UNIVERSITY OF TORONTO
Faculty of Arts & Science

Midterm 2 Examination

CSC 304 H1F

Duration: 50 minutes

Aids Allowed: One *8.5″×11″* sheet of *hand*written notes on *one* side.

*Do **not** turn this page until you have received the signal to start.*
In the meantime, write your name and student number below (please do this now!) and *carefully* read *all* the information on the rest of this page.

First (Given) Name(s):

Last (Family) Name(s):

10-Digit Student Number:

UTORid (e.g., `pitfra12`):

- This midterm examination consists of 2 questions on 6 pages (including this one), printed on both sides of the paper. *When you receive the signal to start, please make sure that your copy of the examination is complete.*
- Answer each question directly on the examination paper, in the space provided, and use a "blank" page for rough work. If you need more space for one of your solutions, use one of the "blank" pages and *indicate clearly the part of your work that should be marked.*
- As a student, you help create a fair and inclusive writing environment. If you possess an unauthorized aid during an exam, you may be charged with an academic offence.

Marking Guide

Nº 1: _____/25

Nº 2: _____/25

TOTAL: _____/50

STUDENTS MUST HAND IN ALL EXAMINATION MATERIALS AT THE END

MIDTERM 2 EXAMINATION

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Question 1. VCG [25 MARKS]

Suppose we want to sell two goods a and b, and there are 3 agents with the following values for different bundles (subsets of goods). Run the VCG auction on this instance.

Bundles	Agent 1	Agent 2	Agent 3
$\emptyset$	0	0	0
$\{a\}$	9	6	3
$\{b\}$	6	2	5
$\{a, b\}$	11	10	13

Part (a) [10 MARKS]

What allocation would VCG choose? What is the social welfare under this allocation? No justification is needed.

Part (b) [15 MARKS]

What payments would be charged to the three agents by VCG? Show your calculation by identifying, for each agent, the allocation that would be implemented and the welfare it would yield to the other agents in the absence of the agent.

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Question 2. Bayes-Nash Equilibria [25 marks]

Two companies are fighting to obtain mining rights near Mountain Plentiful. One side of the mountain contains v_1 thousand tonnes of copper, while the other side contains v_2 thousand tonnes of zinc. Company 1 is more interested in copper than zinc, so it values the mining rights at $v_1 + v_2/2$. Company 2 is more interested in zinc than in copper, so it values the mining rights at $v_1/2 + v_2$. Only one company can win the mining rights.

Both companies know that v_1 and v_2 are drawn iid from the uniform distribution $U[0, 1]$. The state holds a second-price auction. Each company $i \in \{1, 2\}$ will bid an amount b_i, and the company with the higher bid will win the mining rights and pay the second highest bid.

Since company 1 is more interested in copper, it invests effort to determine the precise value of v_1, but still knows nothing about v_2 other than that it is drawn from $U[0, 1]$. Similarly, company 2 has figured out the exact value of v_2, but knows nothing about v_1 other than that it is drawn from $U[0, 1]$.

Show that each bidder bidding $b_i = 1.5v_i$ is a Bayes-Nash equilibrium in this auction. Due to symmetry, you only need to show that company 1 would want to bid $b_1 = 1.5v_1$ if it assumes that company 2 will bid $b_2 = 1.5v_2$, whatever the value of v_2 is. So in the questions below, assume that company 2 bids $b_2 = 1.5v_2$, and derive the optimal bid b_1 of company 1.

Part (a) [5 marks]

Argue why the optimal bid b_1 must be at most 1.5.

Part (b) [5 marks]

Derive the probability of company 1 winning the auction if it bids b_1.

Part (c) [5 marks]

Derive the expected value that company 1 will derive conditioned on it winning the auction.

Part (d) [5 marks]

Derive the expected payment that company 1 will have to pay conditioned on it winning the auction.

Part (e) [5 marks]

Using the calculations in parts (b), (c), and (d), show that the optimal bid for company 1 is $b_1 = 1.5v_1$ by maximizing its expected utility.

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Use the space on this "blank" page for scratch work, or for any solution that did not fit elsewhere.
Clearly label each such solution with the appropriate question and part number.

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