1. What is the computational complexity (as a function of n) of solving a matrix A of size $n \times n$, where

   (a) A is diagonal?

   (b) A is triangular?

   (c) The general case?

2. Given two nonsingular matrices T and A such that the matrix TA has an LU factorization $TA = LU$, describe an **efficient** algorithm for solving the system of linear equations

$$Ax = b \tag{1}$$

3. Let

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix} \tag{2}$$

   You may use, without proof, the fact that the eigenvalues λ_1, λ_2 of A are equal to $5 \pm \sqrt{24}$. Unless otherwise specified, assume the Euclidean/2 norm is being discussed.

   (a) What is the Frobenius norm of A?

   (b) Is A singular or nonsingular? What are the eigenvalues of A^{-1}, if it exists?

   (c) Which vector x causes the greatest increase in the length of the vector Ax? What is the operator norm of A?

   (d) Which vector x causes the greatest increase in the length of the vector $A^{-1}x$? What is the operator norm of A^{-1}?

   (e) What is the condition number of A?

   (f) How can you generalize part (e) to arbitrary matrices where you know the eigenvalues?