

CSC304 Lecture 2

Game Theory (Basic Concepts)

Game Theory

- How do rational, self-interested agents act?
- Each agent has a set of possible actions
- Rules of the game:
 - Rewards for the agents as a function of the actions taken by different agents
- We focus on noncooperative games
 - No external force or agencies enforcing coalitions

Normal Form Games

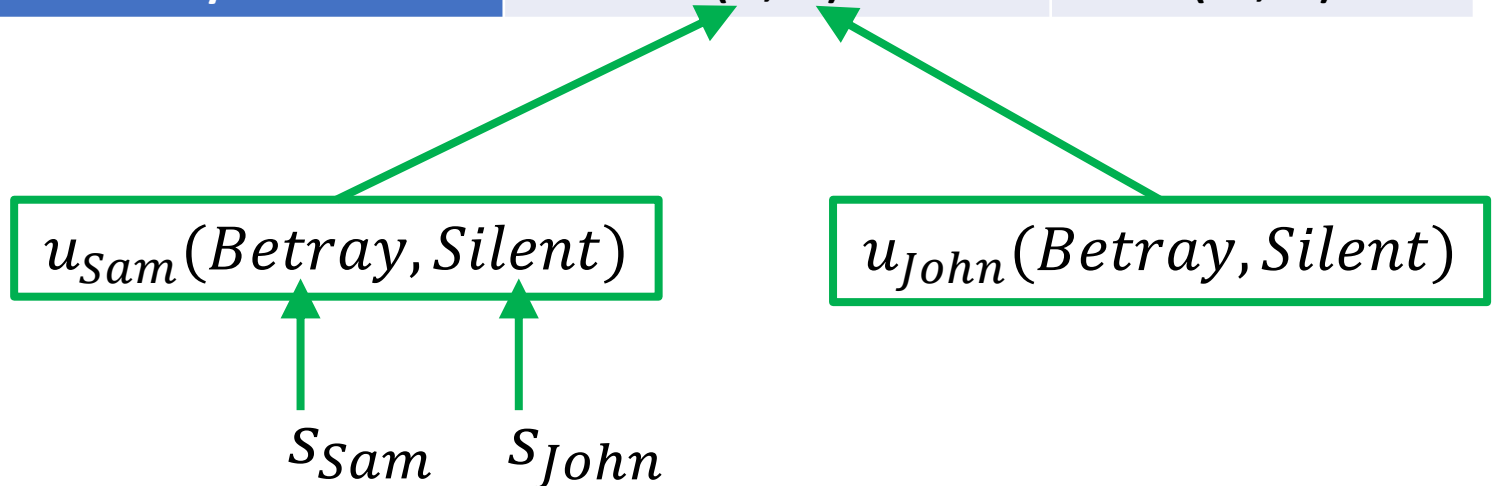
- A set of players $N = \{1, \dots, n\}$
- A set of actions S
 - Action of player $i \rightarrow s_i$
 - Action profile $\vec{s} = (s_1, \dots, s_n)$
- For each player i , utility function $u_i: S^n \rightarrow \mathbb{R}$
 - Given action profile $\vec{s} = (s_1, \dots, s_n)$, each player i gets reward $u_i(s_1, \dots, s_n)$

Normal Form Games

Recall: Prisoner's dilemma

$$S = \{\text{Silent}, \text{Betray}\}$$

Sam's Actions \ John's Actions	John's Actions	
	Stay Silent	Betray
Stay Silent	$(-1, -1)$	$(-3, 0)$
Betray	$(0, -3)$	$(-2, -2)$



Player Strategies

- Pure strategy
 - Choose an action to play
 - E.g., “Betray”
 - For our purposes, simply an action.
 - In repeated or multi-move games (like Chess), need to choose an action to play at every step of the game based on history.
- Mixed strategy
 - Choose a probability distribution over actions
 - Randomize over pure strategies
 - E.g., “Betray with probability 0.3, and stay silent with probability 0.7”

Domination among Strategies

- s_i dominates s'_i if player i is always “better off” playing s_i than s'_i , regardless of the strategies of other players.
- Two variants: weak and strict domination
 - $u_i(s_i, \vec{s}_{-i}) \geq u_i(s'_i, \vec{s}_{-i}), \forall \vec{s}_{-i}$ (needed for both)
 - Strict inequality for **some** \vec{s}_{-i} $\leftarrow s_i$ weakly dominates s'_i
 - Strict inequality for **all** \vec{s}_{-i} $\leftarrow s_i$ strictly dominates s'_i

Example

P1 \ P2	b_1	b_2
a_1	(2 , 3)	(4 , 1)
a_2	(2 , 5)	(6 , 3)
a_3	(3 , 1)	(5 , 2)

- P1

- a_1 vs a_2 ?
- a_1 vs a_3 ?
- a_2 vs a_3 ?

- P2

- b_1 vs b_2 ?

Dominant Strategies

- s_i is a strictly (weakly) dominant strategy for player i if it strictly (weakly) dominates **every other strategy**
- Strict dominance is a strong concept
 - A player who has a strictly dominant strategy has no reason *not* to play it
 - If every player has a strictly dominant strategy, such strategies will very likely dictate the outcome of the game

Example

P1 \ P2		
	b_1	b_2
a_1	(2 , 3)	(4 , 1)
a_2	(2 , 5)	(6 , 3)
a_3	(3 , 1)	(5 , 2)

- Does either player have a dominant strategy?

Example

P1 \ P2	P2		
	b_1	b_2	b_3
a_1	(2 , 3)	(4 , 1)	(2 , 3)
a_2	(2 , 5)	(6 , 3)	(3 , 5)
a_3	(3 , 1)	(5 , 2)	(4 , 3)

- How about now?

Example

P1 \ P2			
	b_1	b_2	b_3
a_1	(2 , 3)	(4 , 1)	(2 , 4)
a_2	(2 , 5)	(6 , 3)	(3 , 6)
a_3	(3 , 1)	(5 , 2)	(4 , 3)

- How about now?

Example: Prisoner's Dilemma

- Recap:

John's Actions		Stay Silent	Betray
Sam's Actions	Stay Silent	$(-1, -1)$	$(-3, 0)$
	Betray	$(0, -3)$	$(-2, -2)$

- Betraying is a strictly dominant strategy for each player

Iterated Elimination

- What if there are no dominant strategies?
 - No single strategy dominates every other strategy
 - But some strategies might still be dominated
- Assuming everyone knows everyone is rational...
 - Can remove their dominated strategies
 - Might reveal a newly dominant strategy
- Two variants depending on what we eliminate:
 - Only strictly dominated? Or also weakly dominated?

Iterated Elimination

- Toy example:
 - Microsoft vs Startup
 - Enter the market or stay out?

Microsoft \ Startup		Enter	Stay Out
		Enter	Stay Out
Enter	Enter	(2 , -2)	(4 , 0)
	Stay Out	(0 , 4)	(0 , 0)

- Q: Is there a dominant strategy for startup?
- Q: Do you see a rational outcome of the game?

Iterated Elimination

- More serious: “Guess $2/3$ of average”
 - Each student guesses a real number between 0 and 100 (inclusive)
 - The student whose number is the closest to $2/3$ of the average of all numbers wins!
- In-class poll!
- Recall: We have a unique optimal strategy only if everyone is rational, and everyone thinks everyone is rational, and so on.

Nash Equilibrium

- What if we don't find a unique outcome after iterated elimination of dominated strategies?

<div>Students \ Professor</div>		Attend	Be Absent
		Attend	Be Absent
Students	Attend	(3 , 1)	(-1 , -3)
	Be Absent	(-1 , -1)	(0 , 0)

Nash Equilibrium

- Nash Equilibrium

- A strategy profile \vec{s} is in Nash equilibrium if s_i is the best action for player i given that other players are playing \vec{s}_{-i}

$$u_i(s_i, \vec{s}_{-i}) \geq u_i(s'_i, \vec{s}_{-i}), \forall s'_i$$



No quantifier on \vec{s}_{-i}

- Each player's strategy is only best *given* the strategies of others, and not *regardless*.

Recap: Prisoner's Dilemma

John's Actions		Stay Silent	Betray
Sam's Actions	Stay Silent	$(-1, -1)$	$(-3, 0)$
	Betray	$(0, -3)$	$(-2, -2)$

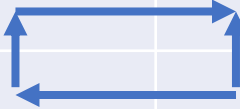


The diagram illustrates the strategic form of the Prisoner's Dilemma. It shows a 2x2 payoff matrix where the rows represent Sam's actions and the columns represent John's actions. The payoffs are given as (Sam, John). The matrix shows that for Sam, betraying is always better than staying silent, regardless of what John does. Similarly, for John, betraying is always better than staying silent, regardless of what Sam does. This leads to a Nash equilibrium at (Betray, Betray) with payoffs (-2, -2). The diagram also shows a cycle of best responses: if Sam stays silent, John's best response is to betray; if John betrays, Sam's best response is to betray; if Sam betrays, John's best response is to betray; and if John betrays, Sam's best response is to betray. This cycle indicates that there is no pure strategy Nash equilibrium in this game.

- Nash equilibrium?
- Food for thought:
 - What is the relation between iterated elimination of weakly/strictly dominated strategies and Nash equilibria?

Recap: Microsoft vs Startup

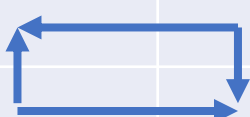
Startup		Enter	Stay Out
Microsoft	Enter	(2 , -2)	(4 , 0)
	Stay Out	(0 , 4)	(0 , 0)



- Nash equilibrium?

Recap: Attend or Not

<div>Students \ Professor</div>		Professor	
		Attend	Be Absent
Students	Attend	(3 , 1)	(-1 , -3)
	Be Absent	(-1 , -1)	(0 , 0)



- Nash equilibrium?

Example: Stag Hunt

Hunter 2 \ Hunter 1		Stag	Hare
		Stag	Hare
Hunter 2	Stag	(4 , 4)	(0 , 2)
	Hare	(2 , 0)	(1 , 1)

- Game:
 - Each hunter decides to hunt stag or hare
 - Stag = 8 days of food, hare = 2 days of food
 - Catching stag requires both hunters, catching hare requires only one
 - If they catch one animal together, they share
- Nash equilibrium?