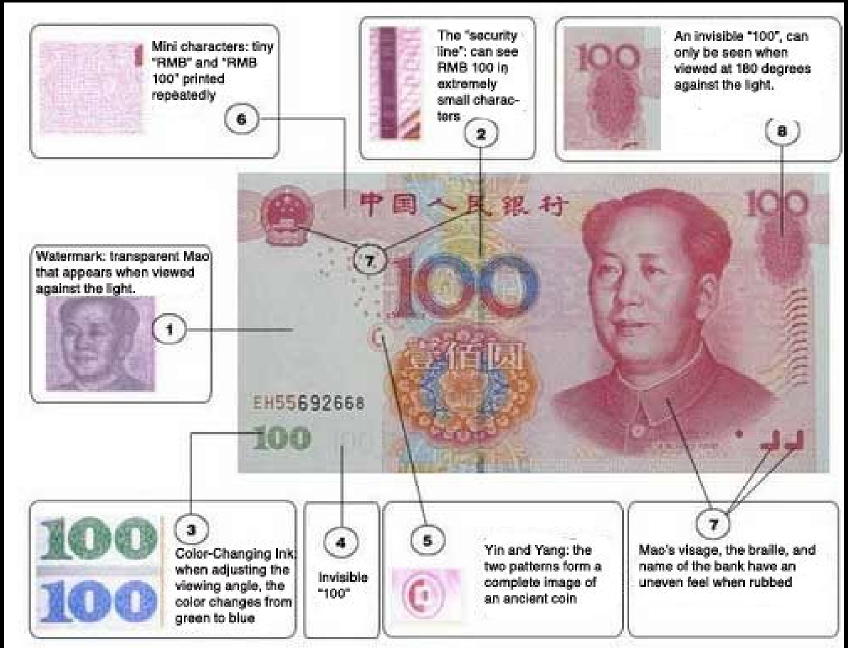
Intuitions

Crook

Generative Adversarial Network (GAN)

Intuitions

Generator



Teller

Google

Generative Adversarial Network (GAN)

Intuitions

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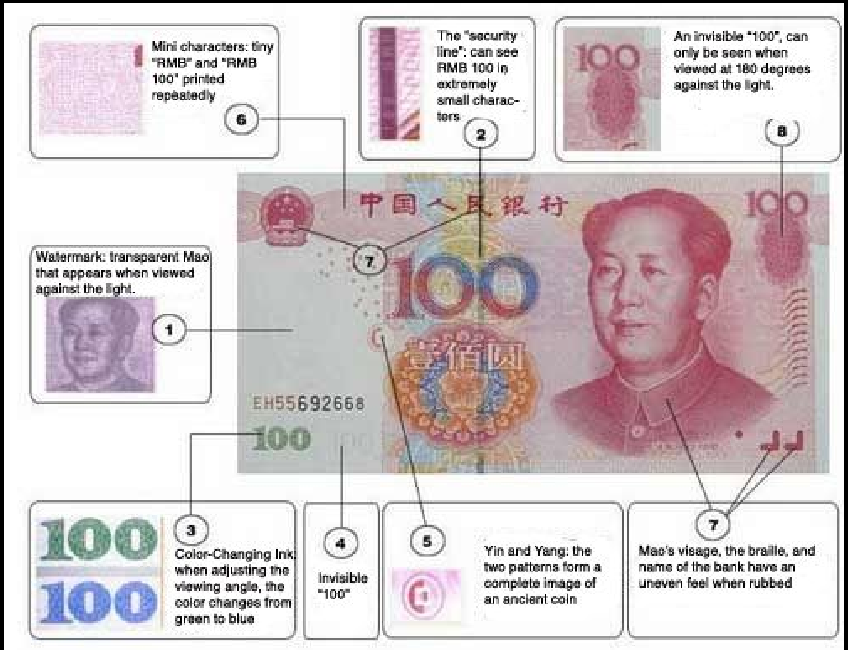

Teller

Generative Adversarial Network (GAN)

Intuitions

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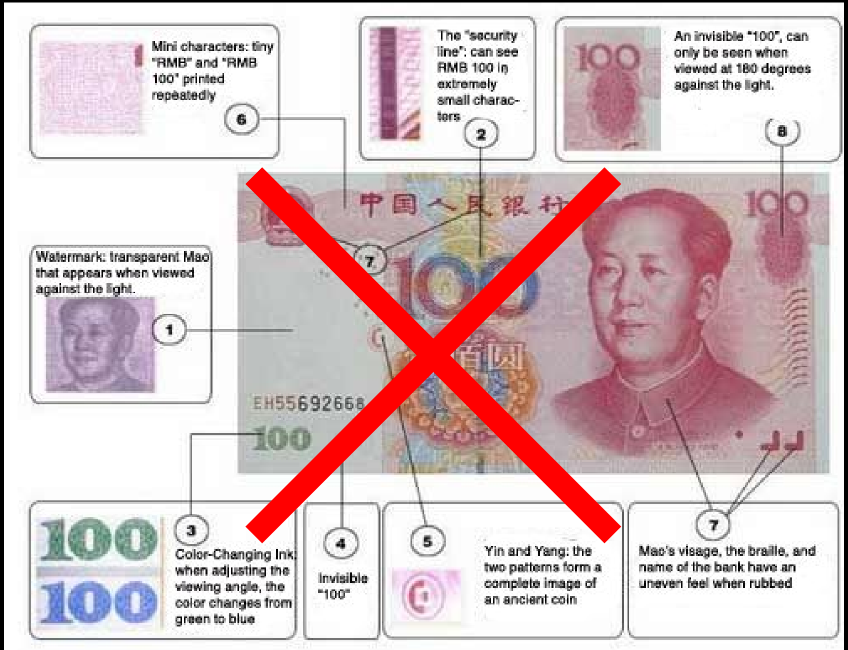


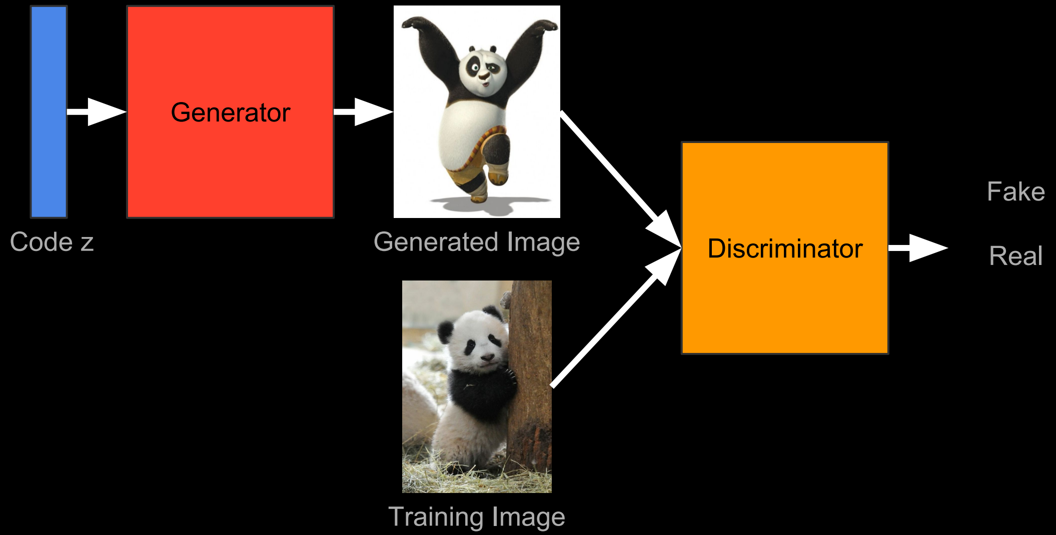
Teller

Google

Generative Adversarial Network (GAN)

Intuitions:

- Generator tries the best to cheat the discriminator by generating more realistic images
- Discriminator tries the best to distinguish whether the image is generated by computers or not



Generative Adversarial Network (GAN)

Objective function:

$$\min_G \max_D V(D, G) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}(\mathbf{x})}[\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{z} \sim p_{\mathbf{z}}(\mathbf{z})}[1 - \log D(G(\mathbf{z}))]$$

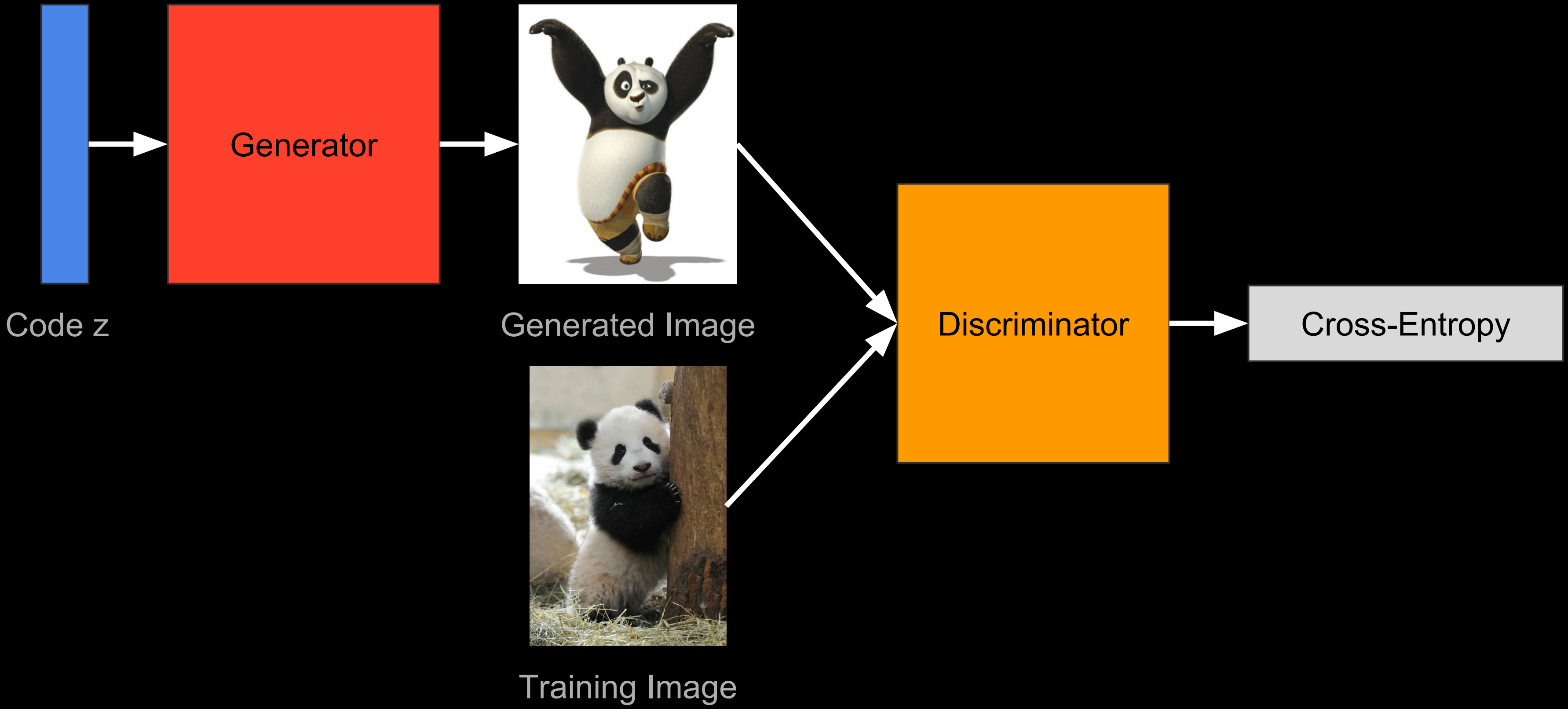
For each iteration:

- Sample a mini-batch of fake images and true images
- Update G using back-prop
- Update D using back-prop

Very difficult to optimize:

- Min-max problem: finding a saddle point instead of a local optimum, unstable

Generative Adversarial Network (GAN)



GANs for face and bedroom

Credit: Denton

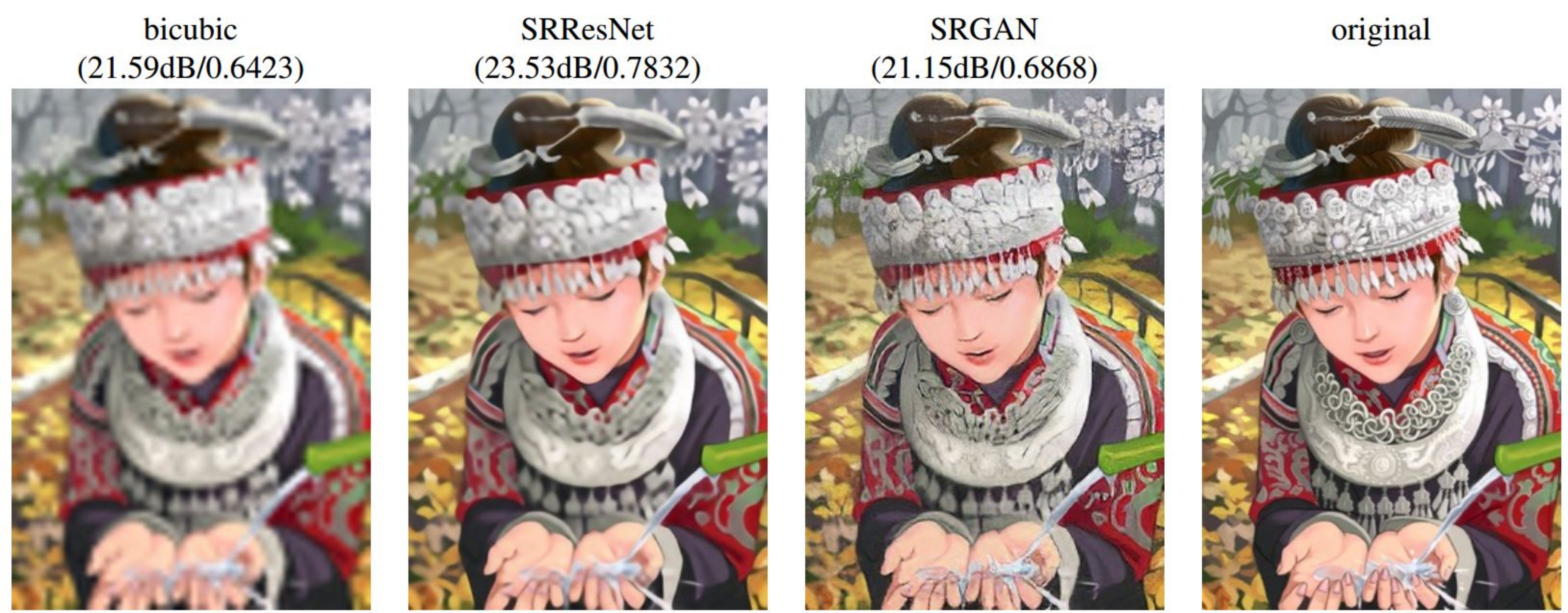
GANs for Japanese Anime

Credit: Radford

GANs for Videos

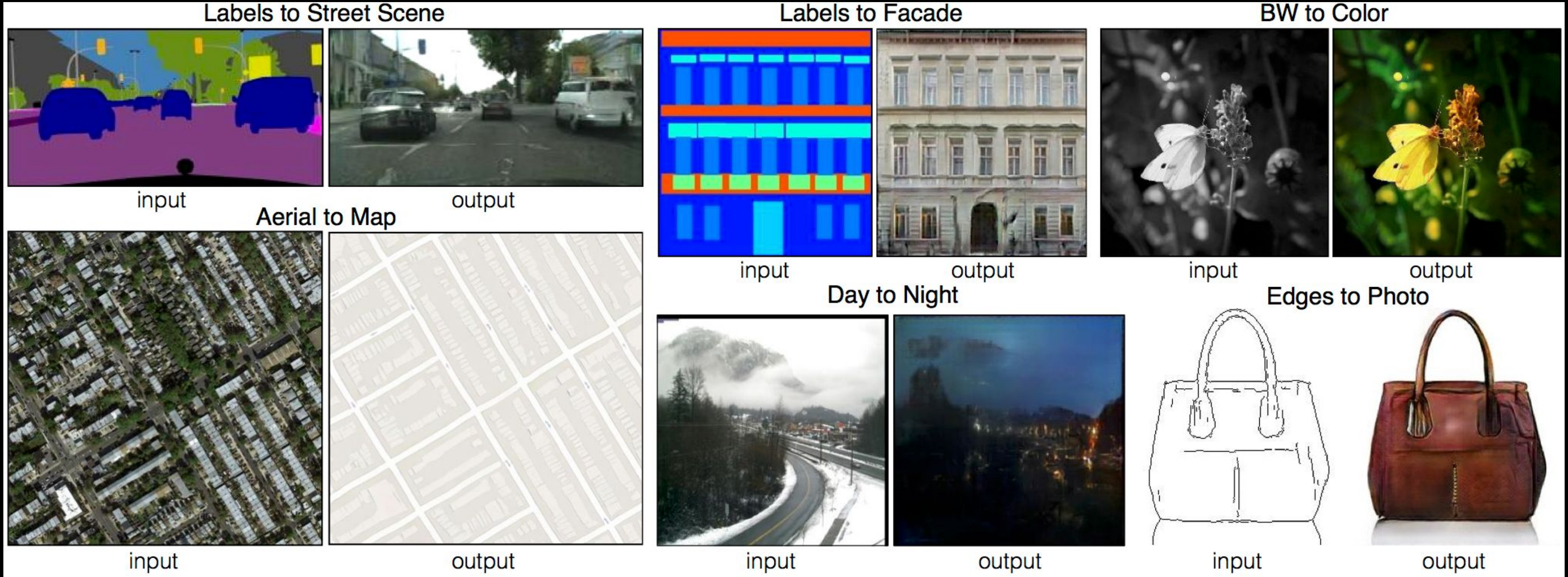
Credit: Vondrick

GANs for Image Upsampling



Credit: Ledig

Conditional GAN



Credit: Zhu et al.

Generative Adversarial Network (GAN)

Extensions:

- DCGANs: some hacks that work well
- LAPGANs: coarse-to-fine conditional generation through Laplacian pyramids
- f-GANs: more general GANs with different loss other than cross-entropy
- infoGANs: additional objective that maximize mutual-information between the latent and the sample
- EBGANs: Discriminative as energy functions
- GVMs: using GANs as an energy term for interactive image manipulation
- Conditional GANs: not random z, instead z is some data from other domain
- ...

Generative Adversarial Network (GAN)

Hacks:

- How to train a GAN?
- 17 hacks that make the training work.
- https://github.com/soumith/ganhacks

Generative Adversarial Network (GAN)

GAN Demo

GANs vs VAEs

GANs:

- High-quality visually appealing result
- Difficult to train
- The idea of adversarial training can be applied in many other domains

VAEs:

- Easy to train
- Blurry result due to minimizing the MSE based reconstruction error
- Nice probabilistic formulation, easy to introduce prior

Demos

VAEs:

- https://github.com/oduerr/dl_tutorial/blob/master/tensorflow/vae/vae_demo.ipynb

GANs:

- https://github.com/ericjang/genadv_tutorial/blob/master/genadv1.ipynb
- https://gist.github.com/wiseodd/b2697c620e39cb5b134bc6173cfe0f56

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Thanks