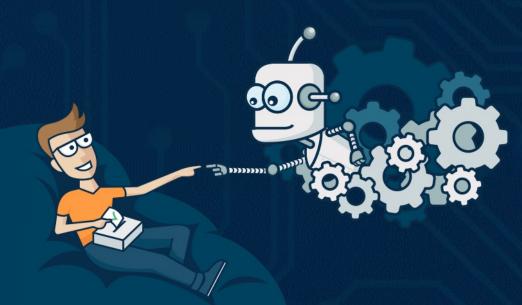
### AAMAS 2022 Tutorial Distortion in Social Choice & Beyond

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### Outline

#### Introduction

- Applications of voting
- Motivating the distortion framework

#### • Utilitarian distortion framework

- Model
- Known results

#### • Metric distortion framework

- Model
- Known results
- Applications beyond voting

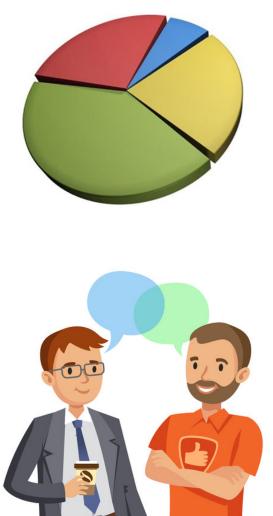
### Voting

# Algorithm for aggregating individual preferences to make collective decisions



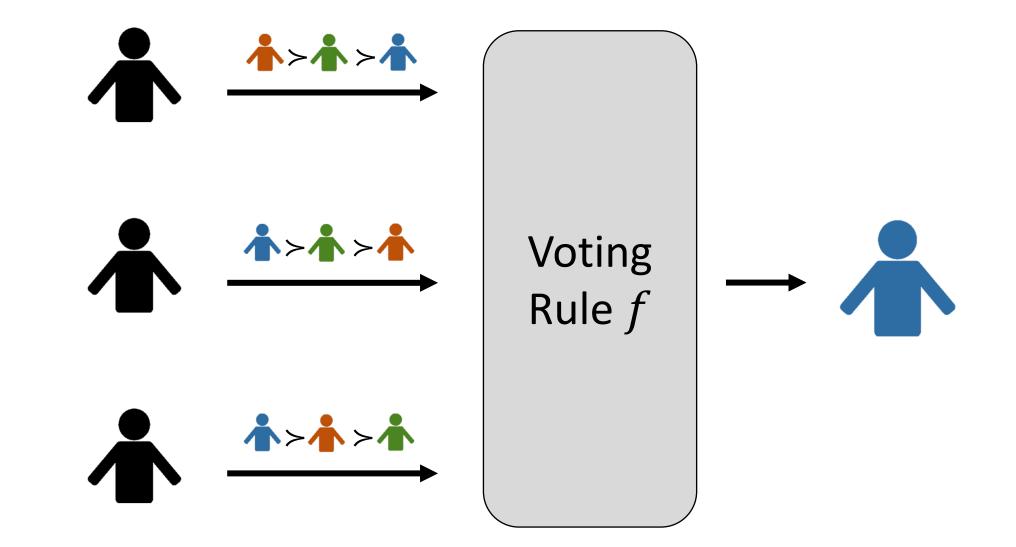
### **Applications of Voting**







#### Voting with Ranked Ballots



### **Axiomatic Framework**

#### • Condorcet consistency

• Whenever there exists an alternative *a* such that for every other alternative *b* a strict majority prefer *a* to *b*, the voting rule must select *a*.

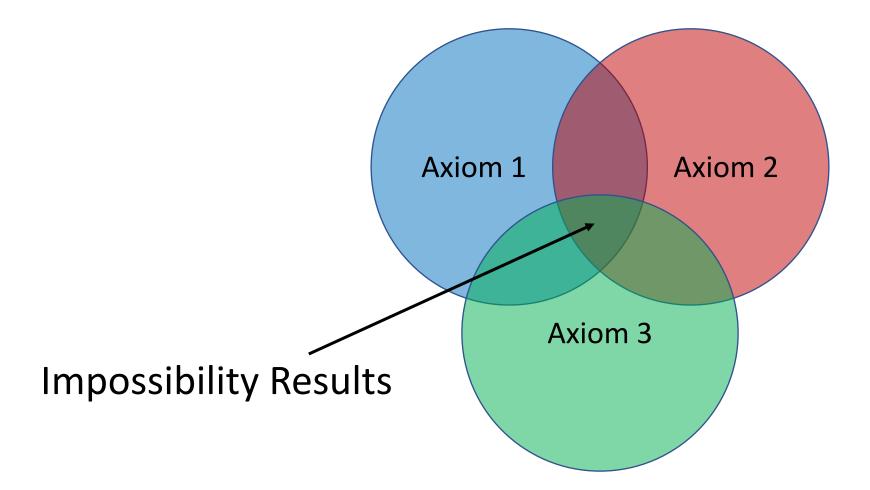
#### • Weak monotonicity

• If the voting rule selects alternative *a* in an instance and *a* moves up in the rankings of some of the voters, the voting rule must continue to select *a*.

#### • Axioms are qualitative

• A voting rule either satisfies an axiom or it does not

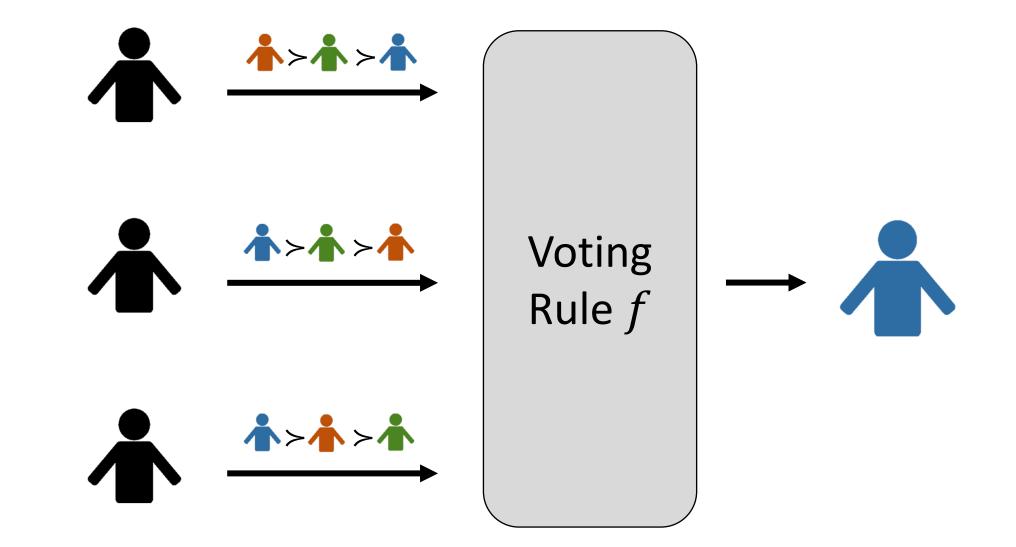
#### Axiomatic Framework



#### Axiomatic Framework

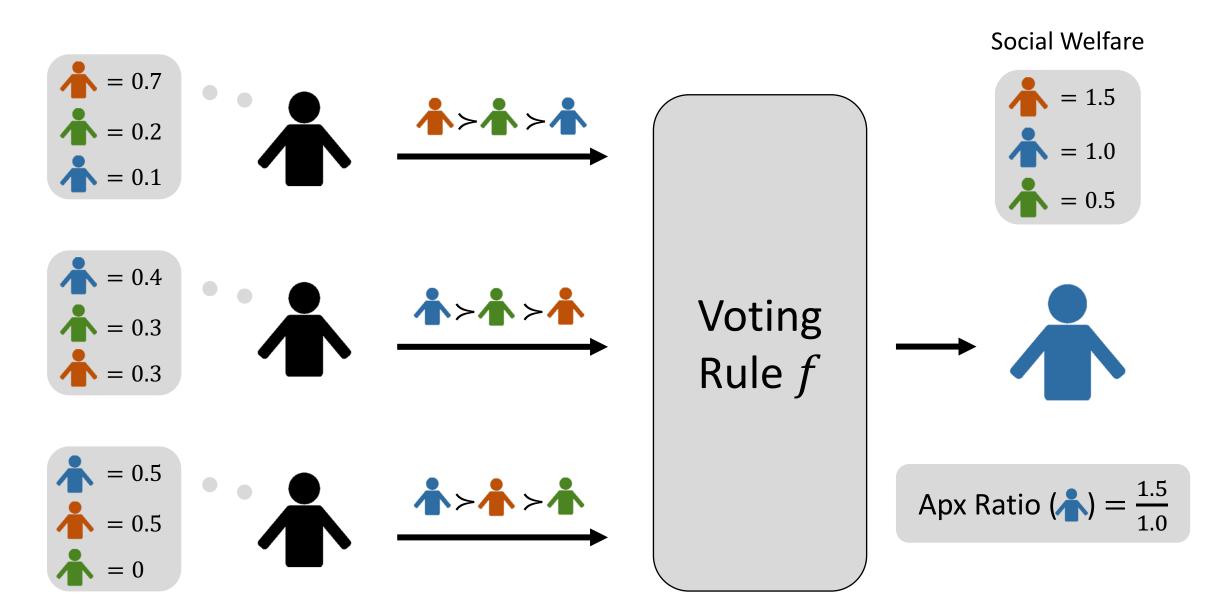
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Criterion		Maj.	Mutual	C. I. I.	Cond.	Smith/			<u></u>		C	Destination	Reversal	Polyti	ime/	C	Later-no-		No	Ballot	Ra	Ranks	
Method	Majority	loser	<u>maj.</u>	Condorcet	loser	ISDA	LIIA	IIA	Cloneproof	wonotone	Consistency	Participation	symmetry	resolv	able	Summable	Harm	Help	favorite betrayal	type	=	>2	
Approval	Rated <sup>[a]</sup>	No	No	No <sup>[b][c]</sup>	No	No <sup>[b]</sup>	Yes	Yes <sup>[d]</sup>	Yes <sup>[e]</sup>	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes <sup>[f]</sup>	Yes	Approvals	Yes	No	
Borda count	No	Yes	No	No <sup>[b]</sup>	Yes	No	No	No	Teams	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	No	Ranking	No	Yes	
Bucklin	Yes	Yes	Yes	No	No	No	No	No	No	Yes	No	No	No	O(N)	Yes	O(N)	No	Yes	If equal preferences	Ranking	Yes	Yes	
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	No	No <sup>[b]</sup>	Teams, crowds	Yes	No <sup>[b]</sup>	No <sup>[b]</sup>	Yes	O(N <sup>2</sup> )	No	O(N <sup>2</sup> )	No <sup>[b]</sup>	No	No <sup>[b]</sup>	Ranking	Yes	Yes	
IRV (AV)	Yes	Yes	Yes	No <sup>[b]</sup>	Yes	No <sup>[b]</sup>	No	No	Yes	No	No	No	No	O(N <sup>2</sup> )	Yes <sup>[g]</sup>	O(N!) <sup>[h]</sup>	Yes	Yes	No	Ranking	No	Yes	
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No <sup>[b]</sup>	Spoilers	Yes	No <sup>[b]</sup> [i]	No <sup>[b]</sup>	Yes	O(N!)	Yes	O(N <sup>2</sup> ) <sup>[j]</sup>	No <sup>[b]</sup>	No	No <sup>[b]</sup>	Ranking	Yes	Yes	
Majority judgment <sup>[k]</sup>	Rated <sup>[I]</sup>	Yes <sup>[m]</sup>	No <sup>[n]</sup>	No <sup>[b][c]</sup>	No	No <sup>[b]</sup>	Yes	Yes <sup>[d]</sup>	Yes	Yes	No <sup>[0]</sup>	No <sup>[p]</sup>	Depends <sup>[q]</sup>	O(N)	Yes	O(N) <sup>[r]</sup>	No <sup>[s]</sup>	Yes	Yes	Scores <sup>[1]</sup>	Yes	Yes	
Minimax	Yes	No	No	Yes <sup>[u]</sup>	No	No	No	No <sup>[b]</sup>	Spoilers	Yes	No <sup>[b]</sup>	No <sup>[b]</sup>	No	O(N <sup>2</sup> )	Yes	O(N <sup>2</sup> )	No <sup>[b][u]</sup>	No	No <sup>[b]</sup>	Ranking	Yes	Yes	
Plurality/FPTP	Yes	No	No	No <sup>[b]</sup>	No	No <sup>[b]</sup>	No	No	Spoilers	Yes	Yes	Yes	No	O(N)	Yes	O(N)	N/A <sup>[v]</sup>	N/A <sup>[v]</sup>	No	Single mark	N/A	No	
Score voting	No	No	No	No <sup>[b][c]</sup>	No	No <sup>[b]</sup>	Yes	Yes <sup>[d]</sup>	Yes	Yes	Yes	Yes	Yes	O(N)	Yes	O(N)	No	Yes	Yes	Scores	Yes	Yes	
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No <sup>[b]</sup>	Yes	Yes	No <sup>[b]</sup>	No <sup>[p][b]</sup>	Yes	O(N <sup>3</sup> )	Yes	O(N <sup>2</sup> )	No <sup>[b]</sup>	No	No <sup>[p][b]</sup>	Ranking	Yes	Yes	
Runoff voting	Yes	Yes	No	No <sup>[b]</sup>	Yes	No <sup>[b]</sup>	No	No	Spoilers	No	No	No	No	O(N) <sup>[w]</sup>	Yes	O(N) <sup>[w]</sup>	Yes	Yes <sup>[x]</sup>	No	Single mark	N/A	No <sup>[y]</sup>	
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	No	No <sup>[b]</sup>	Yes	Yes	No <sup>[b]</sup>	No <sup>[p][b]</sup>	Yes	O(N <sup>3</sup> )	Yes	O(N <sup>2</sup> )	No <sup>[b]</sup>	No	No <sup>[p][b]</sup>	Ranking	Yes	Yes	
STAR voting	No <sup>[2]</sup>	Yes	No <sup>[aa]</sup>	No <sup>[b][c]</sup>	Yes	No <sup>[b]</sup>	No	No	No	Yes	No	No	Depends <sup>[ab]</sup>	O(N)	Yes	O(N <sup>2</sup> )	No	No	No <sup>[ac]</sup>	Scores	Yes	Yes	
Sortition, arbitrary winner <sup>[ad]</sup>	No	No	No	No <sup>[b]</sup>	No	No <sup>[b]</sup>	Yes	Yes	No	Yes	Yes	Yes	Yes	O(1)	No	O(1)	Yes	Yes	Yes	None	N/A	N/A	
Random ballot <sup>[ae]</sup>	No	No	No	No <sup>[b]</sup>	No	No <sup>[b]</sup>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	O(N)	No	O(N)	Yes	Yes	Yes	Single mark	N/A	No	

#### Voting with Ranked Ballots



### **Utilitarian Voting with Ranked Ballots**

[Procaccia, Rosenschein, 2006]



#### **Optimal Voting Rules with Ranked Ballots**



Minimize distortion (Worst-case approximation ratio for utilitarian social welfare)

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- Known results

#### • Metric distortion framework

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- Applications beyond voting

### Voting with Ranked Ballots

- *N* = set of *n* voters
- A = set of m alternatives
  - $\Delta(A)$  = set of distributions over A
- $\overrightarrow{\succ}$  = observed ranked preference profile
  - $\succ_i$  = preference ranking of voter *i*
  - $a \succ_i b$  means the voter ranks a higher than b
- (Randomized) Voting rule *f* 
  - Maps every preference profile  $\overrightarrow{\succ}$  to a distribution over alternatives  $f(\overrightarrow{\succ}) = x \in \Delta(A)$
  - We say that f is deterministic if  $f(\overrightarrow{\succ})$  has singleton support for every  $\overrightarrow{\succ}$

#### **Utilitarian Distortion**

- 1. There exists an underlying utility profile  $\vec{u}$  such that for each  $i \in N$ :
  - Consistency (denoted  $u_i \triangleright \succ_i$ ):  $\forall a, b : a \succ_i b \Rightarrow u_i(a) \ge u_i(b)$
  - Unit-sum:  $\sum_a u_i(a) = 1$ 
    - [Aziz 2019] provides seven justifications!
  - Risk-neutrality: For  $x \in \Delta(A)$ ,  $u_i(x) = \sum_a u_i(a) \cdot x(a)$
- 2. If we knew the utilities, we would want to maximize the (utilitarian) social welfare •  $sw(x, \vec{u}) = \sum_{i \in N} u_i(x)$
- 3. Because this is impossible given the limited ranked information, we want to best approximate the social welfare in the worst case.

#### **Utilitarian Distortion**

• Distortion

dist
$$(x, \overrightarrow{\succ}) = \sup_{\overrightarrow{u} \, \triangleright \, \overrightarrow{\succ}} \frac{\max_{a \in A} sw(a, \overrightarrow{u})}{sw(x, \overrightarrow{u})}$$

• Given voting rule *f* 

$$dist(f) = \max_{\overrightarrow{\succ}} \operatorname{dist}(f(\overrightarrow{\succ}),\overrightarrow{\succ})$$



What is the lowest possible dist(f)? Which voting rule achieves it?

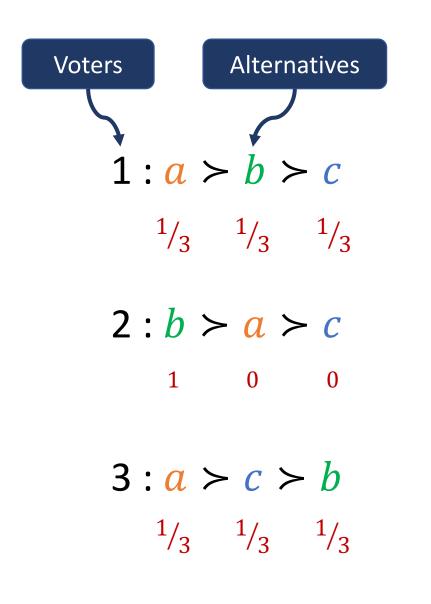
### **Utilitarian Distortion**

- Instance-optimal rules
  - Deterministic  $f_{det}^*$ : Maps every preference profile  $\overrightarrow{\succ}$  to  $a^* \in \arg \min_{a \in A} \operatorname{dist}(a, \overrightarrow{\succ})$
  - Randomized  $f_{rand}^*$ : Maps every preference profile  $\overrightarrow{\succ}$  to  $x^* \in \arg \min_{x \in \Delta(A)} \operatorname{dist}(x, \overrightarrow{\succ})$
  - Have the lowest distortion on each  $\overrightarrow{\succ}$ , and therefore in the worst case over all  $\overrightarrow{\succ}$



Are the instance-optimal rules polytime computable? Do they have a nice analytical structure?

### Example



- Suppose we choose *a*:
  - How much better can *b* be?

$$\max_{\vec{u} \succ \vec{\succ}} \frac{sw(b, \vec{u})}{sw(a, \vec{u})} = \frac{\frac{1}{3} + 1 + \frac{1}{3}}{\frac{1}{3} + 0 + \frac{1}{3}} = \frac{5}{2}$$

• How much better can *c* be?

$$\max_{\vec{u} \Join \vec{v}} \frac{sw(c, \vec{u})}{sw(a, \vec{u})} = \frac{\frac{1}{3} + 0 + \frac{1}{3}}{\frac{1}{3} + 0 + \frac{1}{3}} = 1$$

• Hence, 
$$dist(a, \overrightarrow{\succ}) = \frac{11}{5}$$

- Similarly, compute  $dist(b, \overrightarrow{\succ})$  and  $dist(c, \overrightarrow{\succ})$ 
  - *a* has the lowest distortion

### **Optimal Deterministic Distortion**

- Theorem [Caragiannis, Procaccia, 2011; Caragiannis, Nath, Procaccia, S, 2017]
  - For deterministic aggregation of ranked ballots, the optimal distortion is  $\Theta(m^2)$  and the instance-optimal rule  $f_{det}^*$  is polytime computable.
- Proof (lower bound):
  - High-level approach:
    - Take an arbitrary voting rule f
    - Construct a preference profile  $\overrightarrow{\succ}$
    - Let f choose a winner a on  $\overrightarrow{\succ}$
    - Reveal a bad utility profile  $\vec{u}$  consistent with  $\overrightarrow{\succ}$  in which a is  $\Omega(m^2)$  factor worse than the optimal alternative

#### **Deterministic Rules**

- Proof (lower bound):
  - Let *f* be any deterministic voting rule
  - Consider  $\overrightarrow{\succ}$  on the right
  - Case 1:  $f(\overrightarrow{\succ}) = a_m$ 
    - Infinite distortion. Why?
  - Case 2:  $f(\overrightarrow{\succ}) = a_i$  for some i < m
    - Bad utility profile  $\vec{u}$  consistent with  $\overrightarrow{\succ}$ 
      - Voters in column i have utility 1/m for every alternative
      - All other voters have utility 1/2 for their top two alternatives

• 
$$\operatorname{sw}(a_i, \vec{u}) = \frac{n}{m-1} \cdot \frac{1}{m}$$
,  $\operatorname{sw}(a_m, \vec{u}) \ge \frac{n-n/(m-1)}{2} = \Omega(n)$ 

• Distortion =  $\Omega(m^2)$ 

n/(m-1) voters per column								
$a_1$	$a_2$		$a_{m-1}$					
$a_m$	$a_m$		$a_m$					
•	•	:	•					

#### **Deterministic Rules**

- Proof (upper bound):
  - Plurality rule: Select an alternative *a* that is the top choice of the most voters
  - For this plurality winner:
    - At least n/m voters have a as their top choice (pigeonhole principle)
    - Every voter has utility at least 1/m for their top choice (pigeonhole principle)
  - Hence, for every consistent utility profile  $\vec{u}$ :
    - $sw(a, \vec{u}) \ge n/m^2$
    - $sw(a^*, \vec{u}) \leq n$  for every alternative  $a^*$
  - $dist(a, \overrightarrow{\succ}) = O(m^2)$

### **Optimal Randomized Distortion**

- Theorem [Boutilier, Caragiannis, Haber, Lu, Procaccia, and Sheffet, 2015]
  - For randomized aggregation of ranked ballots, the optimal distortion is  $O(\sqrt{m} \cdot \log^* m)$  but  $\Omega(\sqrt{m})$ , and the instance-optimal rule  $f_{rand}^*$  is polytime computable.
- Proof (lower bound):
  - Same high-level approach:
    - Take an arbitrary randomized voting rule f
    - Construct a preference profile  $\overrightarrow{\succ}$
    - Let *f* choose a distribution *x* over alternatives
    - Reveal a bad utility profile  $\vec{u}$  consistent with  $\overrightarrow{\succ}$  in which the expected social welfare under x is  $\Omega(\sqrt{m})$  factor worse than the optimal social welfare

### **Randomized Rules**

- Proof (lower bound):
  - Let f be an arbitrary rule
  - Consider  $\overrightarrow{\succ}$  on the right with  $\sqrt{m}$  special alternatives
  - f returns distribution x in which at least one special alternative (say  $a_i$ ) must be chosen w.p. at most  $\frac{1}{\sqrt{m}}$
  - Bad utility profile  $\vec{u}$  consistent with  $\overrightarrow{\succ}$ :
    - All voters ranking  $a_i$  first have utility 1 for  $a_i$
    - All other voters have utility 1/m for every alternative
    - $sw(a_i, \vec{u}) = \Theta\left(\frac{n}{\sqrt{m}}\right)$  but  $sw(a, \vec{u}) \le \frac{n}{m}$  for every other alternative a
    - $sw(x, \vec{u}) \leq \left(\frac{1}{\sqrt{m}}\right) \cdot \Theta\left(\frac{n}{\sqrt{m}}\right) + \left(1 \frac{1}{\sqrt{m}}\right) \cdot \left(\frac{n}{m}\right) = O(\frac{n}{m})$
    - Hence,  $dist(x, \vec{u}) = \Omega(\sqrt{m})$

$n/\sqrt{m}$ voters per column									
$a_1$	<i>a</i> <sub>2</sub>		$a_{\sqrt{m}}$						
:	:	:	:						

### **Optimal Randomized Distortion**

#### • Harmonic Rule

• The rule that achieves  $O(\sqrt{m} \cdot \log^* m)$  distortion is complicated, but they propose a simpler harmonic rule that achieves  $O(\sqrt{m} \cdot \log m)$  distortion

#### **Harmonic Rule**

- Each voter *i* awards 1/r points to her  $r^{th}$  ranked alternative for every  $r \in \{1, ..., m\}$
- Harmonic score of alternative a, denoted  $hsc(a, \overrightarrow{\succ})$ , is the total point awarded to a
- W.p.  $\frac{1}{2}$ , choose each  $a \in A$  with probability proportional to  $hsc(a, \overrightarrow{\succ})$
- W.p.  $\frac{1}{2}$ , choose each  $a \in A$  uniformly at random
  - Key proof idea:
    - $hsc(a, \overrightarrow{\succ}) \ge sw(a, \overrightarrow{u})$  for every a, while  $\sum_a hsc(a, \overrightarrow{\succ}) = O(\log m) \cdot \sum_a sw(a, \overrightarrow{u})$

### **Optimal Randomized Distortion**

- Theorem [Ebadian, Kahng, Peters, S, 2022]
  - For randomized aggregation of ranked ballots, the optimal distortion is  $\Theta(\sqrt{m})$ .
- Proof via three steps:
  - I. Define "stable lotteries"
  - II. Prove the existence (and efficient computation) of stable lotteries via the minimax theorem
  - III. Derive  $O(\sqrt{m})$  distortion using stable lotteries

#### **Step I: Define Stable Lotteries**

• For a set of alternatives  $S = \{$ , ,, ,,  $\}$  and an alternative a =

 $V(a,S) = |\{i \in N : a \succ_i b, \forall b \in S\}| = 2$ 

• Lottery S over sets of size k is stable if  $\mathbb{E}_{S \sim S}[V(a, S)] \leq n/k$  for every  $a \in A$ 

#### **Step II: Prove Stable Lotteries Exist**

- Theorem: For every k, a stable lottery over committees of size k exists.
- Proof (skip):

• 
$$\min_{\mathcal{S}} \max_{a \in A} \mathbb{E}_{S \sim \mathcal{S}}[V(a, S)] \leq \min_{\mathcal{S}} \max_{x \in \Delta(A)} \mathbb{E}_{S \sim \mathcal{S}, a \sim x}[V(a, S)]$$
$$= \max_{x \in \Delta(A)} \min_{\mathcal{S}} \mathbb{E}_{S \sim \mathcal{S}, a \sim x}[V(a, S)] \leq \frac{n}{k}$$

- For any  $x \in \Delta(A)$ , consider the lottery  $S^*$ , where we sample k alternatives i.i.d. according to x and replace any duplicates with arbitrary other alternatives
- For each voter *i*:

$$\Pr_{S \sim \mathcal{S}^*, a \sim x}[a \succ_i b, \forall b \in S] \le \frac{1}{k+1}$$

• Hence:

$$\mathbb{E}_{S \sim S^*, a \sim x}[V(a, S)] \le \frac{n}{k+1} < \frac{n}{k} \quad \blacksquare$$

## Step III: Proof of $O(\sqrt{m})$ Distortion

#### **Stable Lottery Rule**

- W.p. ½, find a stable lottery S over sets of size  $\sqrt{m}$ , sample  $S \sim S$ , choose  $a \in S$  uniformly at random
- W.p.  $\frac{1}{2}$ , choose  $a \in A$  uniformly at random
- Theorem: Stable lottery rule achieves  $O(\sqrt{m})$  distortion.
  - Let  $a^*$  be an alternative maximizing social welfare
  - For any  $S: sw(a^*, \vec{u}) \le V(a^*, S) + \sum_{b \in S} sw(b, \vec{u})$
  - Taking expectation over  $S \sim S$ :

$$\begin{split} sw(a^*, \vec{u}) &\leq \mathbb{E}_{S \sim \mathcal{S}} [V(a^*, S)] + \mathbb{E}_{S \sim \mathcal{S}} [\sum_{b \in S} sw(b, \vec{u})] \\ &\leq 2\sqrt{m} \cdot \left(\frac{1}{2} \cdot \frac{n}{m} + \frac{1}{2} \cdot \mathbb{E}_{S \sim \mathcal{S}} \left[\frac{1}{|S|} \cdot \sum_{b \in S} sw(b, \vec{u})\right]\right) \\ &= 2\sqrt{m} \cdot sw(f(\overrightarrow{\succ}), \vec{u}) \blacksquare \end{split}$$

#### Notes

#### • Stable lotteries

- Introduced by [Cheng, Jiang, Munagala, Wang, 2020], who show the existence of a stronger form of stable lotteries which bounds V(S', S) for all  $S' \subseteq A$
- Requires a much more intricate proof

#### • Stable committees

- 16-stable committees exist [Jiang, Munagala, Wang, 2020]:  $V(a, S) \le 16 \cdot \frac{n}{k}$  for all  $a \in A$
- Factor 16 cannot be improved to any lower than 2
- Open question: Do 2-approximately stable committees exist?
- Lower bound
  - The lower bound from before is  $\frac{\sqrt{m}}{2}$
  - Open question: A gap of factor 4 between this lower bound and the  $2\sqrt{m}$  upper bound by stable lottery rule

#### Extensions

- Other utility classes and objective functions
- Incentives
- Ballot formats other than ranked ballots
- Committee selection
- Optimal ballot design
- Participatory budgeting
- Social welfare functions

### **Other Objective Functions**

- Nash social welfare
  - $sw(x, \vec{u}) = \sum_{i \in N} u_i(x)$
  - $nsw(x, \vec{u}) = (\prod_{i \in N} u_i(x))^{1/n}$
  - Nash social welfare is independent of individual scales
    - Any distortion upper bound with respect to unit-sum utilities holds for arbitrary utilities
- Theorem [Ebadian, Kahng, Peters, S, 2022]:
  - With respect to the Nash social welfare:
    - The distortion of harmonic rule is  $\Theta(\sqrt{m \cdot \log m})$
    - The distortion of stable lottery rule is  $\Theta(\sqrt{m})$
    - There is a randomized rule with distortion  $O(\log m)$
    - No randomized rule has distortion better than  $\left(\frac{m^m}{m!}\right)^{1/m} \rightarrow e$
- Open question: Close the gap between  $O(\log m)$  and e

#### **Other Objective Functions**

- Additive distortion
  - $sw(x, \vec{u}) = (1/n) \cdot \sum_{i \in N} u_i(x)$
  - $dist^+(x, \overrightarrow{\succ}) = \max_{\overrightarrow{u} \, \triangleright \, \overrightarrow{\succ}} \left[ \max_{a \in A} sw(a, \overrightarrow{u}) sw(x, \overrightarrow{u}) \right]$
- Theorem [Caragiannis, Nath, Procaccia, S, 2017]:
  - For deterministic rules, the optimal additive distortion is 1/2.
  - For randomized rules, the optimal additive distortion is between 1/4 and  $1/2 \cdot (1 1/m^2)$ .
- Theorem [Kahng, Kehne, 2022]:
  - For randomized rules, the optimal additive distortion is between  $\frac{5}{18}$  and  $\frac{11}{27}$ .
- Open question: Close the gap for randomized rules

### **Other Objective Functions**

- If we knew the utility profile  $\vec{u}$ :
  - Efficiency would ask us to select  $x^* \in \arg \max_x sw(x, \vec{u})$
  - What about fairness?
- Proportional Fairness:  $PF(x, \vec{u}) = \sup_{y} \sum_{i} \frac{u_i(y)}{u_i(x)}$ 
  - Maximum total % change in utilities when moving to any other distribution y
  - Folklore: If we knew  $\vec{u}$ , choosing  $x^* \in \arg \max_x \prod_i u_i(x)$  would guarantee  $PF(x^*, \vec{u}) = 1$
  - Folklore:  $\alpha$ -PF implies  $\alpha$ -approximation to the core
    - Any subgroup of x % of voters cannot find an α factor Pareto improvement over x by allocating x % of the probability mass (or budget), for any x
- Theorem [Ebadian, Kahng, Peters, S, 2022]:
  - With unit-sum utilities, the optimal randomized rule achieves  $\Theta(\log m)$  proportional fairness.
- Open question: Can the core approximation be improved to 2?

### **Other Utility Classes**

- Unit range utilities:
  - $u_i(a) \in [0,1]$  for all  $a \in A$ ,  $\max_a u_i(a) = 1$ ,  $\min_a u_i(a) = 0$
- Theorem [Ebadian, Kahng, Peters, S, 2022]:
  - With respect to unit range utilities:
    - The distortion of harmonic rule increases to  $O(m^{2/3} \cdot \log^{1/3} m)$
    - The distortion of stable lottery rule remains  $O(\sqrt{m})$
    - Every randomized rule has distortion  $\Omega(\sqrt{m})$

#### Incentives

#### Strategyproofness

- A randomized rule is strategyproof if a voter cannot increase her expected utility by misreporting her preference ranking in any instance.
- Theorem [Bhaskar, Dani, Ghosh, 2018]:
  - With respect to unit-sum utilities, the best distortion subject to strategyproofness is  $\Theta(\sqrt{m \cdot \log m})$ .
  - Upper bound is achieved by harmonic rule, which turns out to be strategyproof.
- Theorem [Filos-Ratsikas, Bro Miltersen, 2014; Lee 2019]:
  - With respect to unit-range utilities, the best distortion subject to strategyproofness is  $\Theta(m^{2/3})$ .
  - Note: This explains why the distortion of harmonic rule, which is strategyproof, increases to  $\tilde{O}(m^{2/3})$  for unit-range utilities
    - Harmonic rule achieves near-optimal distortion subject to strategyproofness with respect to both unit-sum and unit-range utilities!

#### **Committee Selection**

- Goal: Select a set of alternatives of given size k
  - Representation utilities:  $u_i(S) = \max_{a \in S} u_i(a)$
  - Apriori, it is not clear if the best possible distortion increases or decreases with k
- Theorem [Caragiannis, Nath, Procaccia, S, 2017]
  - The optimal distortion of deterministic rules is  $\Theta\left(1 + \frac{m \cdot (m-k)}{k}\right)$ .
  - Optimal distortion of randomized rules:

2. Distortion, randomized rules: There exists a randomized voting rule  $f^*$  such that

$$\texttt{dist}(f^*) \leqslant \begin{cases} 2\sqrt{m \cdot H_m} & \text{ if } k \leqslant \frac{2 \cdot m \cdot H_m}{m + H_m}, \\ 4\sqrt{m \cdot k} & \text{ if } \frac{2 \cdot m \cdot H_m}{m + H_m} < k \leqslant \left(\frac{m}{4}\right)^{\frac{1}{3}} \\ \frac{m}{k} & \text{ otherwise}, \end{cases}$$

where  $H_m = \Theta(\log m)$  is the  $m^{th}$  harmonic number. Moreover, for every randomized voting rule f,

$$\label{eq:dist} {\rm dist}(f) \geqslant \begin{cases} \frac{\sqrt{m}}{2} & \mbox{if } k \leqslant \frac{m \cdot (\sqrt{m} - 1)}{m - 1} \approx \sqrt{m}, \\ \\ \frac{m}{k + m/k} & \mbox{otherwise.} \end{cases}$$
 These bounds are tight up to a factor of  $6.35 \cdot m^{1/6}$ .

#### **Committee Selection**

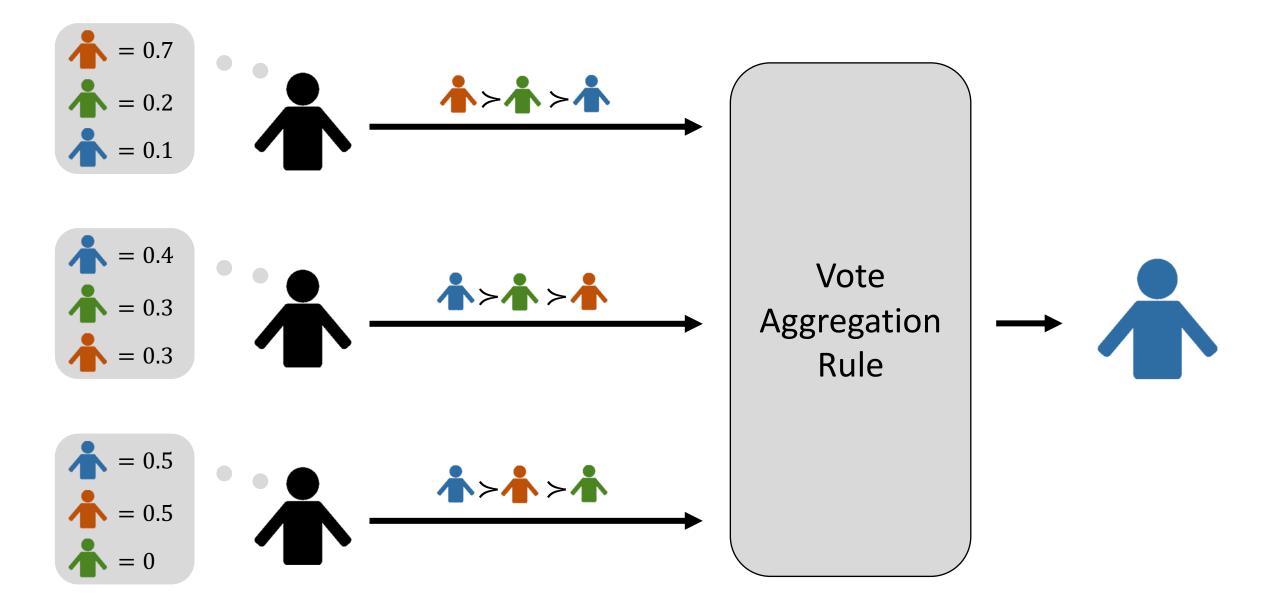
#### **Stable Lottery Rule for Committees**

- If  $k \leq \sqrt{m}$ :
  - W.p. ½, find a stable lottery S over sets of size k · √m, sample S ~ S, and choose S' ⊆ S of size |S'| = k uniformly at random
  - W.p.  $\frac{1}{2}$ , choose  $S \subseteq A$  of size |S| = k uniformly at random
- If  $k \ge \sqrt{m}$ 
  - Choose  $S \subseteq A$  of size |S| = k uniformly at random
- Theorem [Borodin, Halpern, Latifian, S, '22]:
  - Stable lottery rule for committees of size k achieves the optimal distortion of  $\Theta\left(\min\left(\sqrt{m}, \frac{m}{k}\right)\right)$
- Corollary:
  - The best possible distortion (asymptotically) weakly decreases with  $\boldsymbol{k}$

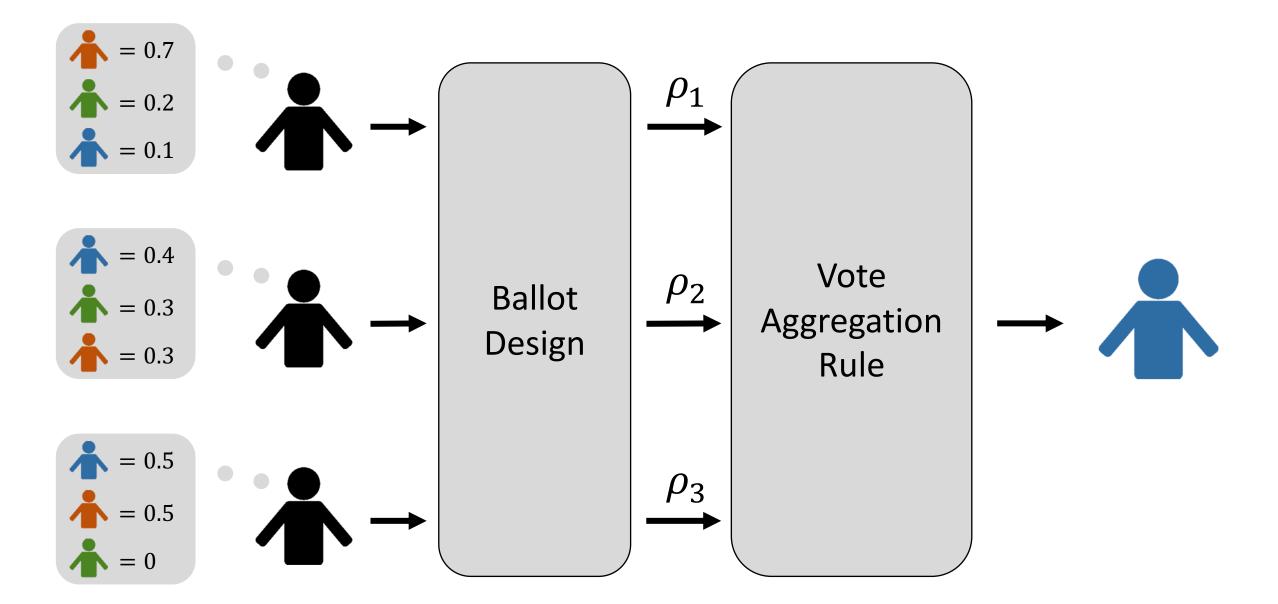
- Top-t preferences (less information than ranked ballots)
  - Each voter ranks her *t* most favorite alternatives
- Theorem [Borodin, Halpern, Latifian, S, '22]:
  - Stable lottery rule for committees has distortion  $O\left(\min\left(\max\left(\sqrt{m}, \frac{m}{t}\right), \frac{m}{k}\right)\right)$ 
    - Apply the rule after arbitrarily completing partial preferences to ranked ballots!
  - Every randomized voting rule has distortion  $\Omega\left(\min\left(\max\left(\sqrt{m}, \frac{m}{k \cdot t}\right), \frac{m}{k}\right)\right)$
  - Open question: Close this gap!
- Corollary:
  - For k = 1 (single-winner), the bound is  $\Theta\left(\max\left(\sqrt{m}, \frac{m}{t}\right)\right)$
  - Optimal  $O(\sqrt{m})$  distortion is already achieved at  $t = \sqrt{m}$ 
    - No benefit from asking voters to rank more than their top  $\sqrt{m}$  alternatives!

- Ranked ballots + additional queries (more information than ranked ballots)
  - Value query: What is  $u_i(a)$ ?
  - Comparison query: Is  $u_i(a) \ge \alpha \cdot u_i(b)$ ?
  - We measure the number of queries *per voter*
- Theorem [Amanatidis, Birmpas, Filos-Ratsikas, Voudouris, 2021]:
  - For any k, it is possible to achieve distortion  $O({}^{k+1}\sqrt{m})$  with  $O(k \cdot \log m)$  value queries
  - It is possible to achieve O(1) distortion using  $O(\log^2 m)$  comparison queries
  - The best distortion with  $\lambda$  value queries is  $\Omega\left(\frac{1}{\lambda+1} \cdot m^{\frac{1}{2(\lambda+1)}}\right)$
  - ...
- Many open questions:
  - E.g., O(1) distortion with  $O(\log m)$  value queries?

#### **Utilitarian Voting with Ranked Ballots**



#### **Utilitarian Voting with Generic Ballots**



#### Ballots

Ranked Ballot	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
А	$\bigcirc$		$\bigcirc$	$\bigcirc$
В		$\bigcirc$	$\bigcirc$	$\bigcirc$
С	$\bigcirc$	$\bigcirc$	$\bigcirc$	
D	$\bigcirc$	$\bigcirc$		$\bigcirc$

Top- <i>t</i> Ballot	<b>1</b> <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>
А	$\bigcirc$		$\bigcirc$	$\bigcirc$
В		$\bigcirc$	$\bigcirc$	$\bigcirc$
С	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$
D	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\bigcirc$

Range Voting	1 (Worst)	2	3	4 (Best)
А	$\bigcirc$	$\bigcirc$		$\bigcirc$
В	$\bigcirc$	$\bigcirc$		$\bigcirc$
С		$\bigcirc$	$\bigcirc$	$\bigcirc$
D	$\bigcirc$		$\bigcirc$	$\bigcirc$

Approval Ballot	1 <sup>st</sup>
А	
В	
С	$\bigcirc$
D	$\bigcirc$

## **Optimal Voting with Optimal Ballot Design**

• Tradeoff





• "Expressiveness" / "cognitive difficulty" imposed

• Crude measure: #bits communicated by each voter



How many bits of information does each voter need to communicate for us to achieve distortion d?

## **Optimal Voting with Optimal Ballot Design**

- Theorem [Mandal, Procaccia, S, Woodruff, 2019; Mandal, S, Woodruff, 2020]
  - For any *d*, the optimal ballot (combined with its optimal randomized aggregation rule) elicits the following number of bits of information from each voter to achieve distortion *d*:
    - Deterministic ballot:  $\widetilde{\Theta}(m/_{kd})$
    - Randomized ballot:  $\widetilde{\Theta}(m/_{kd^3})$
- Comparison to ranked ballots
  - Ranked ballots achieve  $d = \Theta(\min(\sqrt{m}, \frac{m}{k}))$  distortion by eliciting  $\Theta(m \cdot \log m)$  bits
  - Optimal ballot achieves d = O(1) distortion already by eliciting only  $\tilde{O}(m/k)$  bits

## **Participatory Budgeting**

[Benade, Procaccia, Nath, S, 2021]

• Ranking by value



• Ranking by VFM



 Knapsack voting (budget = 4)



 Threshold approval (threshold = 3)







## **Participatory Budgeting**

- Additive utilities
  - $u_i(S) = \sum_{a \in S} u_i(a)$
  - Previously mentioned results were for representation utilities:  $u_i(S) = \max_{a \in S} u_i(a)$
- Theorem [Benade, Nath, Procaccia, S, 2017]:
  - The best possible distortion using randomized aggregation rule is as follows:
    - Knapsack ballot:  $\Theta(m)$
    - Ranking by value:  $\widetilde{\Theta}(\sqrt{m})$
    - Ranking by VFM:  $\widetilde{\Theta}(\sqrt{m})$
    - Threshold approval votes:  $O(\log^2 m)$ ,  $\Omega\left(\frac{\log m}{\log \log m}\right)$

#### **Social Welfare Functions**

- Output: a ranking of the alternatives  $\succ^*$ 
  - How do we define the utility of a voter for a ranking?
  - Each voter *i* has weights  $w_{i,j}$  such that  $w_{i,j} \ge 0$  for all *j* and  $\sum_{j=1}^{m} w_{i,j} = 1$ 
    - $w_{i,j}$  = how much voter *i* cares about which alternative gets ranked  $j^{\text{th}}$  in  $>^*$
    - $u_i(\succ^*) = \sum_{j=1}^m w_{i,j} \cdot u_i(a_j)$ , where  $a_j$  is the  $j^{\text{th}}$  ranked alternative in  $\succ^*$
  - Distortion  $\rightarrow$  worst case over the choice of both voter utilities *and* voter weights
    - Strictly harder than single-winner selection ( $w_{i,1} = 1$ )
- Theorem [Benade, Procaccia, Qiao, 2019]:
  - The best distortion of any randomized social welfare function is  $O(\sqrt{m \cdot \log^3 m})$ .
  - Only polylogarithmically higher than single-winner selection!

## Many, Many Open Questions

- Combining extensions
  - Strategyproofness +
    - Nash welfare distortion, additive distortion, other ballots, committee selection, participatory budgeting, ...
  - Committee selection +
    - Nash welfare distortion, additive distortion, ...
  - Unit-range utilities +

• ...

- Additive distortion, other ballots, committee selection, participatory budgeting, ...
- Social welfare functions?

## Outline

#### • Introduction

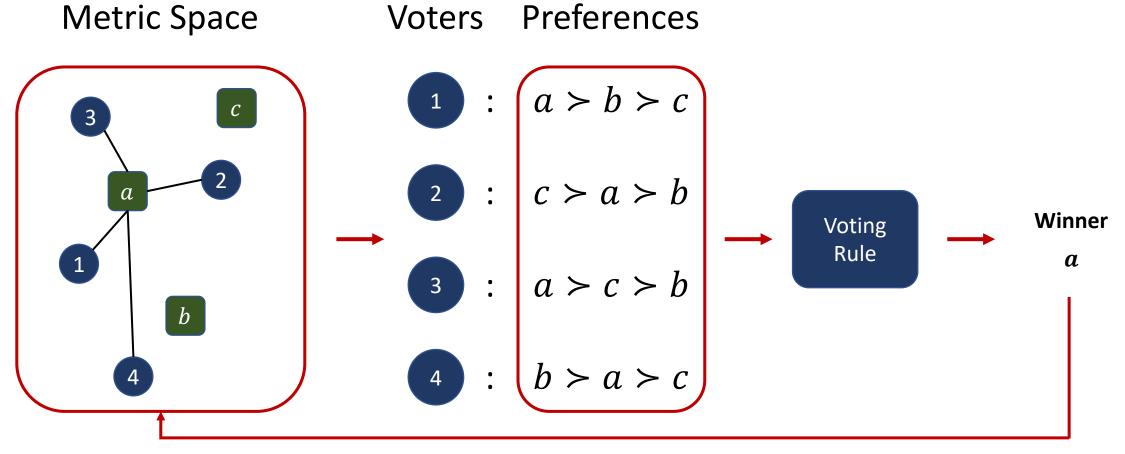
- Applications of voting
- Motivating the distortion framework

#### • Utilitarian distortion framework

- Model
- Known results
- Metric distortion framework
  - Model
  - Known results
- Applications beyond voting

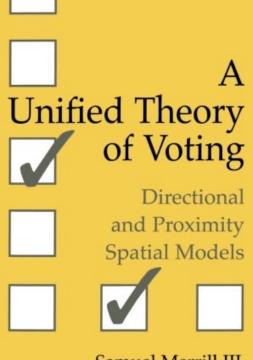
**Metric Distortion** 

[Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]



Assess quality using the underlying metric

## Why The Metric?



Samuel Merrill III & Bernard Grofman ADVANCES IN THE Spatial Theory of Voting

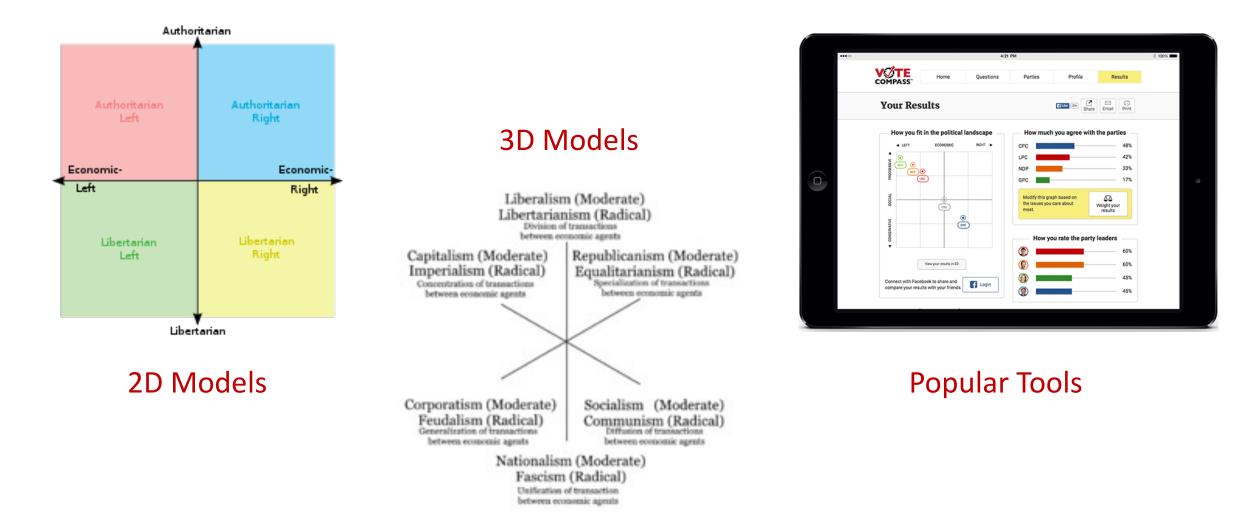
EDITED BY James M. Enelow AND Melvin J. Hinich Ideology and Spatial Voting in AMERICAN ELECTIONS

CAMBRIDGE

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Stephen A. Jessee

## Why The Metric?



#### **Metric Distortion**

- 1. There exists an underlying metric d over voters and alternatives such that:
  - Consistency (denoted  $d \triangleright \overrightarrow{\succ}$ ) :  $\forall a, b : a \succ_i b \Rightarrow d(i, a) \le d(i, b)$
  - Triangle inequality:  $\forall x, y, z, d(x, y) + d(y, z) \ge d(x, z)$
  - Risk-neutrality: For  $x \in \Delta(A)$ ,  $c_i(x) = \sum_a d(i, a) \cdot x(a)$
- 2. If we knew the costs, we would minimize the social cost
  - $sc(x,d) = \sum_{i \in N} d(i,x)$
- 3. Because this is impossible given the limited ranked information, we want to best approximate the social cost in the worst case.

#### **Metric Distortion**

• Distortion

dist
$$(x, \overrightarrow{\succ}) = \sup_{d \rhd \overrightarrow{\succ}} \frac{sc(x, d)}{\min_{a \in A} sc(a, d)}$$

• Given voting rule *f* 

$$dist(f) = \max_{\overrightarrow{\succ}} \operatorname{dist}(f(\overrightarrow{\succ}),\overrightarrow{\succ})$$

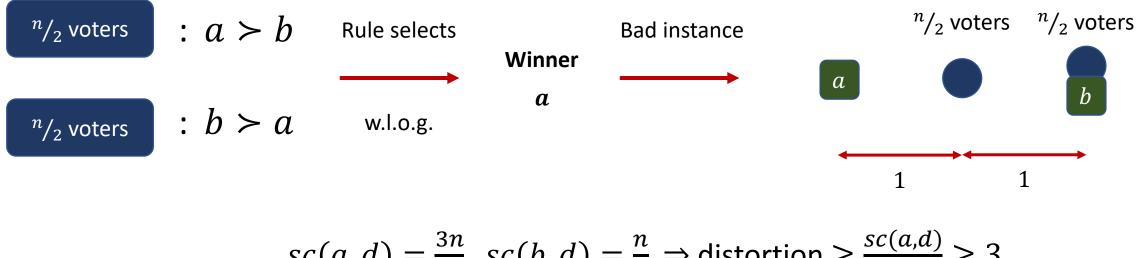


What is the lowest possible distortion of deterministic and randomized rules? Which voting rules achieves it?

#### Lower Bound

[Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]

• A simple lower bound of 3 (deterministic rules) with just two candidates



$$sc(a,d) = \frac{3n}{2}$$
,  $sc(b,d) = \frac{n}{2} \Rightarrow \text{distortion} \ge \frac{sc(a,a)}{sc(b,d)} \ge 3$ 

Can a deterministic rule achieve distortion 3?

## **Deterministic Rules**

• Theorem [Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]:

Rule	Distortion
k-approval ( $k > 2$ )	Unbounded
Plurality, Borda count	$\Theta(m)$
Harmonic rule*	$O\left(\frac{m}{\sqrt{\log m}}\right)$ , $\Omega\left(\frac{m}{\log m}\right)$
Best positional scoring rule	$\Omega(\sqrt{\log m})$
STV	$O(\log m), \ \Omega(\sqrt{\log m})$
Copeland's rule	5
Best deterministic rule	≥ 3

\*Deterministic version of the harmonic rule,

which simply picks an alternative with the largest harmonic score

- The instance-optimal deterministic rule can be computed in polynomial time by solving a number of linear programs.
- Open question: What is the best distortion achievable by any positional scoring rule?

## **Copeland's Rule**

#### • Lemma [Kempe 2020b]:

• If  $(a_1, a_2, ..., a_\ell)$  is a sequence of alternatives such that a (weak) majority of voters prefer  $a_i$  to  $a_{i+1}$  for each  $i = 1, ..., \ell - 1$ , then  $sc(a_1, d) \leq (2\ell - 1) \cdot sc(a_\ell, d)$  for every metric d consistent with the preference profile.

#### • Corollary:

- It is known that Copeland's winner is in the uncovered set:
  - If a<sub>1</sub> is Copeland's winner, then for every other alternative a, either sequence (a<sub>1</sub>, a) or (a<sub>1</sub>, a<sub>2</sub>, a) for some a<sub>2</sub> satisfies the condition above.
- This explains distortion 5 of Copeland's rule
- Lemma quite powerful, later used by [Anagnostides, Fotakis, Patsilinakos, 2021]

#### • Copeland's rule is Condorcet consistent

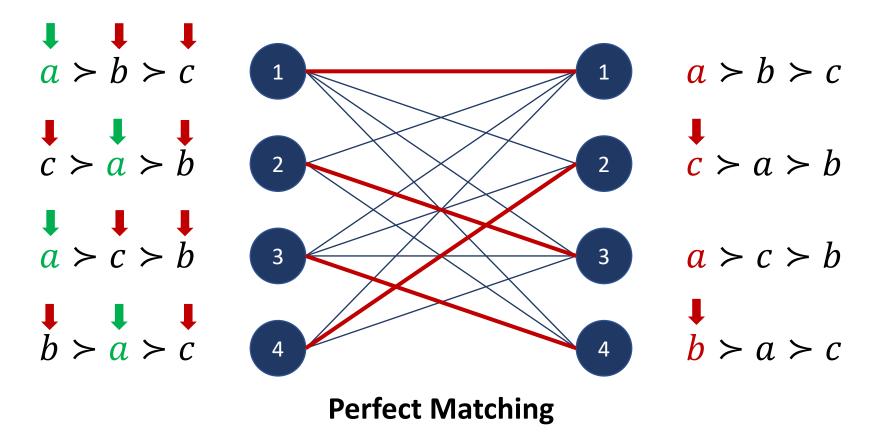
 [Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]: Any voting rule can be made Condorcet consistent without losing distortion because the Condorcet winner is always a 3approximation

#### **Deterministic Rules**

- Theorem [Kempe 2020a]:
  - The distortion of ranked pairs and Schulze's rule is  $\Theta(\sqrt{m})$ .
  - Analysis via a powerful LP duality approach
- Theorem [Munagala, Wang, 2019]:
  - There exists a deterministic voting rule with distortion  $2 + \sqrt{5} \approx 4.236$ .
- Theorem [Gkatzelis, Halpern, S, 2020]:
  - There exists a deterministic voting rule, PluralityMatching, with distortion 3.
  - Proof by proving a (stronger version of a) conjecture by [Munagala, Wang, 2019]

#### **Domination Graph of Candidate** *a*

Edge (i, j) exists when, in *i*'s vote, *a* weakly defeats the top choice of *j* 



## PluralityMatching

- Lemma [Gkatzelis, Halpern, S, 2020]:
  - There always exists an alternative whose domination graph admits a perfect matching, and PluralityMatching outputs any such alternative.
- Proof of distortion 3 (skip):

$$\begin{split} \operatorname{SC}(a) &= \sum_{i \in V} d(i, a) \\ &\leq \sum_{i \in V} d(i, \operatorname{top}(M(i))) & (\because a \succcurlyeq_i \operatorname{top}(M(i)), \forall i \in V) \\ &\leq \sum_{i \in V} \left( d(i, b) + d(b, \operatorname{top}(M(i))) \right) & (\because \operatorname{triangle inequality}) \\ &= \sum_{i \in V} \left( d(i, b) + d(b, \operatorname{top}(i)) \right) & (\because M \text{ is a perfect matching}) \\ &\leq \sum_{i \in V} \left( d(i, b) + d(b, i) + d(i, \operatorname{top}(i)) \right) & (\because \operatorname{triangle inequality}) \\ &\leq \sum_{i \in V} \left( d(i, b) + d(b, i) + d(i, b) \right) \\ &= 3 \cdot \operatorname{SC}(b). \end{split}$$

## **Randomized Rules**

- Theorem [Anshelevich, Bhardwaj, Elkind, Postl, Skowron, 2018]:
  - No randomized rule has distortion better than 2.
  - RandomDictatorship has distortion  $3 \frac{2}{n}$ .
- Theorem [Kempe 2020a]:
  - There is a randomized voting rule with access to only plurality votes with distortion 3 2/m.
- Theorem [Charikar, Ramakrishnan, 2022; Pulyassary, Swamy, 2021]:
  - No randomized rule has distortion better than 2.1126 for all *m*.
    - Weaker lower bounds for fixed, finite *m*
- Open question: What is the optimal metric distortion of randomized rules?
- Open question: Is the instance-optimal randomized rule polytime computable?

#### Extensions

- Other objective functions
- Ballot formats other than ranked ballots
- Committee selection
- Information-distortion tradeoff

#### **Other Objective Functions**

- Bounding higher moments of distortion [Fain, Goel, Munagala, Sakshuwong, 2017; Fain, Goel, Munagala, Prabhu, 2019; Fain, Fan, Munagala, 2020]
  - k<sup>th</sup> moment

$$dist^{k}(x, \overrightarrow{\succ}) = \sup_{d \vDash \overrightarrow{\succ}} \frac{\left(\mathbb{E}_{a \sim x} sc(a, d)^{k}\right)^{1/k}}{\min_{a^{*} \in A} sc(a^{*}, d)}$$

#### • Motivation:

- Bounding, e.g., the 2<sup>nd</sup> moment ("squared distortion") bounds not only the expectation of the social cost approximation ratio, but also its variance
- Filters out rules like RandomDictatorship that achieve terrible social cost with low probability
  - Unbounded squared distortion [Fain, Goel, Munagala, Sakshuwong, 2017]
- By Markov's inequality, one can obtain high-probability bounds on social cost approximation
- By Jensen's inequality, any upper bound on  $dist^k$  is also an upper bound on dist
- Open question: What is the optimal k<sup>th</sup> moment distortion of randomized rules?

- Top-*t* ballots
  - Each voter ranks her *t* most favorite alternatives
  - $t = 1 \Rightarrow$  Plurality is optimal with distortion 2m 1
  - $t = m 1 \Rightarrow$  PluralityMatching is optimal with distortion 3
- Theorem [Kempe 2020a, Kempe 2020b]:
  - The distortion of the optimal deterministic rule for top-t ballots is between  $\frac{2m}{t} 1$  and  $\frac{12m}{t}$ .
- Theorem [Anagnostides, Fotakis, Patsilinakos, 2021]:
  - The upper bound can be improved to  $\frac{6m}{r}$ .
- Open question: Close the gaps!

- Top-*t* ballots
  - Each voter ranks her *t* most favorite alternatives
  - $t = 1 \Rightarrow$  Plurality is optimal with distortion 2m 1
  - $t = m 1 \Rightarrow$  PluralityMatching is optimal with distortion 3
- Theorem [Gross, Anshelevich, Xia, 2017]:
  - The distortion of the optimal randomized rule for top-t ballots is at least  $3 2/\lfloor m/t \rfloor$  when  $t \le m/2$  and at least 2 when  $t \ge m/2$ .
- Open question: Design randomized rules with matching upper bounds!

- More information than ranked ballots
  - $\alpha$ -decisive metric spaces (where  $\alpha \in [0,1]$ ) [Anshelevich, Postl, 2016]:
    - Each voter's distance to her top choice is at most  $\alpha$  times her distance to her 2<sup>nd</sup> choice
    - $\alpha = 1$  provides no additional information
    - $\alpha = 0$  means every voter is co-located with her top choice
- Theorem [Gkatzelis, Halpern, S, 2020]:
  - Deterministic: No rule has distortion better than  $\sim 2 + \alpha \frac{2}{m}$  while PluralityMatching has distortion  $2 + \alpha$ .
  - Randomized: No rule has distortion better than  $\sim {}^{(3+\alpha)}/_2 {}^{(1-\alpha)}/_m$  while there exists a randomized rule (using only plurality votes) with distortion  $2 + \alpha {}^2/_m$ .

#### • Other types of extra information

- "Voter passion" [Abramowitz, Anshelevich, Zhu, 2019]
- Locations of alternatives known [Chen, Li, Wang, 2020; Anshelevich, Zhu, 2021]

#### **Committee Selection**

- Voter costs for committees:
  - Additive costs:  $c_i(S) = \sum_{a \in S} d(i, a)$
  - q-costs:  $c_i(S) = q^{\text{th}} \min_{a \in S} d(i, a)$
- Theorem [Goel, Hulett, Krishnaswamy, 2018]:
  - Under additive costs, applying a single-winner rule with distortion d recursively to choose a committee of size k achieves distortion at most d.
- Theorem [Caragiannis, S, Voudouris, 2022]:
  - Under *q*-costs, the optimal distortion of deterministic rules follows a trichotomy:
    - $q \in [1, k/3]$  :  $\infty$
    - $q \in \binom{k}{3}, \frac{k}{2} : \Theta(n)$
    - $q \in ({^k/_2}, k]$  :3
    - Open question: For  $q > k/_2$ , what distortion can be achieved in polynomial time?
      - Current best is 9

## Many, Many Open Questions

- Extensions for metric distortion less-studied than for utilitarian distortion
  - Participatory budgeting?
  - Strategyproofness?
  - Ranked ballots + additional queries?
  - Information-distortion tradeoff? [Kempe 2020a]
  - ...

## Outline

#### • Introduction

- Applications of voting
- Motivating the distortion framework

#### • Utilitarian distortion framework

- Model
- Known results

#### • Metric distortion framework

- Model
- Known results

#### • Applications beyond voting

## Actually, More Voting First!

#### • Distributed elections

 Voters partitioned into groups that conduct separate elections [Borodin, Lev, S, Strangway, 2019; Filos-Ratsikas, Micha, Voudouris, 2020; Filos-Ratsikas, Voudouris, 2021; Anshelevich, Filos-Ratsikas, Voudouris, 2022]

#### • Representative candidates

• Alternatives sampled from the pool of voters [Cheng, Dughmi, Kempe, 2017; Cheng, Dughmi, Kempe, 2018]

#### Voter abstentions

- What if only a fraction of the voters vote? [Borodin, Lev, S, Strangway, 2019; Seddighin, Latifian, Ghodsi, 2021; Anagnostides, Fotakis, Patsilinakos, 2021]
- Approval-based cost functions for metric distortion [Pierczynski, Skowron, 2019]

## **Beyond Voting**

#### One-Sided Matching

- Match *m* agents to *m* items, where agents have cardinal utilities for the items but only provide ordinal rankings
- Theorem [Filos-Ratsikas, Frederiksen, Zhang, 2014]:
  - The best distortion of any randomized rule is  $\Theta(\sqrt{m})$ .
- Theorem [Amanatidis, Birmpas, Filos-Ratsikas, Voudouris, 2021]:
  - The best distortion of any deterministic rule is  $\Theta(m^2)$ .
  - They also analyze the information-distortion tradeoff via queries.
- Surprisingly, identical bounds as single-winner voting!
- Other work [Ma, Menon, Larson, 2021; Bishop, Chan, Mandal, Tran-Thanh, 2022]

## **Beyond Voting**

- Resource allocation
  - Allocate *m* goods to *n* agents
  - [Halpern, S, 2021]: When every agent ranks the goods
  - [Ebadian, Freeman, S, 2022]: When k agents provide no information while the rest provide cardinal utilities
- Secretary problem [Hoefer, Kodric, 2017]
- Graph-theoretic problems
  - Maximum-weight matching [Anshelevich, Sekar, 2016a]
  - Max k-sum, densest k-subgraph, maximum traveling salesman [Anshelevich, Sekar, 2016b]
  - Min-weight and max-min bipartite matching, facility location, *k*-center, *k*-median [Filos-Ratsikas, Voudouris, 2021; Anshelevich, Zhu, 2021]

#### Future Work: Ballot Design



#### • Common ballot designs

• Pairwise comparisons, "Do you like candidate *a* at least twice as much as candidate *b*?", ...

#### • Better models of cognitive burden

- Psychology, HCI, ...
- Voter errors in answering ballots
  - Expressive ballots can also induce errors
- Intangible aspects of ballot design
  - Barcelona PB team: "Knapsack votes are good because they help voters understand the limitations of the budget."

## Future Work: Distortion vs Other Desiderata





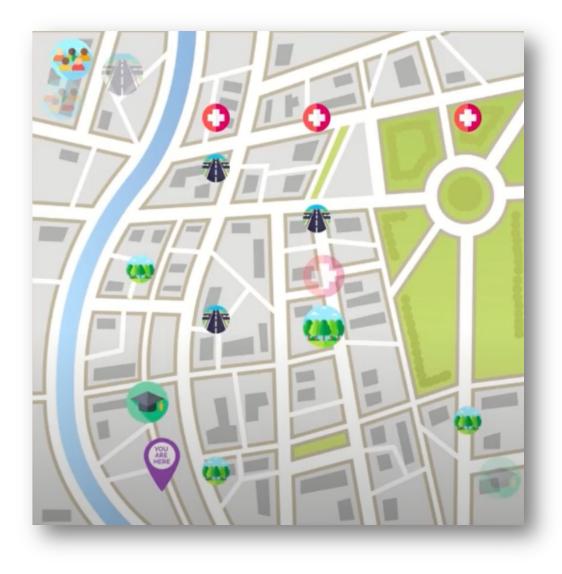
#### • Distortion & Truthfulness

- With ranked ballots, near-optimal distortion can be achieved via truthful aggregation
- What happens with other ballot formats?

#### • Distortion & Axioms

- Can we achieve low distortion together with popular axioms?
- Especially, proportional representation for committee selection
- Distortion & Explainability
  - Explaining the voting rule vs explaining what it does

## Future Work: More Complex Voting Paradigms



- Design optimal voting rules for more complex voting paradigms
  - Participatory budgeting
  - Districting
- Model end-to-end voting
  - In participatory budgeting, voting is but the final step of a year-long process
- Compare models of democracy
  - E.g., direct democracy, representative democracy, and liquid democracy



#### **AI-Driven Decisions**

RoboVote is a free service that helps users combine their preferences or opinions into optimal decisions. To do so, RoboVote employs state-of-the-art voting methods developed in artificial intelligence research. Learn More



#### Poll Types

RoboVote offers two types of polls, which are tailored to different scenarios; it is up to users to indicate to RoboVote which scenario best fits the problem at hand.



#### **Objective Opinions**

In this scenario, some alternatives are objectively better than others, and the opinion of a participant reflects an attempt to estimate the correct order. RoboVote's proposed outcome is guaranteed to be as close as possible — based on the available information — to the best outcome. Examples include deciding which product prototype to develop, or which company to invest in, based on a metric such as projected revenue or market share.



#### Subjective Preferences

In this scenario participants' preferences reflect their subjective taste; RoboVote proposes an outcome that mathematically makes participants as happy as possible overall. Common examples include deciding which restaurant or movie to go to as a group.

Ready to get started?

CREATE A POLL

- Abramowitz, B., Anshelevich, E. and Zhu, W. Awareness of voter passion greatly improves the distortion of metric social choice. WINE, pp. 3-16, 2019.
- Anagnostides, I., Fotakis, D. and Patsilinakos, P. *Metric-distortion bounds under limited information*. SAGT, pp. 299-313, 2021.
- Anshelevich, E., Bhardwaj, O., Elkind, E., Postl, J. and Skowron, P. *Approximating optimal social choice under metric preferences*. AIJ, 264, pp. 27-51, 2018.
- Anshelevich, E., Filos-Ratsikas, A., Shah, N. and Voudouris, A. A. *Distortion in Social Choice Problems: The First 15 Years and Beyond.* IJCAI (Survey Track), pp. 4294-4301, 2021.
- Anshelevich, E., Filos-Ratsikas, A. and Voudouris, A. A. *The distortion of distributed metric social choice*. AlJ, 308, p.103713, 2022.
- Anshelevich, E. and Sekar, S. *Blind, greedy, and random: Algorithms for matching and clustering using only ordinal information.* AAAI, pp. 383-389, 2016a.
- Anshelevich, E. and Sekar, S. *Truthful mechanisms for matching and clustering in an ordinal world.* WINE, pp. 265-278, 2016b.

- Anshelevich, E. and Zhu, W. Ordinal approximation for social choice, matching, and facility location problems given candidate positions. TEAC, 9(2), pp.1-24, 2021.
- Benade, G., Nath, S., Procaccia, A. D. and Shah, N. *Preference elicitation for participatory budgeting*. Management Science, 67(5), pp. 2813-2827, 2021.
- Benade, G., Procaccia, A. D. and Qiao, M. *Low-distortion social welfare functions*. AAAI, pp. 1788-1795, 2019.
- Bhaskar, U., Dani, V. and Ghosh, A. *Truthful and near-optimal mechanisms for welfare maximization in multi-winner elections.* AAAI, pp. 925-932, 2018.
- Bishop, N., Chan, H., Mandal, D. and Tran-Thanh, L. *Sequential Blocked Matching*. AAAI, pp. 4834-4842, 2022.
- Borodin, A., Halpern, D., Latifian, M. and Shah, N. *Distortion in voting with top-t preferences*. IJCAI, 2022 (forthcoming).
- Borodin, A., Lev, O., Shah, N. and Strangway, T. *Primarily about primaries*. AAAI, pp. 1804-1811, 2019.

- Boutilier, C., Caragiannis, I., Haber, S., Lu, T., Procaccia, A. D. and Sheffet, O. *Optimal social choice functions: A utilitarian view.* AIJ, 227, pp. 190-213, 2015.
- Caragiannis, I., Nath, S., Procaccia, A. D. and Shah, N. *Subset selection via implicit utilitarian voting.* JAIR, 58, pp. 123-152, 2017.
- Caragiannis, I. and Procaccia, A. D. *Voting almost maximizes social welfare despite limited communication*. AlJ, 175(9-10), pp. 1655-1671, 2011.
- Caragiannis, I., Shah, N. and Voudouris, A. A. *The metric distortion of multiwinner voting.* AAAI, pp. 4900-4907, 2022.
- Charikar, M. and Ramakrishnan, P. *Metric distortion bounds for randomized social choice*. SODA, pp. 2986-3004, 2022.
- Chen, X., Li, M. and Wang, C. *Favorite-candidate voting for eliminating the least popular candidate in a metric space.* AAAI, pp. 1894-1901, 2020.
- Cheng, Y., Dughmi, S. and Kempe, D. *Of the people: voting is more effective with representative candidates.* EC, pp. 305-322, 2017.

- Cheng, Y., Dughmi, S. and Kempe, D. *On the distortion of voting with multiple representative candidates.* AAAI, pp. 973-980, 2018.
- Cheng, Y., Jiang, Z., Munagala, K. and Wang, K. *Group fairness in committee selection*. TEAC, 8(4), pp. 1-18, 2020.
- Ebadian, S., Freeman, R. and Shah, N. *Efficient Resource Allocation with Secretive Agents*. IJCAI, 2022 (forthcoming).
- Ebadian, S., Kahng, A., Shah, N. and Peters, D. *Optimized distortion and proportional fairness in voting*. EC, 2022 (forthcoming).
- Fain, B., Fan, W. and Munagala, K. *Concentration of distortion: the value of extra voters in randomized social choice.* IJCAI, pp. 110-116, 2020.
- Fain, B., Goel, A., Munagala, K. and Prabhu, N. *Random dictators with a random referee: Constant sample complexity mechanisms for social choice.* AAAI, pp. 1893-1900, 2019.
- Fain, B., Goel, A., Munagala, K. and Sakshuwong, S. *Sequential deliberation for social choice*. WINE, pp. 177-190, 2017.

- Filos-Ratsikas, A. and Miltersen, P. B. *Truthful approximations to range voting.* WINE, pp. 175-188, 2014.
- Filos-Ratsikas, A., Frederiksen, S. K. S. and Zhang, J. *Social welfare in one-sided matchings: Random priority and beyond.* SAGT, pp. 1-12, 2014.
- Filos-Ratsikas, A., Micha, E. and Voudouris, A. A. *The distortion of distributed voting.* AlJ, 286, p. 103343, 2020.
- Filos-Ratsikas, A. and Voudouris, A. A. *Approximate mechanism design for distributed facility location.* SAGT, pp. 49-63, 2021.
- Gkatzelis, V., Halpern, D. and Shah, N. *Resolving the optimal metric distortion conjecture*. FOCS, pp. 1427-1438, 2020.
- Goel, A., Hulett, R. and Krishnaswamy, A. K. *Relating metric distortion and fairness of social choice rules*. NetEcon, pp. 1-1, 2018.
- Gross, S., Anshelevich, E. and Xia, L. *Vote until two of you agree: Mechanisms with small distortion and sample complexity.* AAAI, pp. 544-550, 2017.

- Halpern, D. and Shah, N. *Fair and efficient resource allocation with partial information*. IJCAI, pp. 224-230, 2021.
- Hoefer, M. and Kodric, B. *Combinatorial secretary problems with ordinal information*. ICALP, pp. 1-14, 2017.
- Jiang, Z., Munagala, K. and Wang, K. *Approximately stable committee selection*. STOC, pp. 463-472, 2020.
- Kahng, A. and Kehne, G. Worst-case voting when the stakes are high. AAAI, pp. 5100-5107, 2022.
- Kempe, D. Communication, distortion, and randomness in metric voting. AAAI, pp. 2087-2094, 2020a.
- Kempe, D. An analysis framework for metric voting based on LP duality. AAAI, pp. 2079-2086, 2020b.
- Lee, S. *Maximization of relative social welfare on truthful cardinal voting schemes.* arXiv:1904.00538, 2019.
- Ma, T., Menon, V. and Larson, K. *Improving welfare in one-sided matchings using simple threshold queries.* IJCAI, pp. 321-327, 2021.

- Mandal, D., Procaccia, A. D., Shah, N. and Woodruff, D. P. *Efficient and thrifty voting by any means necessary*. NeurIPS, pp. 7180-7191, 2019.
- Mandal, D., Shah, N. and Woodruff, D. P. *Optimal communication-distortion tradeoff in voting*. EC, pp. 795-813, 2020.
- Munagala, K. and Wang, K. *Improved metric distortion for deterministic social choice rules.* EC, pp. 245-262, 2019.
- Pierczynski, G. and Skowron, P. *Approval-based elections and distortion of voting rules*. IJCAI, pp. 543-549, 2019.
- Procaccia, A. D. and Rosenschein, J. S. *The distortion of cardinal preferences in voting.* CIA, pp. 317-331, 2006.
- Pulyassary, H. and Swamy, C. *On the randomized metric distortion conjecture.* arXiv:2111.08698, 2021.
- Seddighin, M., Latifian, M. and Ghodsi, M. *On the distortion value of elections with abstention.* JAIR, 70, pp. 567-595, 2021.

# Thank you!

Questions?