CSC304 Lecture 13

Mechanism Design w/o Money: Facility Location

Lack of Money

- Mechanism design with money:
  - VCG can implement welfare maximizing outcome because it can charge payments

- Mechanism design without money:
  - Suppose you want to give away a single item, but cannot charge any payments
  - Impossible to get meaningful information about valuations from strategic agents
  - How would you maximize welfare as much as possible?

Lack of Money

- One possibility: Give the item to each of n bidders with probability $1/n$.
- Does not maximize welfare
  - It's impossible to maximize welfare without money
- Achieves an n-approximation of maximum welfare
  - $\frac{\max_i v_i}{(1/n)\sum_i v_i} \le n$
- Can't do better than n-approximation without money

MD w/o Money Theme

1. Define the problem: agents, outcomes, valuations
2. Define the goal (e.g., maximizing social welfare)
3. Check if the goal can be achieved using a strategyproof mechanism
4. If not, find the strategyproof mechanism that provides the best worst-case approximation ratio
   - Worst-case approximation ratio is similar to the price of anarchy (PoA)

Facility Location

- Set of agents N
- Each agent i has a true location $x_i \in \mathbb{R}$
- Mechanism f
  - Takes as input reports $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$
  - Returns a location $y \in \mathbb{R}$ for the new facility
- Cost to agent i : $c_i(y) = |y - x_i|$
- Social cost $C(y) = \sum_i c_i(y) = \sum_i |y - x_i|$

Facility Location

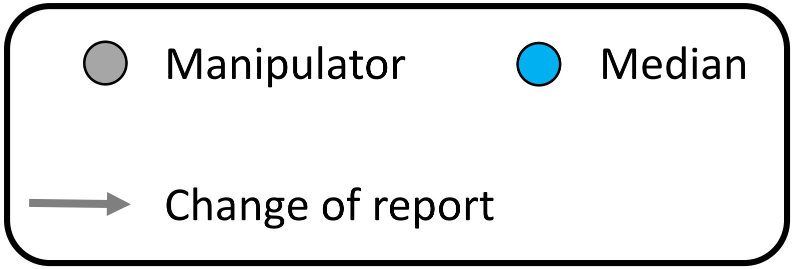
- Social cost $C(y) = \sum_i c_i(y) = \sum_i |y - x_i|$
- Q: Ignoring incentives, what choice of y would minimize the social cost?
- A: The median location $\text{med}(x_1, \dots, x_n)$
  - n is odd → the unique "(n+1)/2"th smallest value
  - n is even → "n/2"th or "(n/2)+1"st smallest value
  - Why?

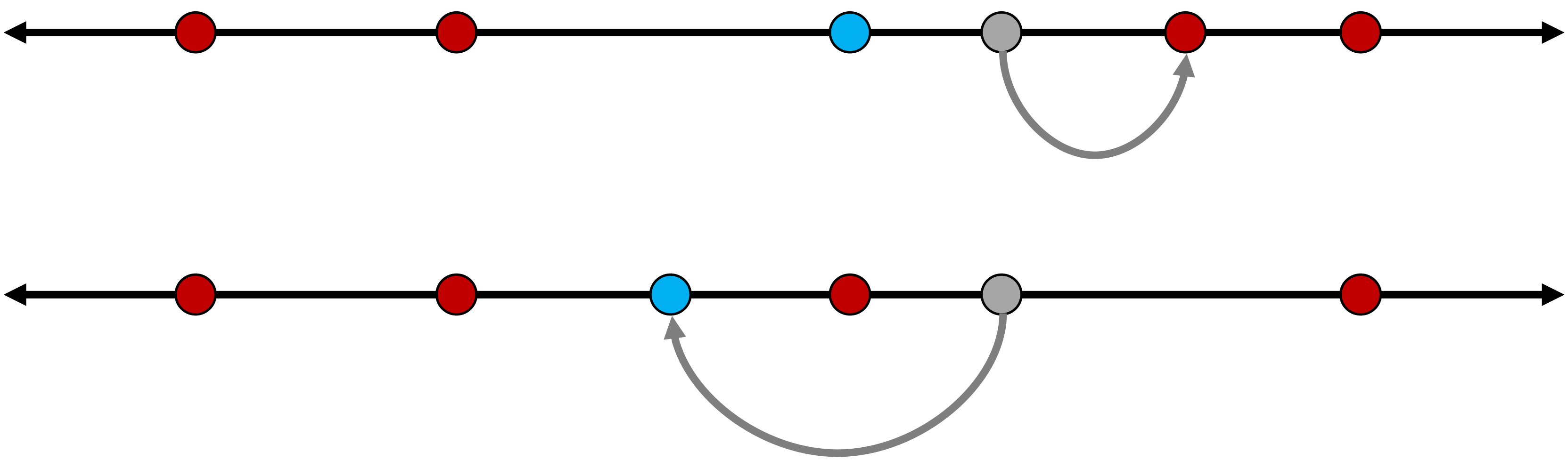
Facility Location

- Social cost $C(y) = \sum_i c_i(y) = \sum_i |y - x_i|$
- Median is optimal (i.e., 1-approximation)
- What about incentives?
  - Median is also strategyproof (SP)!

Median is SP

No manipulation can help





Max Cost

- A different objective function $C(y) = \max_i |y - x_i|$
- Q: Again ignoring incentives, what value of y minimizes the maximum cost?
- A: The midpoint of the leftmost ($\min_i x_i$) and the rightmost ($\max_i x_i$) locations (WHY?)
- Q: Is this optimal rule strategyproof?
- A: No! (WHY?)

Max Cost

- $C(y) = \max_i |y - x_i|$
- We want to use a strategyproof mechanism.
- Question: What is the approximation ratio of median for maximum cost?
  1. $\in [1,2)$
  2. $\in [2,3)$
  3. $\in [3,4)$
  4. $\in [4,\infty)$

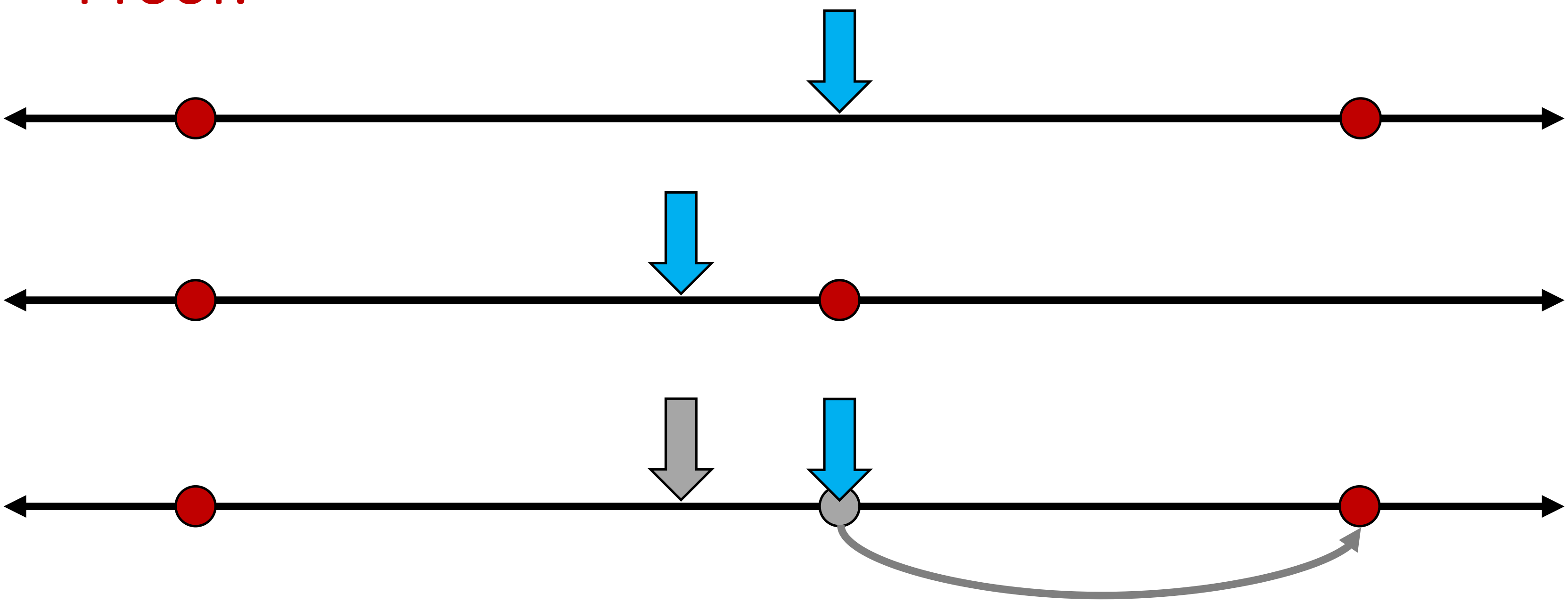
Max Cost

- Answer: 2-approximation

- Other SP mechanisms that are 2-approximation
  - Leftmost: Choose the leftmost reported location
  - Rightmost: Choose the rightmost reported location
  - Dictatorship: Choose the location reported by agent 1
  - …

Max Cost

- Theorem [Procaccia & Tennenholtz, '09]
  No deterministic SP mechanism has approximation ratio < 2 for maximum cost.
- Proof:



Max Cost [For later reference]

- Theorem [Procaccia & Tennenholtz, '09]
  No deterministic SP mechanism has approximation ratio < 2 for maximum cost.
- Proof:
  - Suppose the two agents report $x_1 = 0$ and $x_2 = 1$.
    - For approximation ratio < 2, the facility must be at $0 < y < 1$.
  - Now, suppose the true preferences of the agents are $x_1 = 0$ and $x_2 = y$, and they report honestly.
    - Again, the facility must be at $0 < y' < y$.
    - Then agent 2 has strict incentive to report 1 instead of y so the facility shifts to his true location y.
  - QED!

Max Cost + Randomized

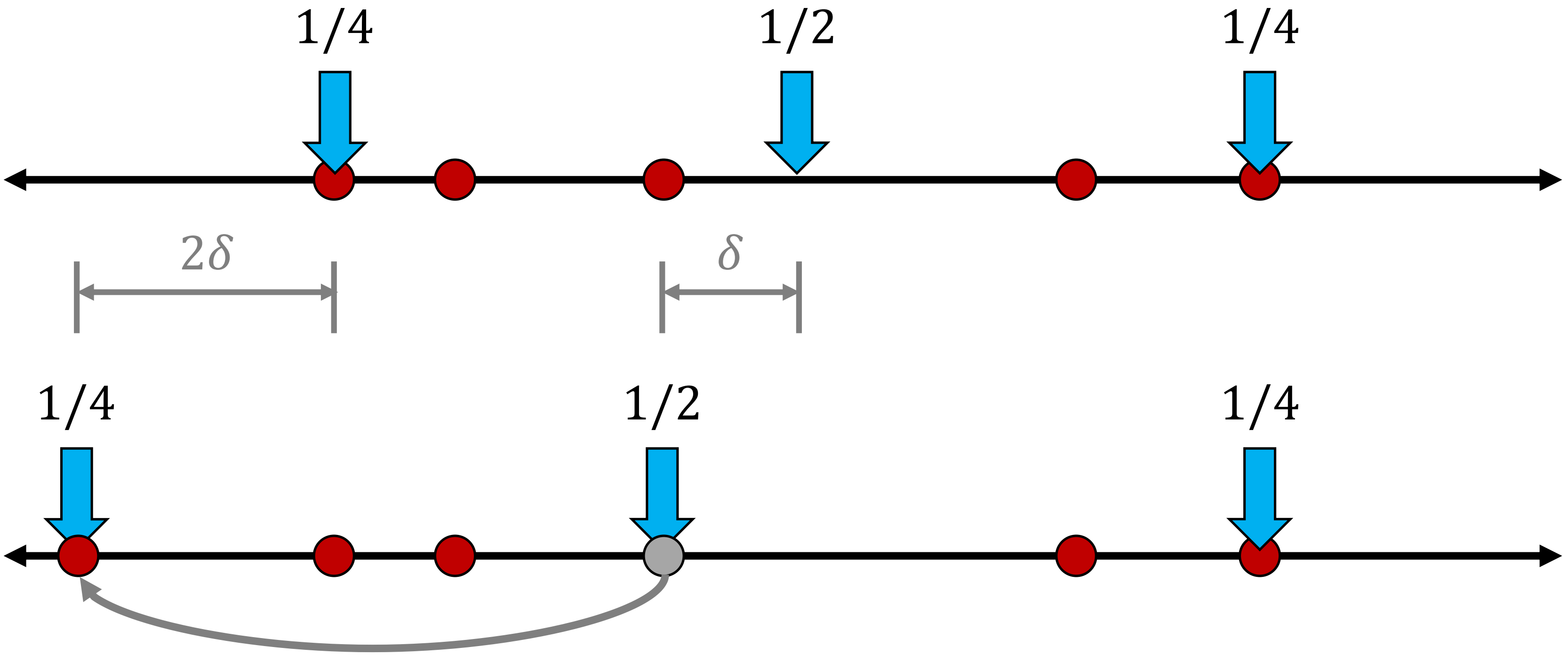
- The Left-Right-Middle (LRM) Mechanism
  - Choose $\min_i x_i$ with probability ¼
  - Choose $\max_i x_i$ with probability ¼
  - Choose $(\min_i x_i + \max_i x_i)/2$ with probability ½

- Question: What is the approximation ratio of LRM for maximum cost?

- At most $\frac{(1/4)*2C+(1/4)*2C+(1/2)*C}{C} = \frac{3}{2}$

Max Cost + Randomized

- Theorem [Procaccia & Tennenholtz, '09]:
  The LRM mechanism is strategyproof.
- Proof Sketch:



Max Cost + Randomized

- Exercise!
  Try showing that no randomized SP mechanism can achieve approximation ratio $< 3/2$