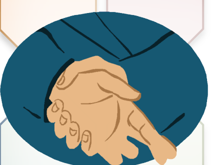
Mergers and Collusion in All-Pay Auctions and Crowdsourcing Contests



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Perliminaries

All-pay auctions

Bidders bid **and pay their bid** to the auctioneer

Auction winner is one which submitted the highest bid

Why all-pay auctions?

Explicit all-pay auctions are rare, but implicit ones are extremely common:

Competition for patents between firms

Crowdsourcing competitions (e.g., Netflix challenge, TopCoder, etc.)

Hiring employees

Employee competition ("employee of the month")

Perliminaries

Auctioneer types

“sum profit”

Gets the bids from all bidders – regardless of their winning status

E.g., “emloyee of the month”

“max profit”

Gets only the winner’s bid. Other bids are, effectively, “burned”

E.g., hiring an emplyee

Regular all-pay

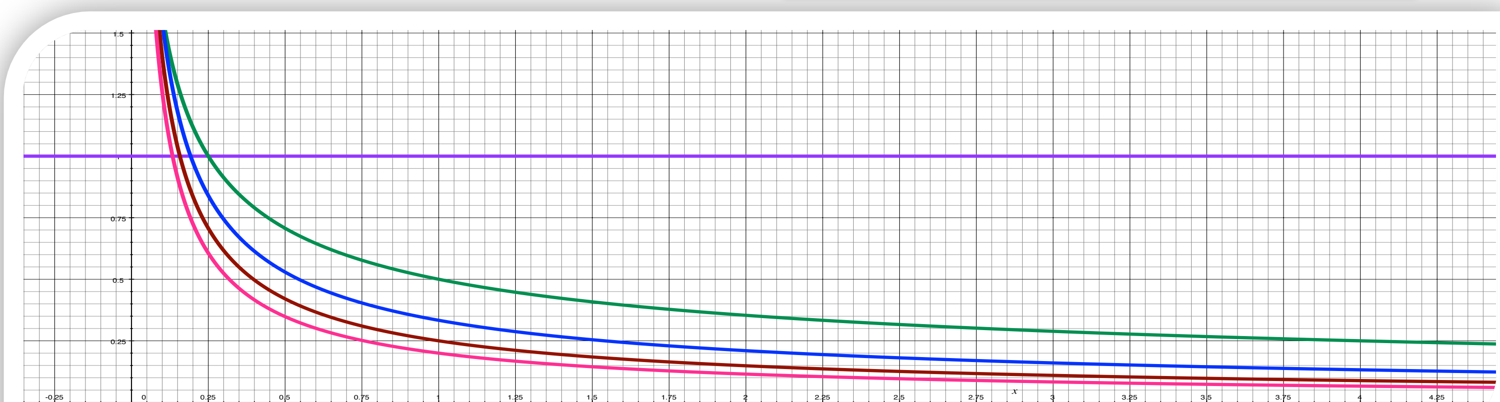
All-pay auction equilibrium

All bidders give the object in question a value of 1

A single symmetric equilibrium – for *n* bidders:

$$F_n(x) = x^{\frac{1}{n-1}}$$

$$f_n(x) = \frac{x^{\frac{2-n}{n-1}}}{n-1}$$



Baye, Kovenock, de Vries

Regular all-pay

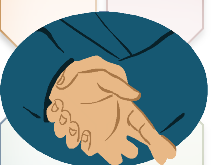
All-pay auction equilibrium

bidder properties

Expected utility:	0
Utility variance:	$\dfrac{3n^2-5n+2}{n(2n-1)(3n-2)}$
Expected bid:	$\dfrac{1}{n}$
Bid variance:	$\dfrac{1}{2n-1}-\dfrac{1}{n^2}$

Baye, Kovenock, de Vries

Regular all-pay



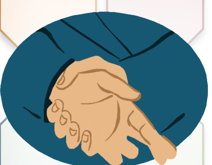
All-pay auction equilibrium

auctioneer properties

Sum profit expected profit:	1
Sum profit profit variance:	$\frac{n}{2n-1} - \frac{1}{n}$
Max profit expected profit:	$\frac{n}{2n-1}$
Max profit profit variance:	$\frac{n(n-1)^2}{(3n-2)(2n-1)^2}$

Baye, Kovenock, de Vries

Example no collusion case



3 bidders

Bidders' c.d.f is $\sqrt{x}$ and the expected bid is ⅓, with variance of $\frac{4}{45}$. Expected profit is 0 with variance of $\frac{2}{15}$.

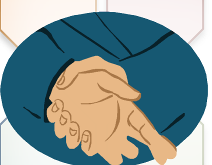
Sum profit auctioneer has expected profit of 1 with variance of $\frac{4}{15}$.

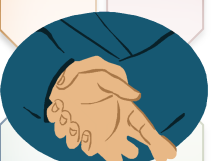
Max profit auctioneer has expected profit of ⅗ with variance of $\frac{12}{175}$.

Baye, Kovenock, de Vries

Mergers

k bidders (out of the total n) collaborate, having a joint strategy. **All other bidders are aware of this.**





Mergers
(collaboration public knowledge)

Merger properties

Equilibrium remains the same – but with smaller *n*

Bidder

- Expected Utility: 0
- Utility variance:
- Expected bid:
- Bid variance:

Sum Profit

- Expected profit: 1
- Profit variance:

Max Profit

- Expected profit:
- Profit variance:

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Mergers
(collaboration public knowledge)

Example merger case

3 bidders, 2 of them merged

Bidders' c.d.f is uniform, and the expected bid is $\frac{1}{2}$, with variance of $\frac{1}{12}$. Expected profit is 0 with variance of $\frac{1}{6}$.

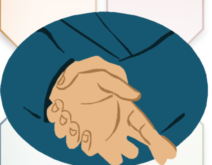
Sum profit auctioneer has expected profit of 1 with variance of $\frac{1}{6}$.

Max profit auctioneer has expected profit of $\frac{2}{3}$ with variance of $\frac{1}{18}$.

Collusions

Collusion
(collaboration private knowledge)

k bidders (out of the total *n*) collaborate, having a joint strategy. **Other bidders are not aware of this and continue to pursue their previous strategies.**



Collusion
(collaboration private knowledge)

Collusion colluders

Colluders have a pure, optimal strategy

$$b^* = \left(\frac{n-k}{n-1}\right)^{\frac{n-1}{k-1}}$$

k: $\downarrow$ n: $\overset{e^{-1}}{\uparrow}$

Producing an expected profit of:

$$\left(\frac{n-k}{n-1}\right)^{\frac{n-1}{k-1}} \left(\frac{k-1}{n-1}\right)$$

k: $\uparrow$ n: $\downarrow$

Colluders' profit **per colluder** increases as number of colluders grows

Profit variance: $\left(\frac{n-k}{n-1}\right)^{\frac{n-k}{k-1}} - \left(\frac{n-k}{n-1}\right)^{\frac{2(n-k)}{k-1}}$

Collusion auctioneers

Collusion
(collaboration private knowledge)

Sum profit: $$\frac{n-k}{n} + \left(\frac{n-k}{n-1}\right)^{\frac{n-1}{k-1}}$$

k: $\downarrow$ n: $\uparrow$

Max profit: $$\frac{n-k}{2n-k-1}\left(1 + \left(\frac{n-k}{n-1}\right)^{\frac{2(n-k)}{k-1}}\right)$$

k: $\downarrow$ n: $\uparrow$

For large enough n exceed non-colluding profits

Collusion non-colluding bidders

Utility for non-colluding bidders is:

$$\frac{k}{n(n-k)} - \frac{\left(\frac{n-k}{n-1}\right)^{\frac{n-k}{k-1}}}{n-k}$$

For large enough k (e.g., $\frac{n}{2}$) this expression is positive.

I.e., non-colluders profit from collusion

If a non-colluder discovers the collusion, best to bid a bit above colluders

Example no collusion case

3 bidders

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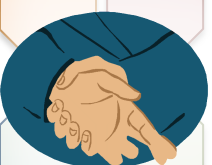

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Collusion
(collaboration private knowledge)

Example collusion case

Collusion
(collaboration private knowledge)

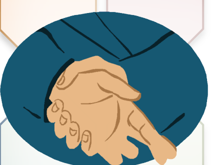
3 bidders, 2 of them collude

One bidder has c.d.f of $\sqrt{x}$ (expected bid of ⅓), colluders bid ¼. Colluders' expected profit is ¼, while the non-colluder expected *profit* is ⅙.

Sum profit auctioneer expected profit only $\frac{7}{12}$.

Max profit auctioneer has expected profit of $\frac{10}{24}$.

Future directions



Adding bidders' skills to model

Detecting collusions by other bidders

Designing crowdsourcing mechanisms less susceptible to collusion

Adding probability to win based on effort

The End

Thanks for listening!