Due: Friday, January 26, beginning of tutorial

NOTE: Each problem set counts 10% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Give a Turing machine M which adds 1 in binary. Assume that the input to M is a nonempty string w representing a number k in binary. After M halts, the tape should consist of a string representing $k+1$ in binary, followed by blanks.

   You should assume the standard input convention for w, and stick to the convention that M's tape has a left-most tape square and cannot be extended to the left. You may assume that M has extra symbols in its tape alphabet if you want (but the slickest solution doesn't need any).

   Present your TM either as a state diagram in the style used in the text book, or in the style of the CSC 365 course notes "Turing Machines and Reductions", page 3 (except it would be helpful to annotate some of the TM instructions). Explain clearly how your TM works. **Solutions without clear explanations will not be marked.**

2. Show that **SD** is closed under union and intersection. That is, show that if A and B are each semi-decidable, then so are $A \cup B$ and $A \cap B$. NOTE: This problem is solved on page 4 of the Notes "Computability and Noncomputability". Therefore you should give a different proof, based on enumerators: Given enumerators for A and B, show how to use them to get enumerators for $A \cup B$ and $A \cap B$.

3. Recall from page 6 of the Notes "Computability and Noncomputability" that DIAG $= \{\langle M\rangle \mid \langle M\rangle \notin L(M)\}$. Modify the proof in the Notes showing that DIAG is undecidable to show that the set

$$\text{DD} = \{\langle M\rangle \mid \langle M\rangle\langle M\rangle \notin L(M)\}$$

   is not decidable.

4. Let

$$A = \{\langle M_1\rangle, \langle M_2\rangle, \langle M_3\rangle, \dots\} \tag{1}$$

   be a semidecidable set of Turing machine descriptions. Show that there exists a decidable set B of Turing machine descriptions which has the same set of associated languages $\mathcal{L}(M_i)$ as A.
   Hint: Show how you can pad the description $\langle M\rangle$ of a Turing machine M to get an equivalent Turing machine, but with a much longer description.