

1. Derive Simpson's rule using first principles (i.e. polynomial interpolation and integration by hand)

$$\int_a^b f(x) dx \approx \frac{b-a}{6} \left[ f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \quad (1)$$

2. Write a program that computes an integral numerically, using composite trapezoid and composite Simpson methods. Using  $n = 2, 4, 8, 16, 32$  subintervals, compute the following integrals:

(a)

$$\int_0^1 \sin(x) dx = 1 - \cos(1) \quad (2)$$

(b)

$$\int_0^1 x^{5/2} dx = \frac{2}{7} \quad (3)$$

(c)

$$\int_0^1 x^3 dx = \frac{1}{4} \quad (4)$$

What order of convergence do you observe and why?

3. Derive the composite Simpson method using Richardson Extrapolation and the composite Trapezoidal rule.