

Ex - 5

Q2. Prove or disprove

(a) $\{P \vee Q, \neg P \vee R, \neg Q \vee R\} \models R$

$\neg P \vee R, \neg Q \vee R$ S.F.A.
 $(\neg P \wedge \neg Q) \vee R$
 $\neg(P \vee Q) \vee R, \underline{P \vee Q}$

 R

$$(b) \quad P \rightarrow (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$$

$$\begin{array}{ccc} & \swarrow & \searrow \\ \underline{a \rightarrow b} & & \underline{\sim a \vee b} \end{array}$$

P	Q	R	$P \rightarrow (Q \rightarrow R)$	$(P \wedge Q) \rightarrow R$
0	0	0	1	1
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$$P \rightarrow (Q \rightarrow R)$$

$$(P \wedge Q) \rightarrow R$$

$$P \rightarrow (\sim Q \vee R)$$

$$(\sim P \vee \sim Q) \vee R$$

$$\sim (P \wedge Q) \vee R$$

$$(P \wedge Q) \rightarrow R$$

$$a \rightarrow b$$

$$\sim a \vee b$$

Q5. Prove or refute, $S \models \alpha$ and $S \models \beta$ are formulas

(i) if either $S \models \alpha$ or $S \models \beta$, then $S \models (\alpha \vee \beta)$

(ii) If $S \models (\alpha \vee \beta)$, then $S \models \alpha$ or $S \models \beta$

(i) True

$S \models \alpha$ is True

If S is True α is True

$\Downarrow$
 $\alpha \vee \beta$ is True

$S \models (\alpha \vee \beta)$

$S \models \beta$

False

(ii) $S \models (\alpha \vee \beta)$

S
 $(\alpha \vee \beta) \models (\alpha \vee \beta)$

$(\alpha \vee \beta) \models \alpha$

$\alpha = 0$
 $\beta = 1$

$(\alpha \vee \beta) \models \beta$

$\alpha = 1$
 $\beta = 0$

q6.

0. A is a formula. A^δ dual of A
with $\neg, \vee, \wedge$
formula obtained by
replacing $\wedge \rightarrow \vee$ $\vee \rightarrow \wedge$
and atom $P \rightarrow \neg P$

Prove (by struct. induction)

$$\boxed{A^\delta} \equiv \neg \boxed{A}$$

Ans.

Base Case

$$A = \gamma \quad A^5 \equiv \neg \gamma \equiv \neg A$$

I.H, for P & Q or s.t. $P^\delta = 7P$
 $Q^\delta = 7Q$

Induction Step

$$A = \neg P \quad A^\delta \equiv \neg(P^\delta) \equiv \neg(\neg P) \equiv P \quad \text{I.H.}$$

$$\underbrace{A = P \vee Q} \quad \underbrace{A}^{\delta} \equiv (P \vee Q)^{\delta} \equiv \underbrace{P^{\delta} \wedge Q^{\delta}}_{\text{!!!}}$$

$$\begin{array}{l} A = P \wedge Q \\ \hline \end{array}$$

Q7 PK proofs. Rules

Axiom

$$\boxed{A \Rightarrow A}$$

Weakening

$$\text{left} \quad \frac{\Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A}$$

Exchange

$$\text{left} \quad \frac{\Gamma_1, A, B, \Gamma_2 \Rightarrow \Delta}{\Gamma_1, B, A, \Gamma_2 \Rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \Rightarrow \Delta, A, B, \Delta_2}{\Gamma \Rightarrow \Delta, B, A, \Delta_2}$$

Contraction

$$\text{left} \quad \frac{\Gamma, A, A \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \rightarrow \Delta, A, A}{\Gamma \rightarrow \Delta, A}$$

$\neg$ intro

$$\text{left} \quad \frac{\Gamma \Rightarrow \Delta, A}{\neg A, \Gamma \Rightarrow \Delta}$$

$$\text{right} \quad \frac{A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \neg A}$$

$\wedge$ intro

$$\text{left} \quad \frac{A, B, \Gamma \Rightarrow \Delta}{(A \wedge B), \Gamma \Rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \Rightarrow \Delta, A \quad \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, (A \wedge B)}$$

$\vee$ intro

$$\text{left} \quad \frac{A, \Gamma \Rightarrow \Delta \quad B, \Gamma \Rightarrow \Delta}{(A \vee B), \Gamma \Rightarrow \Delta}$$

$$\text{right} \quad \frac{\Gamma \Rightarrow \Delta, A, B}{\Gamma \Rightarrow \Delta, (A \vee B)}$$

Cut

$$\frac{\Gamma \Rightarrow \Delta, A \quad A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}$$

$$\cdot \Rightarrow \neg(A \wedge \neg A)$$

$$A \Rightarrow A$$

$$\neg A, A \Rightarrow$$

$$\cdot A \wedge \neg A \Rightarrow$$

$$\Rightarrow \neg(\cdot A \wedge \neg A)$$

$$\underline{A, \neg A \Rightarrow}$$

one of Q_i is true or one of P_i is false $\underline{P_1, P_2, \dots, P_n} \Rightarrow Q_1, \dots, Q_n$

$$\cdot A \wedge (B \vee C) \Rightarrow (\underbrace{(A \wedge B)}_D \vee \underbrace{(A \wedge C)}_D)$$

$$\text{i)} \underline{A, B \Rightarrow D}$$

$$\text{ii)} \underline{A, C \Rightarrow D}$$

$$A, B \vee C \Rightarrow D$$

$$A \wedge (B \vee C) \Rightarrow D$$

$$A \Rightarrow A$$

$$B \Rightarrow B$$

$$A, B \Rightarrow A$$

$$A, B \Rightarrow B$$

$$A, B \Rightarrow (A \wedge B)$$

$$A, B \Rightarrow (A \wedge B), (A \wedge C)$$

$$A, B \Rightarrow (A \wedge B) \vee (A \wedge C)$$

(1)

$$\bullet [A \vee (B \wedge C) \Rightarrow ((A \vee B) \wedge (A \vee C))]$$

$$A \Rightarrow A$$

$$A \Rightarrow A$$

$$B \Rightarrow B$$

$$A \Rightarrow A, B$$

$$A \Rightarrow A, C$$

$$B, C \Rightarrow B$$

$$A \Rightarrow A \vee B$$

$$A \Rightarrow A \vee C$$

$$B, C \Rightarrow A, B$$

$$\underline{A \Rightarrow (A \vee B) \wedge (A \vee C)}$$

$$B, C \Rightarrow A \vee B$$

$$B, C \Rightarrow (A \vee B) \wedge (A \vee C)$$

$$C \Rightarrow C$$

$$\underline{B \wedge C \Rightarrow (A \vee B) \wedge (A \vee C)}$$

$$B, C \Rightarrow C$$

$$B, C \Rightarrow A, C$$

$$A \vee (B \wedge C) \Rightarrow (A \vee B) \wedge (A \vee C), B, C \Rightarrow A \vee C$$

$$\bullet [(\neg(A \wedge B) \vee C) \Rightarrow (\neg A \vee (\neg B \vee C))]$$

$$\neg A \wedge B \Rightarrow \neg A \vee (\neg B \vee C)$$

$$C \Rightarrow \neg A \vee (\neg B \vee C)$$

$$A \Rightarrow A$$

$$C \Rightarrow C$$

$$\neg A \Rightarrow \neg A$$

$$C \Rightarrow \neg B, C$$

$$\neg A, B \Rightarrow \neg A, \neg B \vee C$$

$$C \Rightarrow \neg B \vee C$$

$$\neg A \wedge B \Rightarrow \neg A \vee (\neg B \vee C)$$

$$C \Rightarrow \neg A, \neg B \vee C$$

$$C \Rightarrow \neg A \vee (\neg B \vee C)$$

$$\neg(A \wedge B) \vee C \Rightarrow \neg A \vee (\neg B \vee C)$$

TT-1

Q 5.

(a) A is TR $\bar{A}$ is TR.

ST

(b) A, B are TR $A \cup B$ is TR

AT

(c) A is not TR, B is TR $A \cup B$ is TR

ST

d) A, B are not TR $A \cup B$ is not TR

ST

(e) $A \leq_m \bar{A}$, A is decidable

ST

L is undecidable

$\Downarrow$

$$L' = \{0w : w \in L\} \cup \{1w : w \notin L\}$$

$$\bar{L}' = \{0w : w \notin L\} \cup \{1w : w \in L\}$$

$$f: L' \rightarrow \bar{L}'$$

$$f(x) = \begin{cases} 1w & x = 0w \\ 0w & x = 1w \end{cases}$$

$$x \in L' \quad x = 0w \Rightarrow w \in L \Rightarrow 1w \notin L \Rightarrow f(x) \in \bar{L}'$$

$$x = 1w \Rightarrow w \notin L \Rightarrow 0w \in L \Rightarrow f(x) \in \bar{L}'$$