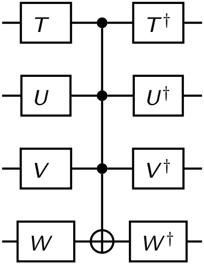
Bounds on the QAC^0 Complexity of Approximating Parity

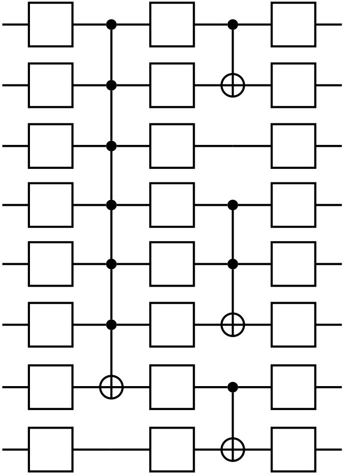


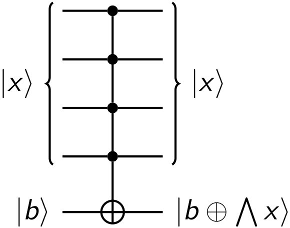
Gregory Rosenthal

University of Toronto

ITCS 2021

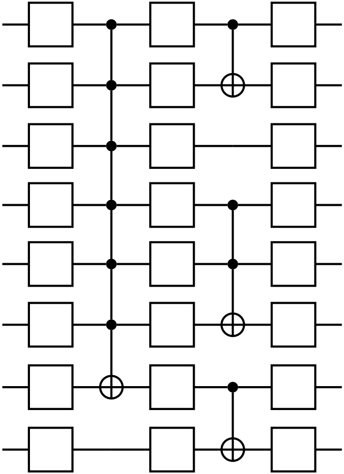
QAC^0

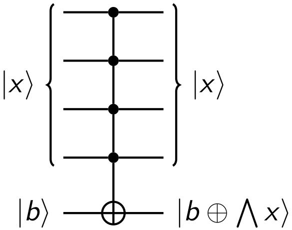




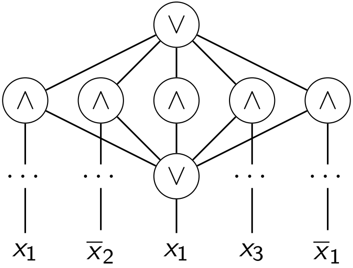
- constant depth

QAC



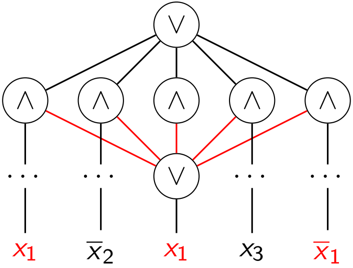


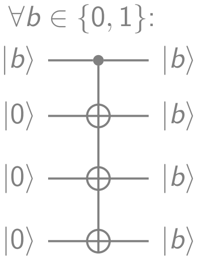
AC^0



$\vee$
$\wedge$ $\wedge$ $\wedge$ $\wedge$ $\wedge$
$\vee$
x_1 $\overline{x}_2$ x_1 x_3 $\overline{x}_1$

Fanout & Motivation





- Is fanout in QAC^0?
- Fanout $\sim$ Parity [GHMP'02]
- Parity $\notin \mathrm{AC}^0$ [H'86]
- $\mathrm{TC}^0 \subseteq \mathrm{QAC}^0[\text{parity/fanout}]$ [HS'05,TT'16]

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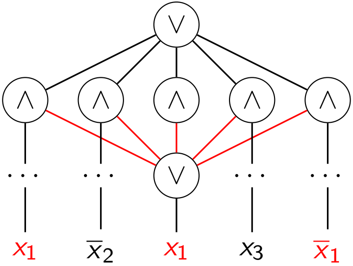

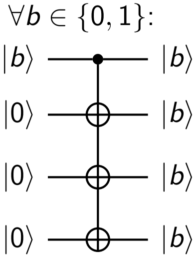

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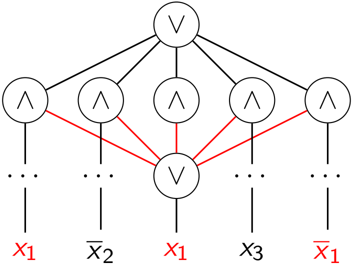
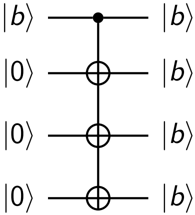


$\forall b \in \{0,1\}$:



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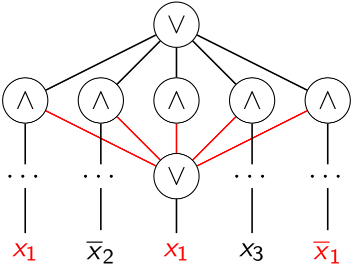

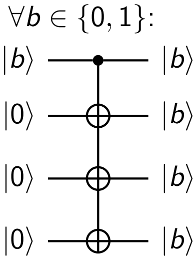

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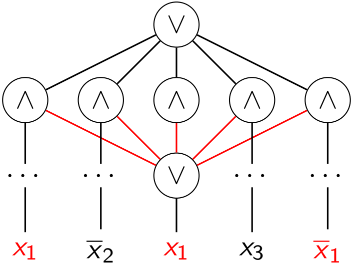

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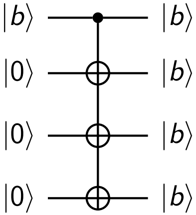

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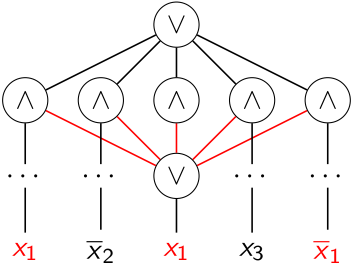

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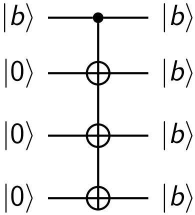

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Bounds for Parity/Fanout

Prior Results

	Depth*	Size*	Ancillae	
U.B.	$\sim \log n$	$O(n)$	$O(n)$	[GHMP'02]
L.B.	$\sim \log(n/(a+1))$	∞	a	[FFGHZ'06]
L.B.	2	∞	∞	[PFGT'20]

Our Results

	Depth*	Size*	Ancillae	Approx.
U.B.	$d \geq 7$	$e^{n^{O(1/d)}} \log(n/\varepsilon)$	$e^{n^{O(1/d)}} \log(n/\varepsilon)$	$1 - \varepsilon$
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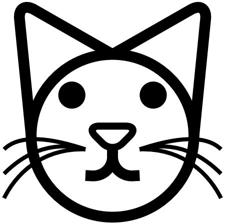
Parity/Fanout ~ Nekomata

Cat State	Nekomata
$\frac{1}{\sqrt{2}} \sum_{b=0}^{1} \lvert b\rangle^n$	$\frac{1}{\sqrt{2}} \sum_{b=0}^{1} \lvert b\rangle^n \lvert\psi_b\rangle$
	

- I.e. some n qubits of a nekomata measure to 0^n and 1^n each with probability $1/2$.
- $\Rightarrow$ [GHMP'02]:
  $\text{Fanout}\left(\frac{\lvert 0\rangle+\lvert 1\rangle}{\sqrt{2}} \lvert 0\rangle^{n-1}\right) = \frac{1}{\sqrt{2}} \sum_{b=0}^{1} \text{Fanout}\left(\lvert b\rangle \lvert 0\rangle^{n-1}\right) = \lvert$ 🐱 $\rangle$.
- $\Leftarrow$: Next slide.

Parity/Fanout $\sim$ Nekomata

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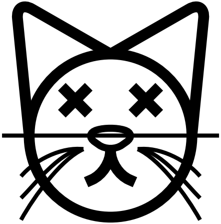	

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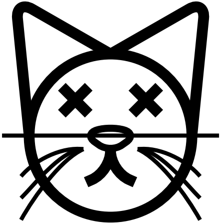	

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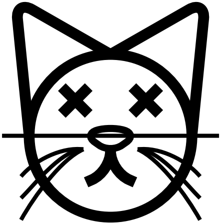	

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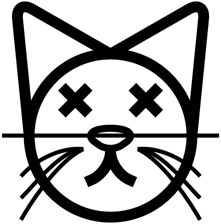	

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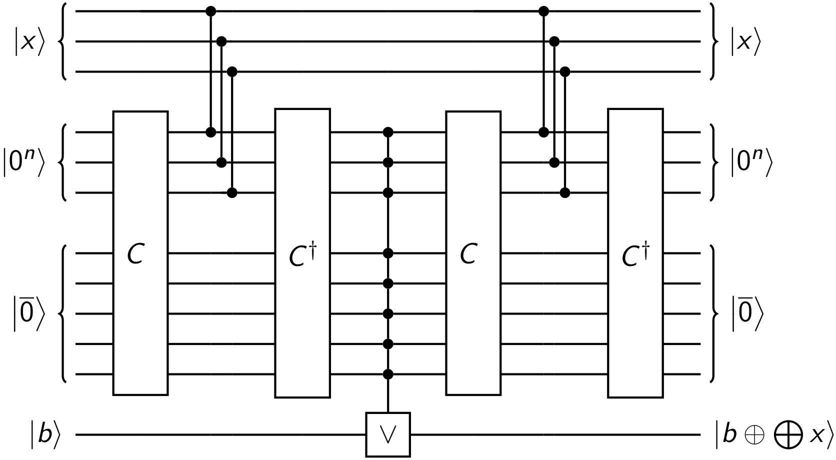
Nekomata $\Rightarrow$ Parity

- If $C|\overline{0}\rangle$ is a nekomata $\left(\frac{1}{\sqrt{2}}\sum_{b=0}^{1}|b\rangle^{n}|\psi_b\rangle\right)$ then:



Upper Bounds for Approximate Parity/Fanout

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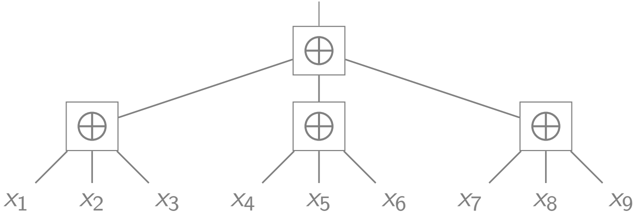
- Further optimization $\Rightarrow$ depth-7 U.B.
- Downward self-reducibility of parity $\Rightarrow$ depth-d U.B.



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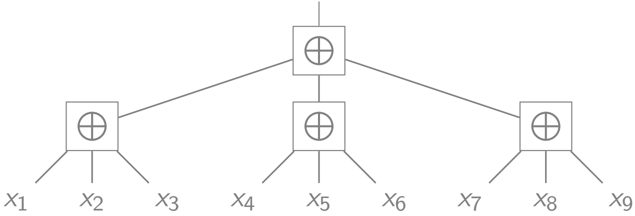

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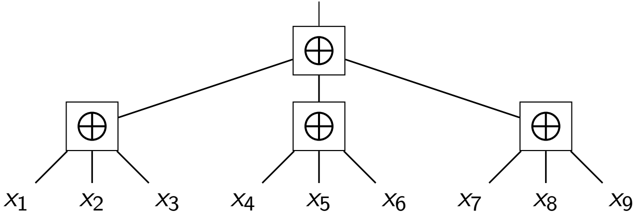
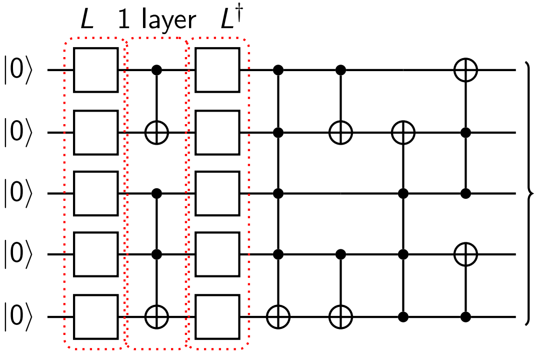


Bounds for Nekomata in “Mostly Classical” Circuits



$\left(\frac{1}{\sqrt{2}}\sum_{b=0}^{1}|b\rangle^{n}|\psi_b\rangle\right)$

	Depth	Size	# Qubits	Approx.
U.B.	2	$e^{O(n\log(n/\varepsilon))}$	$e^{O(n\log(n/\varepsilon))}$	$\geq 1-\varepsilon$
L.B.	$o(\log n)$	$e^{n^{1-\Omega(1)}}$	∞	$\leq 1/2 + e^{-n^{\Omega(1)}}$

- Approx. $=$ max $|\langle\nu|C|\overline{0}\rangle|^2$ over all nekomata $|\nu\rangle$.

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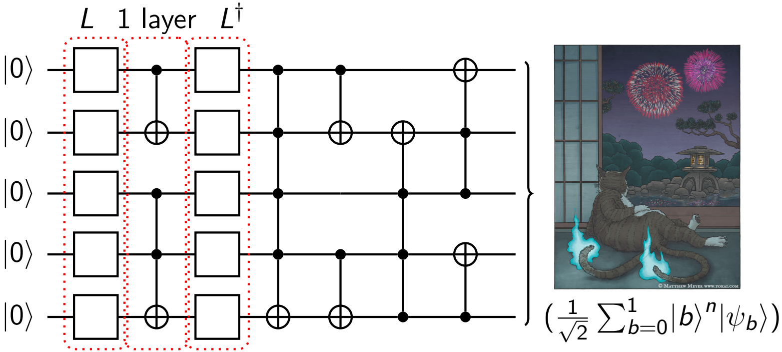

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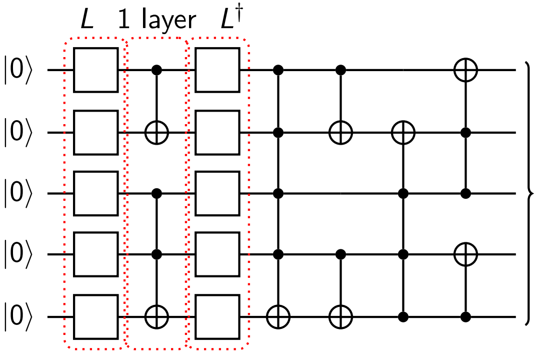

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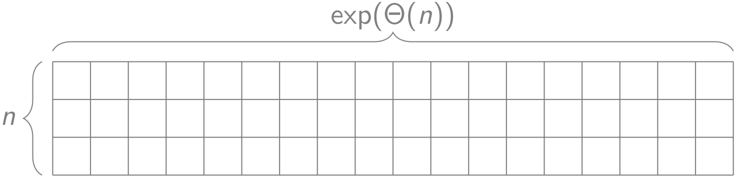
Bounds for Nekomata in M.C. Circuits – Proofs

Upper Bound:

- Goal: sample 0^n and 1^n each with probability $\approx 1/2$.



1. In each column, independently sample 1^n with probability $\exp(-\Theta(n))$ and 0^n otherwise.
   - $(I - 2|\tilde{1}^n\rangle\langle\tilde{1}^n|)|0^n\rangle$ where $|\tilde{1}\rangle \approx |1\rangle$
2. Compute the OR of each row.

Lower Bound:

- Concentration inequality for Hamming weight, via [GLSS'15].

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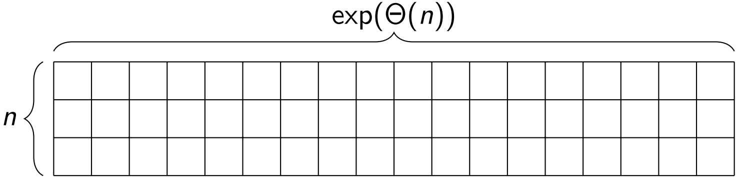

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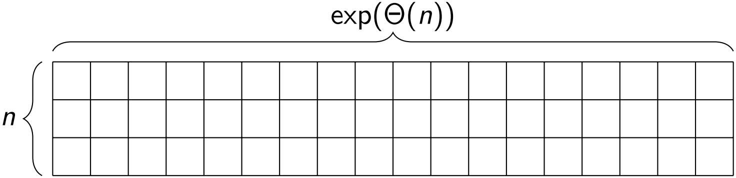

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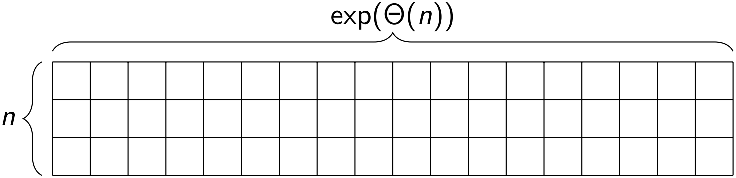

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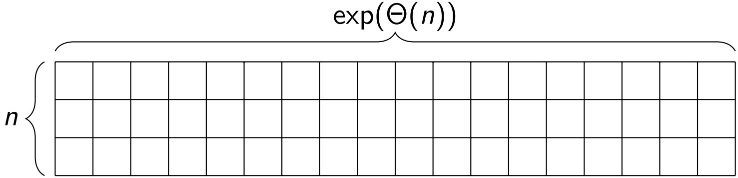

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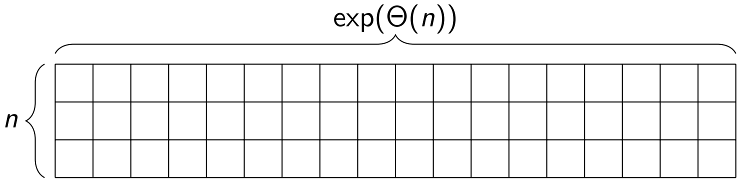

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Remaining Proof Summaries & Result Clarifications

Depth-d, Size-$0.2n/(d+1)$ Circuits C:

- Normal form (see next slide) & triangle inequality $\Rightarrow$ L.B. for nekomata:
- $|\langle \nu | C | \overline{0} \rangle|^2 \leq 1/2 + e^{-\Omega(n/d)}$ for all nekomata $|\nu\rangle$.
- Implies L.B. for parity/fanout.

Depth-2 Circuits C:

- Measure ancillae to kill off gates, apply (a generalization of) the above theorem $\Rightarrow$ L.B. for $|\text{🐱}\rangle$:
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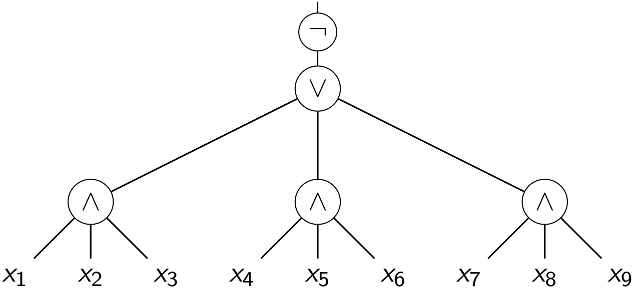
A Normal Form for QAC Circuits



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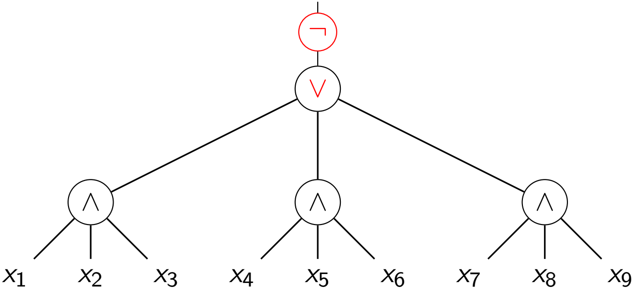

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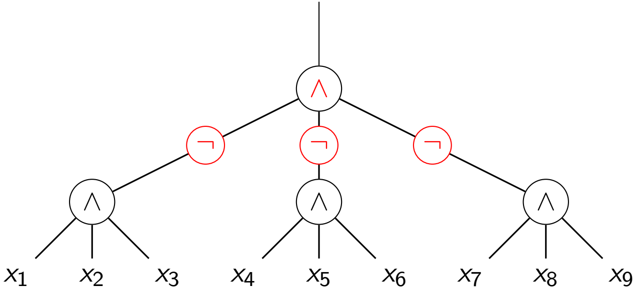

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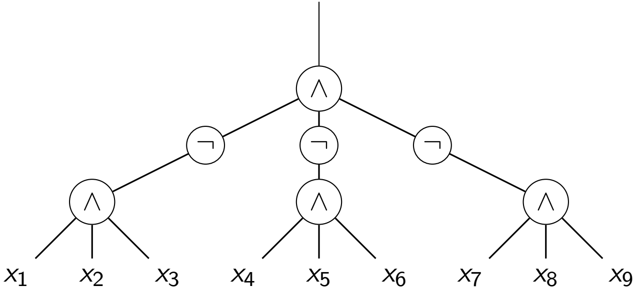

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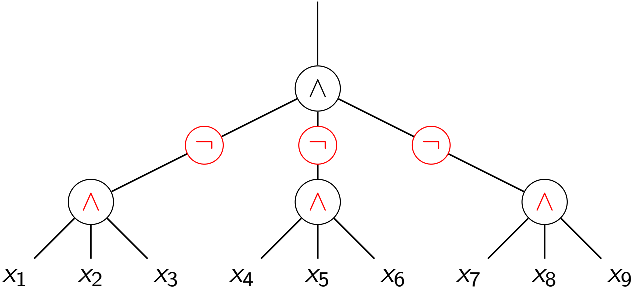

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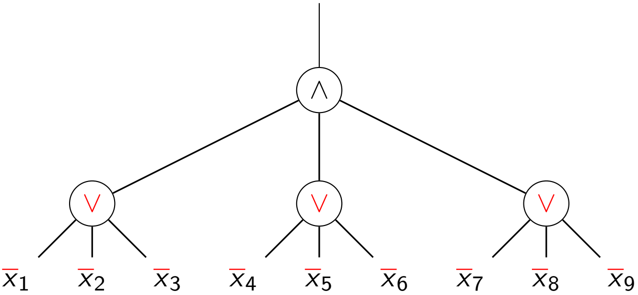

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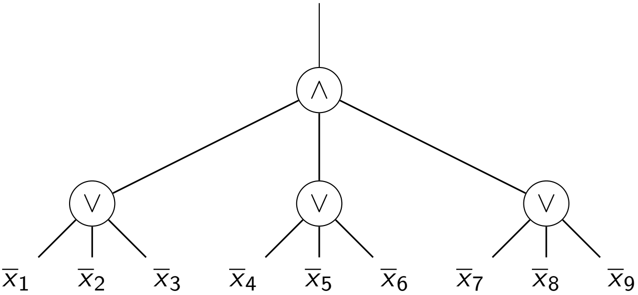

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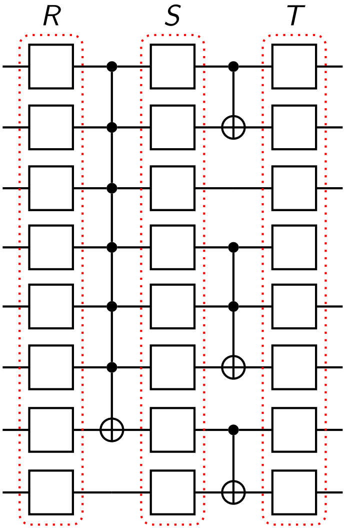

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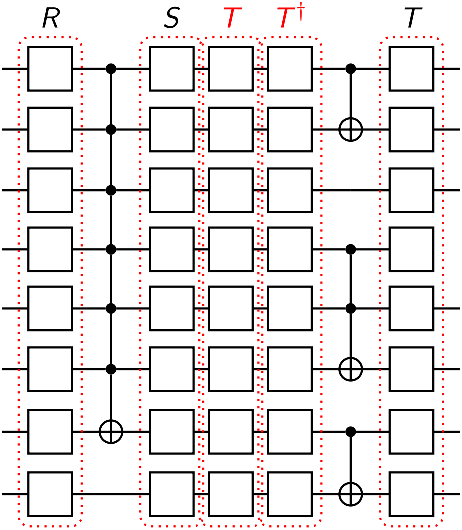

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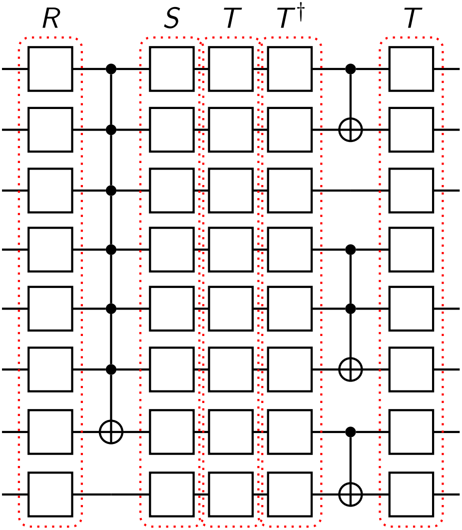

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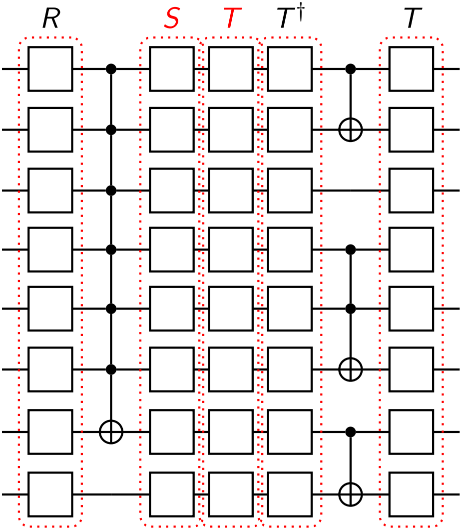

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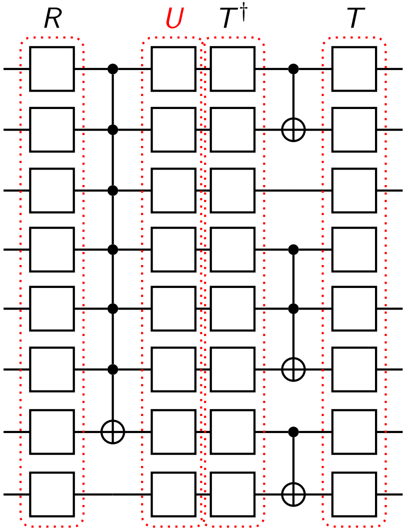

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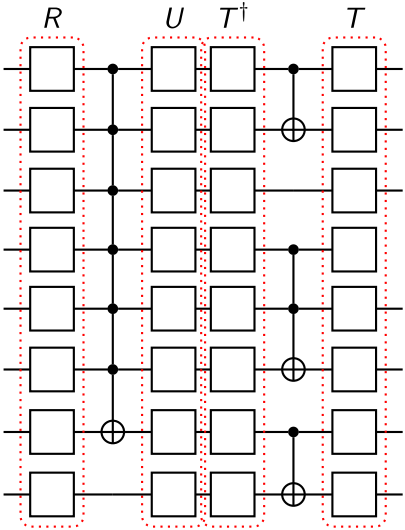

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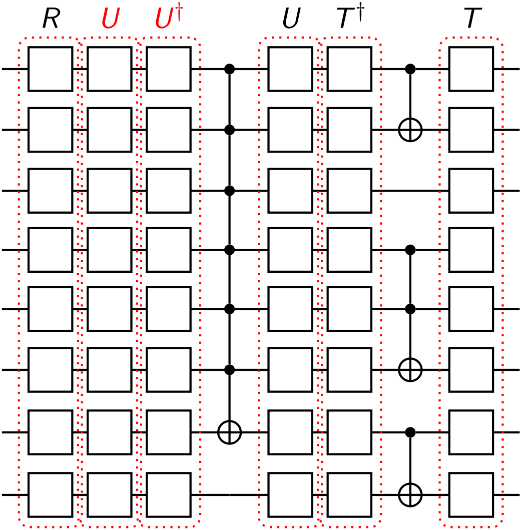

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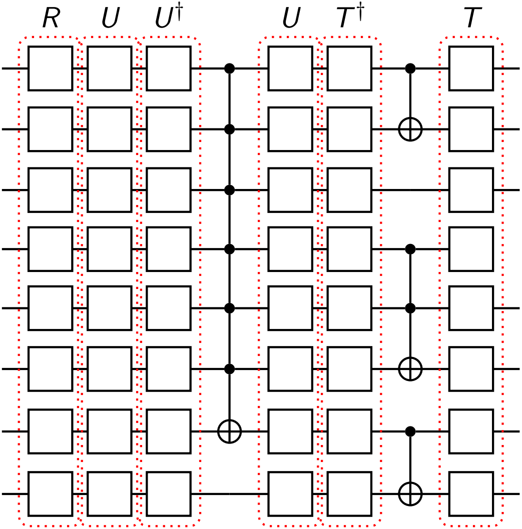

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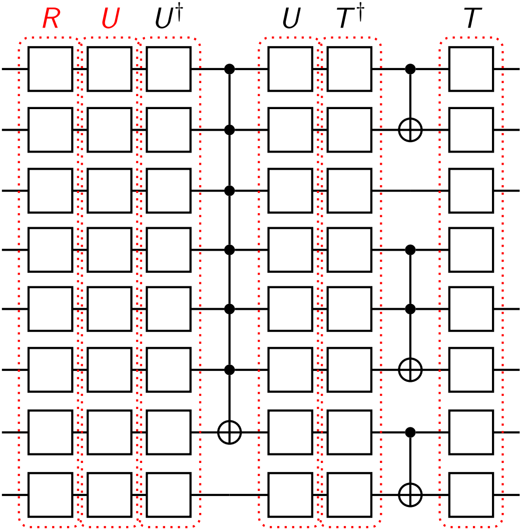

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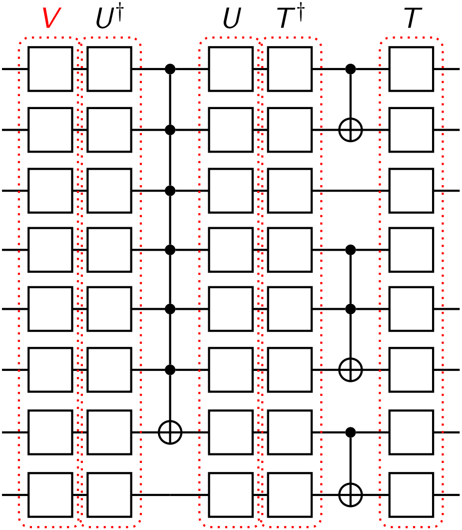

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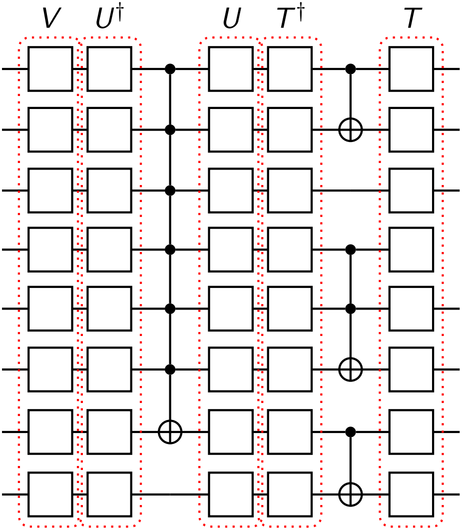

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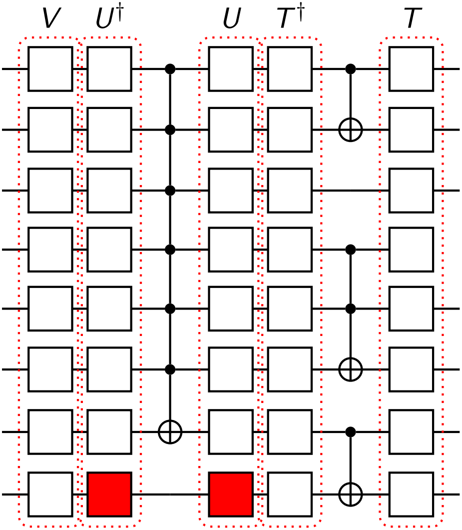

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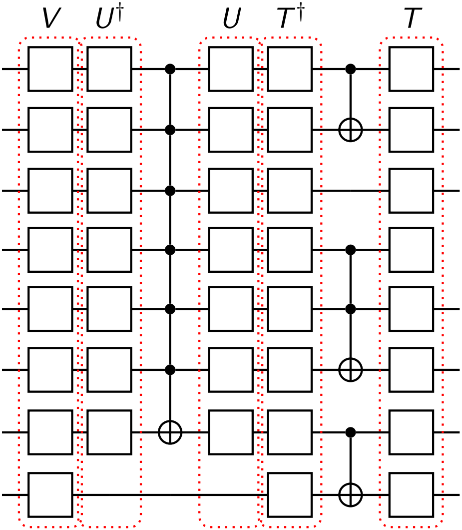

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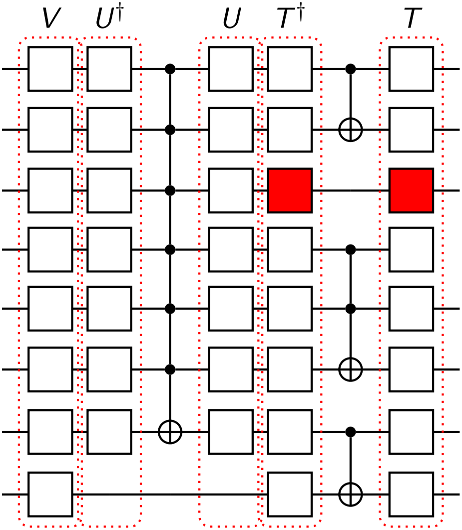

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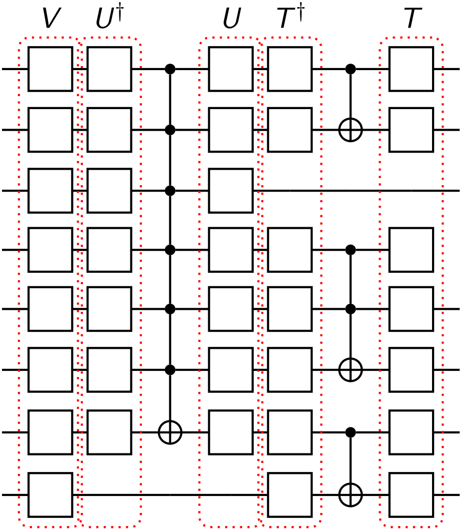
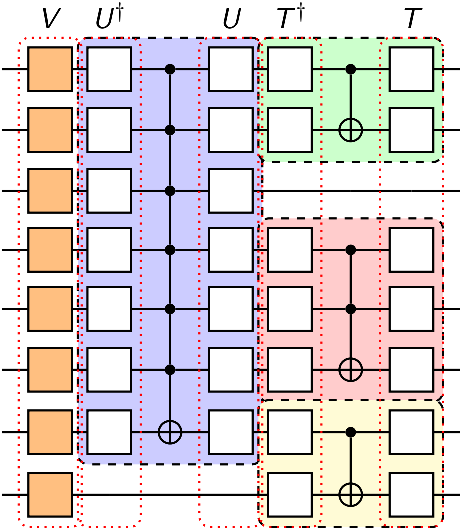


A Normal Form for QAC Circuits

- eigenvalues $1, \ldots, 1, -1$
- -1 eigenvector is product of one-qubit states



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