CSC304 Fall'26
Tutorial 6

Q1 VCG Mechanism for Auction

Suppose we want to sell three goods, a, b, and c. There are three agents, 1, 2, and 3.

- Agent 1 either wants $\{a, b\}$ at \$4 or $\{c\}$ at \$3, or $\{a, b, c\}$ at \$7, and has zero value for other combinations.
- Agent 2 values a, b, and c at \$3, \$2, and \$2, respectively, and has additive values for bundles of goods (i.e., her value for a set of goods is the sum of her values for the individual goods in the set). So, for instance, agent 2's value for $\{a, b\}$ would be $3 + 2 = 5$.
- Agent 3 values a, b, and c at \$2, \$3, and \$2, respectively, and also has additive values for bundles of goods.

Determine how the goods would be allocated and how much the three agents would have to pay under VCG.

Q2 VCG for Public Policy

Suppose a government wants to choose a tax policy from three proposed alternatives A, B, and C. The government asks three lobbying groups ($L1$, $L2$, and $L3$) to submit their values for the three proposals, and wants to run a VCG auction.

If the following bids are submitted, which proposal would be selected, and how much would each lobbying group be charged under VCG?

	A	B	C
$L1$	12	9	2
$L2$	5	8	10
$L3$	4	5	6

Q3 Sponsored Search

Consider the following sponsored search auction with three slots A, B and C, and four bidders W, X, Y and Z. The *clickthrough rate* of each slot and the *value per click* of each bidder is as follows:

Values per Click	Bidders	Slots	Clickthrough Rates
10	W	A	0.6
5	X	B	0.4
3	Y	C	0.3
1	Z		

(a) Suppose we run the VCG auction and the bidders bid their true values. Compute the assignment of slots to bidders, the payment per click charged to each bidder, and the expected payment charged to each bidder. Show your calculation.

(b) Suppose we run the generalized second-price (GSP) auction and the bidders bid their true values. Compute the assignment of slots to bidders, the payment per click charged to each bidder, and the expected payment charged to each bidder. Show your calculation.

(c) Suppose we run the GSP auction. Is it a Nash equilibrium for all bidders to bid their true values? If not, find a bidder who can strictly increase her expected utility by changing only her own bid (while all other bidders still bid their true values), and compute her expected utility before and after the change. Show your calculation.

[Hint: For each bidder, consider the slots she could get by changing her bid, and the payment per click she would be charged for each of them.]

Q4 Sealed-Bid First-Price Auction

In class, we studied the sealed-bid first-price auction for a single item, in which each bidder privately submits a bid, and the one with the highest bid wins the item and pays her bid.

We proved that with $n = 2$ bidders drawing their values from $U[0, 1]$ iid, it is a Bayes-Nash equilibrium for each bidder i to bid $b(v_i) = v_i/2$.

Your goal is to generalize this to n bidders: If each bidder i bidding $b(v_i) = c \cdot v_i$ is a (symmetric) Bayes-Nash equilibrium, then $c = \frac{n-1}{n}$.

Q5 Lobbying

The government is considering n possible tax policies ($\{1, 2, \ldots, n\}$) and there are n companies ($\{1, 2, \ldots, n\}$). Each company i has a private value v_i drawn from $U[0, 1]$, independently of all other companies, such that the company gets value v_i if policy i is implemented and value 0 if any other policy is implemented. The company lobbies the government to implement policy i by giving a donation of d_i. The greedy government announces that it will implement the policy recommended by its biggest donor, but obviously, it will keep all the donations it receives.

Suppose that this game has a symmetric Bayes-Nash equilibrium in which each company donates $d_i = c \cdot v_i^{\alpha}$, for some $c, \alpha > 0$. Find the values of c and α.