1. Consider writing a function to compute the Euclidean norm of a vector x, given by the formula
$$\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}. \tag{1}$$
What numerical issues could arise? How would you resolve them?

2. Derive a fourth-order finite difference formula for $f'(x)$ based on the points $x - 2h$, $x - h$, x, $x + h$, $x + 2h$ using
   (a) Taylor Series expansion.
   (b) Richardson Extrapolation (yes, it can be applied to more than just numerical integration).

3. Midterm, q3

4. Show that if you apply Newton's method to find the root of $(x - 2)^2$, you get linear convergence and determine the rate of convergence ρ.

5. Midterm, q4

6. Let A be a matrix, and $B = A - A^T$ be nonsingular.
   (a) Prove that B is positive definite, or give an example to show that B is not positive definite.
   (b) What method should be used to solve the equation $Bx = y$, for an unknown x and a known y? Justify your answer.

7. How does exact linesearch work? Can you think of a case where it is practical?

8. How do quasi-Newton methods work? What is the motivation behind it?

9. Why does the inverse iteration converge so quickly compared to the power method?

10. Show that it is not necessary to normalize the vector v on each iteration in the power method in the PageRank application.

11. Consider the Matlab script

```
>> a = 0; b = 1;
>> x1 = linspace(a, b, 11);
>> y1 = sqrt(x1);
>> xq = linspace(a, b, 1001);
>> yq = interp1(x1, y1, xq);
>> max(abs(yq - sqrt(xq)))
ans =
    0.0791
>> x2 = linspace(a, b, 21);
>> y2 = sqrt(x2);
>> yq = interp1(x2, y2, xq);
>> max(abs(yq - sqrt(xq)))
```

What output do you expect to get? Explain your reasoning.

12. Let $f(0) = 1.5$, $f'(0) = 1$, and $f(5) = 0$. Suppose we want to find the lowest-degree polynomial that interpolates f and its derivatives.
    (a) What degree polynomial is required?
    (b) Set up, but do not solve the system of linear equations (you may use any basis – I recommend monomial).

13. Consider computing the integral
$$S = \int_0^{\pi} \sin(x)\, dx = 2 \tag{2}$$
    (a) What error do you get for $n = 2, 4$ panels with trapezoidal rule?
    (b) Calculate the order of accuracy. Explain the results.
    (c) Repeat parts (a) and (b) with Simpson's rule.

Recommended formulas and facts on note sheet:

- Machine epsilon for double precision floating point system: 10^{-16}
- Machine epsilon for single precision floating point system: 10^{-8}
- Taylor series (eq 6 lecture 1)
- condition number error amplification (eq 51 lecture 3)
- error of polynomial interpolant (eq 20, lecture 7)