STA 450/2102 — Assignment #2

Due at the beginning of class on March 10. Worth 18% of the final mark.

Below are two maximum likelihood estimation problems. For each problem, you should hand in

- Your derivation of the relevant formulas (eg, the derivatives of the log likelihood).
- A listing of your program to find the maximum likelihood estimate (and standard errors for problem 2), which should be formatted for easy reading, with an appropriate amount of documentation.
- The output of several tests designed to show that your program is correct.
- Discussion as requested for the problem.

Problem 1: Suppose that independent, identically-distributed data points $x_1, \ldots, x_n$ are to be modeled as coming from a Cauchy distribution with scale parameter 1, but with unknown location parameter, θ. The likelihood function will be

$$L(\theta) = \prod_{i=1}^{n} \frac{1}{\pi(1 + (x_i - \theta)^2)}$$

Write an S/R function to find the maximum likelihood estimate for θ, using Newton-Raphson iteration to find a point where the derivative of the log likelihood is zero. The function should take as arguments the data, an initial guess for θ, and the number of iterations to do. Test it on various data sets, with various initial guesses for θ. Does it always converge to an answer? Is the answer always a maximum? When a maximum is found, is it always the global maximum?

Looking at plots of the log likelihood funcition may help in answering these questions.

Discuss whether or not $\sqrt{-1/\ddot{\ell}(\hat{\theta})}$ would give a useful standard error for $\hat{\theta}$.

Problem 2: A quantity of some radioactive substance has been monitored for radioactive decays. At times $t_1, t_2, \ldots, t_m$, the number of decays detected by a gieger counter in a one-second period was recorded to be $n_1, n_2, \ldots, n_m$.

The number of decays detected in one second at time t is expected to have a Poisson distribution with a mean that decreases exponentially over time, as $\exp(a - bt)$, with a and b being unknown parameters. Counts at different times are independent. The likelihood function is therefore

$$L(a, b) = \prod_{i=1}^{m} \frac{[\exp(a - bt)]^{n_i}}{n_i!} \exp(-\exp(a - bt))$$

Write an S/R function to find the maximum likelihood estimates for a and b, using Newton-Raphson iteration. Also, compute the estimated covariance matrix for these estimates based on the observed information, and from this find the standard errors for a and b.

For this problem, are there problems with Newton-Raphson iteration failing to converge, or with it converging to something other than the global maximum?