1. Find the Newton iteration for each of the following systems:

   (a)

$$x^2 + xy^3 = 9$$
$$3x^2y - y^2 = 4$$

   (b)

$$x + y - 2xy = 0$$
$$x^2 + y^2 - 2x + 2y = 1$$

   (c)

$$x^3 - y^2 = 0$$
$$x + x^2y = 2$$

2. Consider the following optimization problem

$$\min \phi(x) = \frac{1}{2}x^T Dx \tag{1}$$

   where $x \in \mathbb{R}^2$ and $D = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

   Surely, you will agree that the minimum is trivial to find and occurs at $x = 0$. However, the purpose of this question is to get you familiar with the dynamics of gradident descent.

   (a) Show that the gradient of $\phi(x)$ is given by

$$\nabla \phi = \begin{bmatrix} x_1 \\ 4x_2 \end{bmatrix} \tag{2}$$

   and derive the gradient update.

   (b) Normally, we do not do exact linesearch, but only a weak linesearch. (Can you think of why?) However, for simple functions such as the one above, we can find a formula for exact linesearch.
   Suppose $x = [1, 2]^T$. Then, what value of α minimizes $\phi(x_k - \alpha p_k)$?

   (c) What direction is $x_{k+1} = x_k - \alpha p_k$ in?