RL Tutorial

CSC2515 Fall 2019

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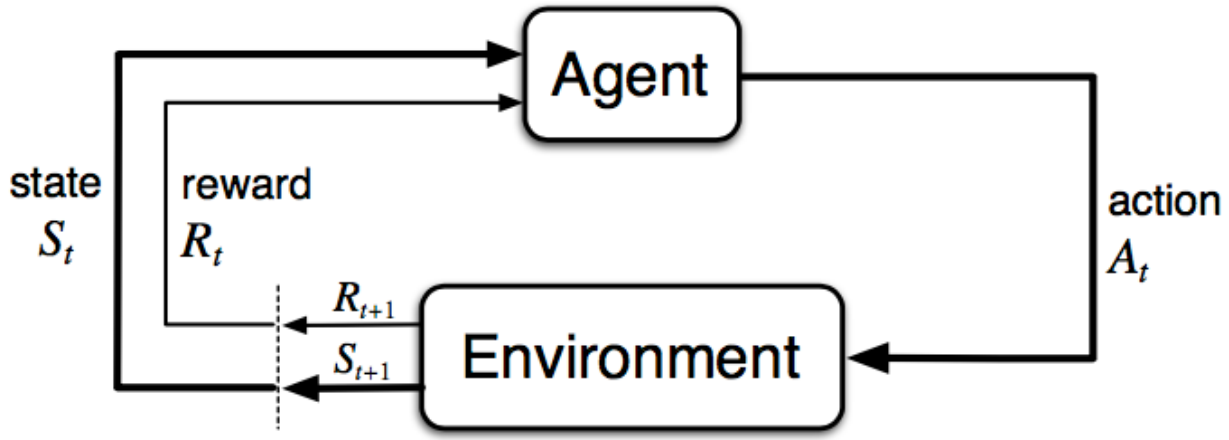
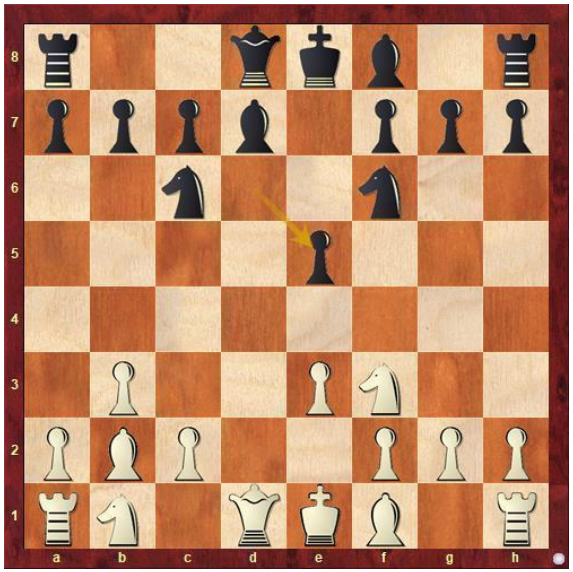
Markov Decision Process (MDP)



- $S_t \in \mathcal{S}, A_t \in \mathcal{A}, R_t \in \mathbb{R}$
- Timesteps: $t \in \mathbb{N}$ (discrete)
- Finite: $\mathcal{S}, \mathcal{A}$ both have finite number of elements (discrete)
- Fully observable: S_t is known exactly
- Trajectory: sequence of state-action-reward $S_0, A_0, R_1, S_1, A_1, ...$

Markov Property

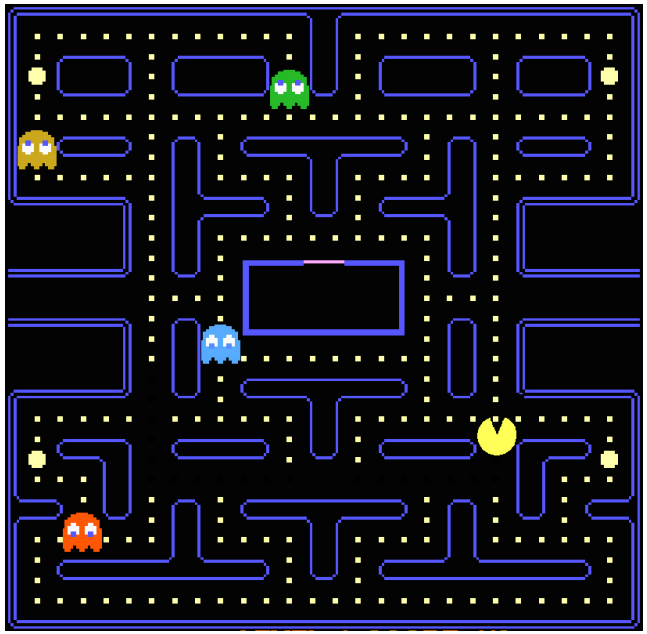


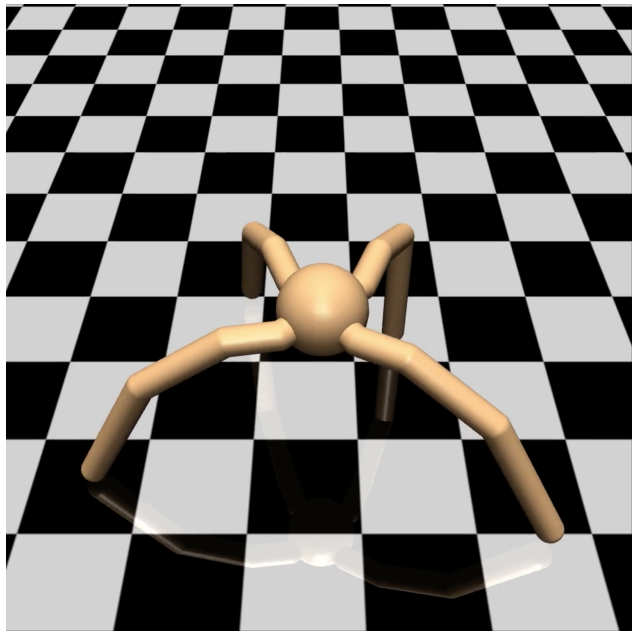
- Dynamics of the MDP are completely captured in the current state and the proposed action
- States include all information about the past that make a difference for the future
- $p(s', r|s, a) \triangleq Pr\{S_t = s', R_t = r|S_{t-1} = s, A_{t-1} = a\}$

MDP Generalizations

- Continuous timesteps
- Continuous states/actions
- Partially observable (POMDP): probability distribution over S_t
- Don't worry about them for now

Episodic vs. Continuing Tasks





Episodic Task:

- $t = 0, 1, ..., T$
- T (length of episode) can vary from episode to episode
- Example: Pacman

Continuing Task:

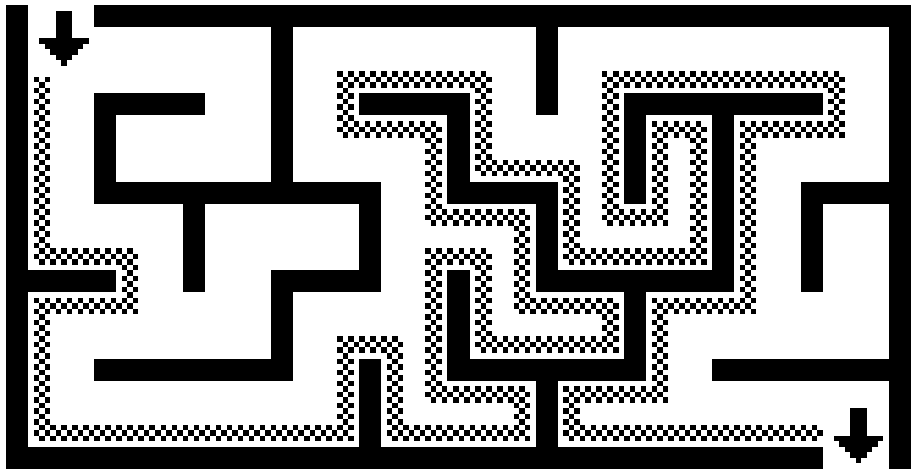
- No upper limit on t
- Example: learning to walk

Expected Return and Unified Task Notation

- G_t is the expected return (expected sum of rewards)
- $G_t \triangleq \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$
- γ is the discount factor, $0 < \gamma \leq 1$
- Episodic Tasks:
  - $R_t = 0$ for all $t > T$, finite sum
  - Often choose $\gamma = 1$
  - Always converges since sum is finite
- Continuing Tasks:
  - Infinite sum
  - $\gamma < 1$ (future rewards are valued less)
  - If $\{R_t\}$ is bounded, the infinite sum converges
- Either way, we can insure that G_t is well-defined

Policies

- Policy π: a mapping from state to actions
- $\pi(a|s) \triangleq Pr\{A_t = a|S_t = s\}$
- Can be deterministic or probabilistic
- Example: deterministic wall-following maze navigation policy
  - Turn right if possible
  - If you can't turn right, go straight if possible
  - If you can't turn right or go straight, turn left if possible
  - Otherwise, turn around

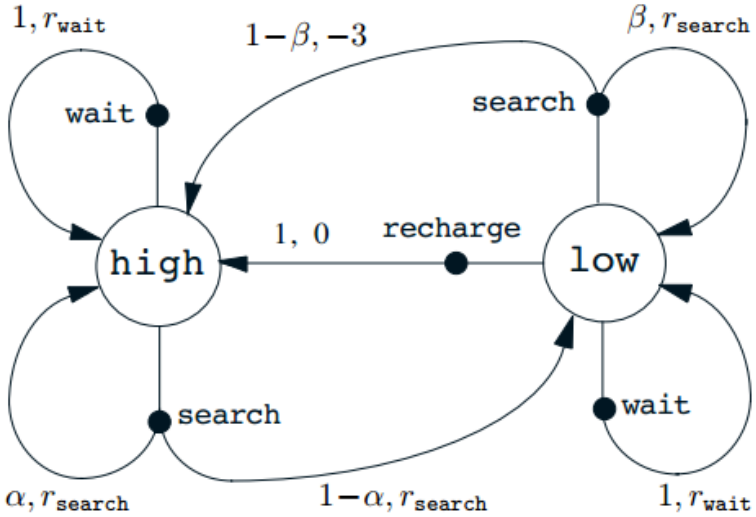


Value Functions

- State-value function $v_\pi(s) \triangleq \mathbb{E}_\pi [G_t | S_t = s]$
  - Intuitively: how "good" a state s is under policy π
  - $v_\pi(s) = \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \middle| S_t = s \right]$
- Action-value function $q_\pi(s, a) \triangleq \mathbb{E}_\pi [G_t | S_t = s, A_t = a]$
  - Intuitively: how "good" an action a is in state s under policy π
  - $q_\pi(s, a) = \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \middle| S_t = s, A_t = a \right]$

MDP Illustration

s	a	s'	$p(s'\|s,a)$	$r(s,a,s')$
high	search	high	α	$r_{\texttt{search}}$
high	search	low	$1-\alpha$	$r_{\texttt{search}}$
low	search	high	$1-\beta$	-3
low	search	low	β	$r_{\texttt{search}}$
high	wait	high	1	$r_{\texttt{wait}}$
high	wait	low	0	-
low	wait	high	0	-
low	wait	low	1	$r_{\texttt{wait}}$
low	recharge	high	1	0
low	recharge	low	0	-



- Note: in this case, we are using $p(s'|s,a)$ to model the environment instead of $p(s',r|s,a)$
- This is because there is no ambiguity in reward r given s,a,s' (but in general there could be)
- Even only two states and three possible actions can be pretty complicated to model

Some Potentially Useful Value Identities

- Recall: $\pi(a|s) \triangleq Pr\{A_t = a|S_t = s\}$
- Recall: $v_\pi(s) \triangleq \mathbb{E}_\pi[G_t|S_t = s]$
- Recall: $q_\pi(s,a) \triangleq \mathbb{E}_\pi[G_t|S_t = s, A_t = a]$
- $v_\pi(s) = \mathbb{E}_\pi[q_\pi(s,a)|S_t = s] = \sum_{a \in A(s)} \pi(a|s) q_\pi(s,a)$
- $q_\pi(s,a) = \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1}|S_t = s, A_t = a]$
- $q_\pi(s,a) = \mathbb{E}_{p(s',r|s,a)}[R_{t+1} + \gamma v_\pi(S_{t+1})|S_t = s, A_t = a]$
- Common theme: value functions are recursive, expand one step to see a useful pattern

Solving RL Problems

Two main classes of solutions:

- Tabular Solution Methods
  - Provide exact solutions
  - Do not scale very well with search space
- Approximate Solution Methods
  - Use function approximators (i.e. neural networks) for value and/or policy functions
  - Policy Gradient Methods fall under this category

Introducing Policy Gradient

- Parameterize policy $\pi(a|s;\theta) \triangleq Pr\{A_t = a|S_t = s;\theta_t = \theta\}$
  - For example, π could be a neural network with weights θ
- Measure policy performance with scalar performance measure $J(\theta)$
- Maximize $J(\theta)$ with gradient ascent on θ
- General update rule: $\theta_{t+1} = \theta_t + \alpha\widehat{\nabla_\theta J(\theta_t)}$
  - α is step size
  - $\widehat{\nabla_\theta J(\theta_t)}$ is an approximation to $\nabla_\theta J(\theta_t)$
- We are increasing the probability of trajectories with large rewards and decreasing the probability of trajectories with small rewards
- For simplicity, we'll only consider episodic tasks for the remainder of the presentation

The Policy Gradient Theorem

- $J(\theta) \triangleq v_\pi(s_0)$, where s_0 is initial state of a trajectory and π is a policy parameterized by θ
- The Policy Gradient Theorem states that
  $\nabla J(\theta) = \nabla_\theta v_\pi(s_0) \propto \sum_s \mu(s) \sum_a q_\pi(s,a) \nabla(\pi(a|s;\theta))$
- $\mu(s) \triangleq \frac{\eta(s)}{\sum_{s'} \eta(s')}$, the on-policy distribution under π
  - $\eta(s)$ is the expected number of visits to state s
  - $\mu(s)$ is the fraction of time spent in state s
- Constant of proportionality is the average length of an episode
- See p. 325 of Sutton RL Book (Second Edition) for concise proof

REINFORCE Update

$$\begin{aligned}
\nabla J(\theta) &\propto \sum_s \mu(s) \sum_a q_\pi(s,a) \nabla(\pi(a|s;\theta)) \\
&= \mathbb{E}_\pi \left[\sum_a q_\pi(S_t,a) \nabla \pi(a|S_t,\theta) \right] \\
&= \mathbb{E}_\pi \left[\sum_a \pi(a|S_t,\theta) q_\pi(S_t,a) \frac{\nabla \pi(a|S_t,\theta)}{\pi(a|S_t,\theta)} \right] \\
&= \mathbb{E}_\pi \left[\mathbb{E}_\pi \left[q_\pi(S_t,A_t) \frac{\nabla \pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta)} \right] \right] \\
&= \mathbb{E}_\pi \left[q_\pi(S_t,A_t) \frac{\nabla \pi(A_t|S_t,\theta)}{\pi(A_t|S_t,\theta)} \right]
\end{aligned}$$

REINFORCE Update (Continued)

$$
\begin{aligned}
\nabla J(\theta) &\propto \mathbb{E}_\pi \left[q_\pi(S_t, A_t) \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \right] \\
&= \mathbb{E}_\pi \left[\mathbb{E}_\pi \left[G_t | S_t, A_t \right] \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \right] \\
&= \mathbb{E}_\pi \left[\mathbb{E}_\pi \left[G_t \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \middle| S_t, A_t \right] \right] \\
&= \mathbb{E}_\pi \left[G_t \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)} \right]
\end{aligned}
$$

Reinforce update: $\theta_{t+1} = \theta_t + \alpha G_t \frac{\nabla \pi(A_t|S_t, \theta)}{\pi(A_t|S_t, \theta)}$

Quick Aside: Monte Carlo

- Way of approximating an integral/expectation
- $\mathbb{E}_{p(x)}[f(x)] = \int_{x \in \mathcal{X}} p(x) f(x) dx$
- If $p(x)$ and/or $f(x)$ is complicated, the integral is difficult to solve analytically
- Simple idea: sample N values from the domain of the distribution $\mathcal{X}$ and take the average
- $\mathbb{E}_{p(x)}[f(x)] \approx \frac{1}{N} \sum_{i=1}^{N} p(x^{(i)}) f(x^{(i)})$
- The larger N is, the better the estimate

REINFORCE Algorithm

REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ $\quad (G_t)$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

- Generate entire trajectories and compute G_t from it
- Gradient estimates only need to be proportional to the true gradient since constant of proportionality can be absorbed in the step size α

REINFORCE as a Score Function Gradient Estimator

- We can rewrite $J(\theta)$ in terms of an expectation over trajectories τ
- $J(\theta) \triangleq v_\pi(s_0) = \mathbb{E}_\tau [G_0(\tau)] = \sum_\tau p(\tau|\pi_\theta) G_0(\tau)$
  - $p(\tau|\pi_\theta)$ is the probability of the trajectory under policy π_θ
  - $G_0(\tau)$ is the sum of the rewards given trajectory τ
- Recall: $\nabla_x \log f(x) = \dfrac{\nabla_x f(x)}{f(x)}$ by chain rule
- We can use the log-derivative trick to derive an expression for $\widehat{\nabla J(\theta)}$
- In statistics, this estimator is called the Score Function Estimator

REINFORCE as a Score Function Gradient Estimator

$$
\begin{aligned}
\nabla J(\theta) &= \nabla \sum_{\tau} p(\tau|\pi_\theta) G_0(\tau) \\
&= \sum_{\tau} \nabla p(\tau|\pi_\theta) G_0(\tau) \\
&= \sum_{\tau} \frac{p(\tau|\pi_\theta)}{p(\tau|\pi_\theta)} \nabla p(\tau|\pi_\theta) G_0(\tau) \\
&= \sum_{\tau} p(\tau|\pi_\theta) \nabla \log p(\tau|\pi_\theta) G_0(\tau) \\
&= \mathbb{E}_\tau \left[\nabla \log p(\tau|\pi_\theta) G_0(\tau) \right] \\
\widehat{\nabla J(\theta)} &= \frac{1}{N} \sum_{i=1}^{N} \log p(\tau^{(i)}|\pi_\theta) G_0(\tau^{(i)})
\end{aligned}
$$

(Monte Carlo approximation)

Properties of REINFORCE/Score Function Gradient Estimator

- Pros:
  - Unbiased: $\mathbb{E}\left[\widehat{\nabla J(\theta)}\right] = J(\theta)$
  - Very general (reward does not need to be continuous/differentiable)
- Cons:
  - High variance (noisy)
  - Requires many sample trajectories to estimate gradient

Reducing REINFORCE Estimator Variance

- Recall: $\nabla J(\theta) \propto \sum_s \mu(s) \sum_a q_\pi(s,a) \nabla(\pi(a|s;\theta))$
- Let $b(s)$ be a baseline (some function of s that does not depend on θ)
- $\sum_a b(s) \nabla \pi(a|s,\theta) = b(s) \nabla \sum_a \pi(a|s,\theta) = b(s) \nabla 1 = 0$
- Thus $\nabla J(\theta) \propto \sum_s \mu(s) \sum_a (q_\pi(s,a) - b(s)) \nabla(\pi(a|s;\theta))$
- New update rule: $\theta_{t+1} = \theta_t + \alpha \left(G_t - b(S_t)\right) \frac{\nabla \pi(A_t|S_t,\theta_t)}{\pi(A_t|S_t,\theta_t)}$
- Natural choice for $b(s)$: an estimate of the state value $\hat{v}(S_t, w)$ parameterized by weights w
  - Want to adjust θ based on how much better (or worse) the path is than the average score of paths starting in the current state ($\hat{v}$)

REINFORCE with Baseline Algorithm

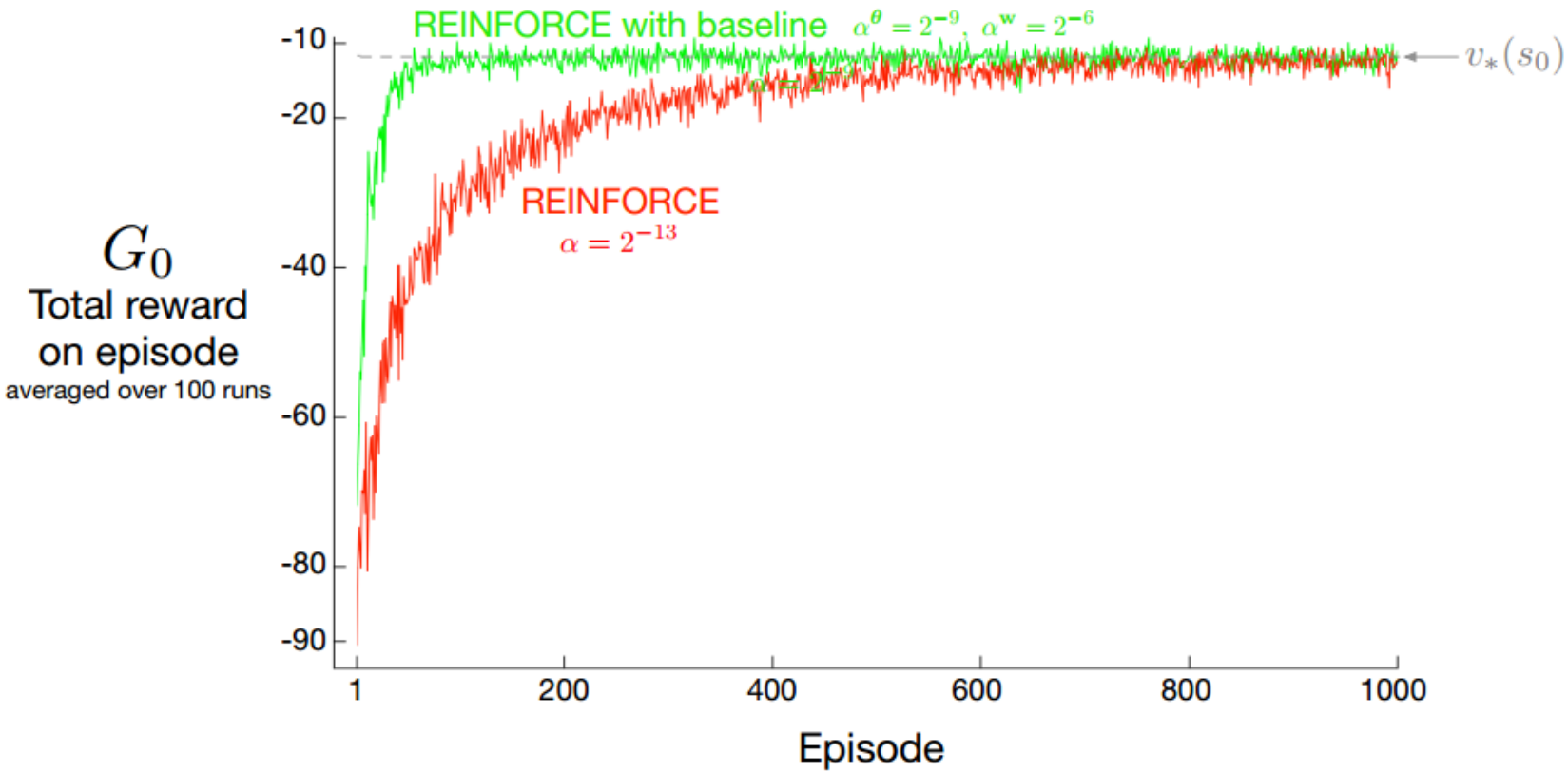
REINFORCE with Baseline (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s, \mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\delta \leftarrow G - \hat{v}(S_t, \mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S_t, \mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

- Two different step sizes: α^{θ} and α^{w}
- $\hat{v}$ is also estimated with Monte Carlo

REINFORCE with Baseline vs. without



Extensions: Actor-Critic

- Use bootstrapping to update the state-value estimate $\hat{v}(s, w)$ for a state s from the estimated values of subsequent states
- Introduces bias but reduces variance
- Can use one-step or n-step return (as opposed to full returns in REINFORCE)

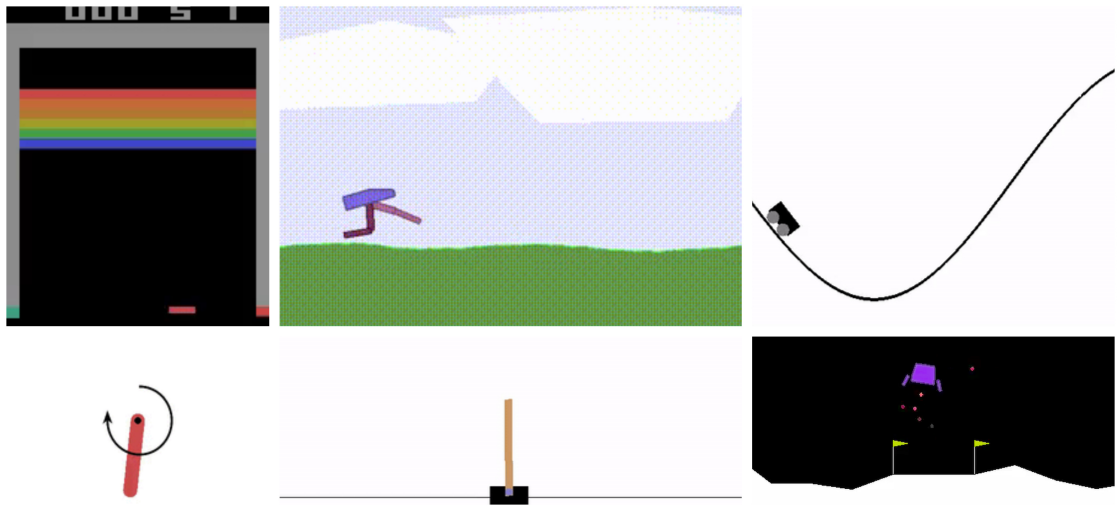
One-step Actor-Critic Algorithm

One-step Actor–Critic (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)
Loop forever (for each episode):
  Initialize S (first state of episode)
  $I \leftarrow 1$
  Loop while S is not terminal (for each time step):
    $A \sim \pi(\cdot|S,\boldsymbol{\theta})$
    Take action A, observe S', R
    $\delta \leftarrow R + \gamma \hat{v}(S',\mathbf{w}) - \hat{v}(S,\mathbf{w})$ (if S' is terminal, then $\hat{v}(S',\mathbf{w}) \doteq 0$)
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} I \delta \nabla \ln \pi(A|S,\boldsymbol{\theta})$
    $I \leftarrow \gamma I$
    $S \leftarrow S'$

OpenAI Gym

- Free python library of environments for reinforcement learning
- Easy to use, good for benchmarking
- Environments include:
  - Classic control problems
  - Atari games
  - 3D simulations (Mujoco)
  - Robotics
- Check out the docs and environments



REINFORCE bipedal walker demo

- OpenAI Gym Environment
- Train a 2D walker to navigate a landscape
- Different levels of difficulty
- Code taken from Michael Guerzhoy's CSC411 course webpage
- Uses policy gradient with REINFORCE
- Note: requires tensorflow 1.x (not 2.x)

References

- Sutton and Barto, "Reinforcement Learning: An Introduction (Second Edition)", 2018
- Xuchan (Jenny) Bao, Intro to RL and Policy Gradient CSC421 Tutorial, 2019 (link)
- Michael Guerzhoy, CSC411 Project 4: Reinforcement Learning with Policy Gradients, 2017 (link)