

**Due: Friday, January 27, beginning of tutorial**

NOTE: Each problem set counts 10% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. Describe a Turing machine  $M$  whose input alphabet is  $\{0, 1\}$  and which accepts an input string  $w$  iff  $w$  has equally many 0s and 1s (and halts on all inputs).

Your description should start by specifying the tape alphabet of  $M$ . Present your Turing machine either in the style of the course notes “Turing Machines and Reductions”, page 3, or as a state diagram like that in the text. Explain in English how your TM works.

**Note:** This is mostly a practice exercise and will count very little.

2. Prove that a set  $A \subseteq \{0, 1\}^*$  is decidable iff it can be enumerated by an enumerator which writes the members of  $A$  on its output tape in lexicographical order.
3. Show that if a set  $A$  is semidecidable, then so is  $A^*$ . (See Definition 1.23 in Sipser’s book for a definition of  $A^*$ .) (You may use results proved in Sipser’s Chapters 3 and 4.)
4. Prove that every infinite semidecidable set has an infinite decidable subset.