

**CSC 2541, Small exercise #1, due in class January 17, worth 5% of the mark**

Suppose we model the observations  $Y_1, Y_2, \dots, Y_n$ , which are positive real numbers, as being independent, with each having the same distribution. We model this distribution as having the following density:

$$P(y) = \begin{cases} 0 & \text{if } 0 < y < \theta \\ c(\theta)/y^p & \text{if } y \geq \theta \end{cases}$$

Here,  $p$  is a known real constant greater than one, and  $\theta$  is an unknown positive real parameter.

**Question 1:** Find the function  $c(\theta)$ , which must be such that the density for  $y$  integrates to one.

**Question 2:** Find the likelihood function for this model, given values of the observations,  $Y_1 = y_1, Y_2 = y_2, \dots, Y_n = y_n$ . Recall that factors not involving the parameter  $\theta$  can be ignored in the likelihood function.

**Question 3:** Find a class of prior distributions for  $\theta$  that is conjugate for this model — ie, for which the posterior distribution is in the same class as the prior — and for which analytically calculating the posterior mean is tractable.