STA 437/1005 — Answers to Quiz #1 — 2010-09-27

1. Consider the following six observations of two variables:

100	20
95	18
110	22
100	20
100	21
95	19

   a) [10 marks] Find the sample mean vector for this data set.

   Answer: $[\,100,\ 20\,]'$

   b) [30 marks] Find the sample covariance matrix for this data. (Use the definition in which the divisor is the number of observations minus one.)

   Answer: $\begin{bmatrix} 30 & 7 \\ 7 & 2 \end{bmatrix}$

2. Let $X = [X_1,\ X_2,\ X_3]'$ be a three-dimensional random vector. Suppose that X has mean vector $\mu = [0,\ 0,\ 1]'$ and covariance matrix Σ and inverse covariance Σ^{-1} as below:

$$\Sigma \;=\; \begin{bmatrix} 1 & -1/2 & 1/2 \\ -1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 4 \end{bmatrix}, \qquad \Sigma^{-1} \;=\; \begin{bmatrix} 5/3 & 1 & -1/3 \\ 1 & 5/3 & -1/3 \\ -1/3 & -1/3 & 1/3 \end{bmatrix}$$

   a) [40 marks] Define $Y = [Y_1,\ Y_2]'$ with $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + 2X_3$. Find the mean vector and covariance matrix for Y.

   Answers: $E(Y) \;=\; [\,0,\ 2\,]', \quad \mathrm{Cov}(Y) \;=\; \begin{bmatrix} 3 & 3/2 \\ 3/2 & 19 \end{bmatrix}$

   b) [20 marks] Suppose the vector X has the multivariate normal distribution with mean $\mu = [0,\ 0,\ 1]'$ and with covariance matrix Σ given above. Which of the points below have the same probability density as the point $[1,\ 0,\ 1]'$? Write "Yes" or "No" for each (no explanation required).

   $[-1,\ 0,\ 1]$ *Answer: Yes*

   $[0,\ 0,\ 1]$ *Answer: No*

   $[0,\ 0,\ 6]$ *Answer: No*

   $[0,\ 1,\ 0]$ *Answer: No*

   $[0,\ -1,\ 1]$ *Answer: Yes*