STA 247 — Assignment #3, Due in class on December 5 at 3:10pm

Late assignments will be accepted only with a valid medical or other excuse.
This assignment is to be done by each student individually.
There is just one page for this assignment.

1. Let the random variables X and Y be independent, and let both X and Y have the Poisson distribution with mean λ. Prove that $Z = X + Y$ has the Poisson distribution with mean 2λ, by computing the probability mass function for Z and comparing it with the Poisson probability formula. Hint: To find $P(Z = z)$, consider all possible ways that z can be written as $x + y$, with x and y being non-negative integers.

2. Suppose you roll a fair six-sided die until it shows the number six. Let X be the sum of the numbers on all these rolls (including the final six). For example, you might rolls the numbers 3, 1, 5, and 6, in which case $X = 15$. Find the mean and standard deviation of X.

3. Suppose the random variable W has the following probability density function, for some value of the constant c:

$$f(w) = \begin{cases} 0 & \text{if } x < 1 \\ c/x^2 & \text{if } 1 \leq x \leq 3 \\ 0 & \text{if } x > 3 \end{cases}$$

Find what the value of c must be in order for this to be a valid probability density function, and find the mean and the median of W. The median is the point, a, for which $P(W \leq a) = 1/2$.

4. Bits sent through a communications channel are sometimes received with the wrong value. For some channels, such errors occur in bursts, with several errors occurring in a row. We can model such error behaviour using a Markov chain. Let $E(i)$ be the random variable having the value 1 if an error occurred in bit i, and 0 otherwise. Suppose that these errors have the Markov property, so that

$$P(E(i) = e_i \,|, E(i-1) = e_{i-1}) \;=\; P(E(i) = e_i \,|\, E(i-1) = e_{i-1},\, E(i-2) = e_{i-2}, \ldots, E(0) = e_0)$$

We can then specify the error behaviour of the channel by the one-step transition probabilities for this Markov chain. Suppose that these transition probabilities are the same at all times (ie, the Markov chain is homogeneous). The one-step transition probabilities will then be determined by just two numbers, P_{01} and P_{11}, defined by

$$P_{01} = P(E(i) = 1 \,|\, E(i-1) = 0), \quad P_{11} = P(E(i) = 1 \,|\, E(i-1) = 1)$$

For this question, we will assume that $P_{01} = 0.01$ and $P_{11} = 0.4$.

   (a) Find $P(E(i+3) = 1 \,|\, E(i) = 1)$ exactly.

   (b) Suppose that data is sent through this channel in blocks of 100 bits, using an error-correcting code that is capable of fixing up to four errors in a block (but no more). Write an R function that simulates the transmission of 1000 such blocks through this channel, and returns the fraction of these blocks that had errors that couldn't be fixed (ie, the fraction of blocks for which more than four of the 100 bits were received with errors). To start your simulation, assume that the bit before the first bit in the first block you simulate was not an error.

   Hand in a listing of your function, and the output of three runs of this function.