(PRINT) Name: _____ Student No.: _____

Signature: _____ Marks: _____ / 30

CSC 304 — Algorithmic Game Theory and Mechanism Design

Midterm 2, November 18, 2019

This test has six (6) pages. Make sure that you have a complete paper.
Answer all questions on this paper. You are allowed to use *one 8.5"x11" aid sheet of handwritten notes on one side*. No other materials or aids of any type are permitted.

Time: 50 minutes; Total Marks: 30

Q1. VCG (10 pts)

The government decides to help out three companies by building a single warehouse that they can all share. The warehouse is to be built at a location $x \in [0, 10]$. Each company $i \in \{1, 2, 3\}$ is located at y_i, where $y_1 = 1$, $y_2 = 5$, and $y_3 = 9$. A warehouse at location x provides value $100 - (y_i - x)^2$ to each company i simultaneously.

Under the VCG mechanism, decide where the warehouse should be built, and how much payment each company should be charged. Show your calculation.

[Note: In this case, the space of possible outcomes is infinite, but VCG still applies.]

Use this page to continue answering any question or as a scrap paper.

Q2. Bayes-Nash Equilibria (10 pts)

Consider an auction between a buyer and a seller. Seller s wants to sell a good for which his own value is v_s. Buyer b has value v_b for the good. Both v_s and v_b are drawn independently from the uniform distribution $U[0, 1]$. Each player knows his own value, but only knows the distribution of the other player's value.

Here's how the auction goes down. The seller and the buyer independently write down some prices (say p_s and p_b, respectively) on different pieces of paper. Then, they compare their prices. If it turns out that $p_s \leq p_b$, then the item is sold at price p_s, the seller gets utility $p_s - v_s$, and the buyer gets utility $v_b - p_s$. On the other hand, if $p_s > p_b$, the item remains unsold and both players get zero utility.

Is there a Bayes-Nash equilibrium in which the buyer honestly writes down her value (i.e. $p_b = v_b$)? If so, what is the seller's price p_s in this equilibrium? Show your calculation.

[Hint: Assume $p_b = v_b$, and derive the optimal price p_s for the seller to write. Then, check whether given this strategy of the seller, $p_b = v_b$ is indeed the optimal price for the buyer to write.]

Use this page to continue answering any question or as a scrap paper.

Q3. Sponsored Search (10 pts)

Consider the following sponsored search auction with three slots A, B and C, and four bidders W, X, Y and Z. The *clickthrough rate* for each slot and the *value per click* for each bidder are as follows:

Values per Click	Bidders	Slots	Clickthrough Rates
10	W	A	0.6
5	X	B	0.4
3	Y	C	0.3
1	Z		

(a) [5 points] Suppose we run the generalized second price (GSP) auction and the bidders bid their true values. Compute the assignment of slots to bidders and the *expected payments* charged to bidders under GSP. Show your calculation.

(b) [5 points] Find a bidder who has an incentive to misreport his value under GSP, assuming the other bidders bid truthfully. Identify the bidder, state the beneficial manipulation, and derive the gain in utility to the bidder due to the manipulation. Show your calculation.

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