CSC 363, Winter 2010 — Short Assignment #4

*Due at **start** of tutorial time (really test time) on March 17. Late assignments will **not** be accepted, as the answers will be gone over in lecture right after. I prefer that you hand the assignment in on paper, but if you need to, you can email it (as a plain text file or PDF attachment, no Microsoft Word files please) to `radford@cdf.utoronto.ca`. (Please use this email address only for assignment submission.)*

This assignment is to be done by each student individually. You are encouraged to discuss the course material in general with other students, but you should not discuss this assignment (verbally, in writing, by email, or in any other way) with people other than the course instructor and tutors, except to clarify the meaning of the question. Handing in work that is not your own is a serious academic offense.

Give a description of a polynomial time verifier (see Definition 7.18 in the text) for each of the following languages, along with an argument that it does indeed operate in time that is polynomial in the length of the string being verified. You needn't find an explicit polynomial bound, however, so it doesn't matter whether you assume computation is done with a one-tape Turing Machine, a k-tape Turing Machine, or any other reasonable model of computation. Similar, except as noted, the exact way in which the input is encoded should not matter, as long as it is reasonable.

1. $\{\langle\phi\rangle \mid \phi$ is a Boolean formula for which both ϕ and $\overline{\phi}$ are satisfiable$\}$.
2. $\{\langle M, k\rangle \mid M$ is a deterministic Turing Machine that accepts some string of length k after running for exactly k^3 steps$\}$

   Here, k has a unary encoding (eg, five is encoded as "11111").
3. $\{w \mid w = 1^n$ for some composite integer $n > 1\}$
4. $(01)^*$
5. $\{\langle G_1, G_2\rangle \mid G_1$ and G_2 are undirected graphs that contain maximal cliques of the same size$\}$.

A clique is a subset of nodes in a graph in which there is an edge between every pair of nodes in the subset. A maximal clique is a clique that is not a subset of any other clique. (Note that this is not the same as a maximum clique, which is a clique that is as big as possible.)