

# CSC304 Lecture 10

Mechanism Design w/ Money:  
Sponsored search; Bayesian framework;  
Bayes-Nash equilibria; First price auction

# Announcements

- **Reminder:**

- Assignment 1 is due on Monday, Oct 14 by 3pm
- You can take up to two late days for the assignment
- On Wednesday, Oct 16, one of the TAs will go over assignment solutions in class
  - Assignment solutions will NOT be posted online!
- The first midterm will be on Monday, Oct 21, 3:10-4pm in your assigned tutorial room

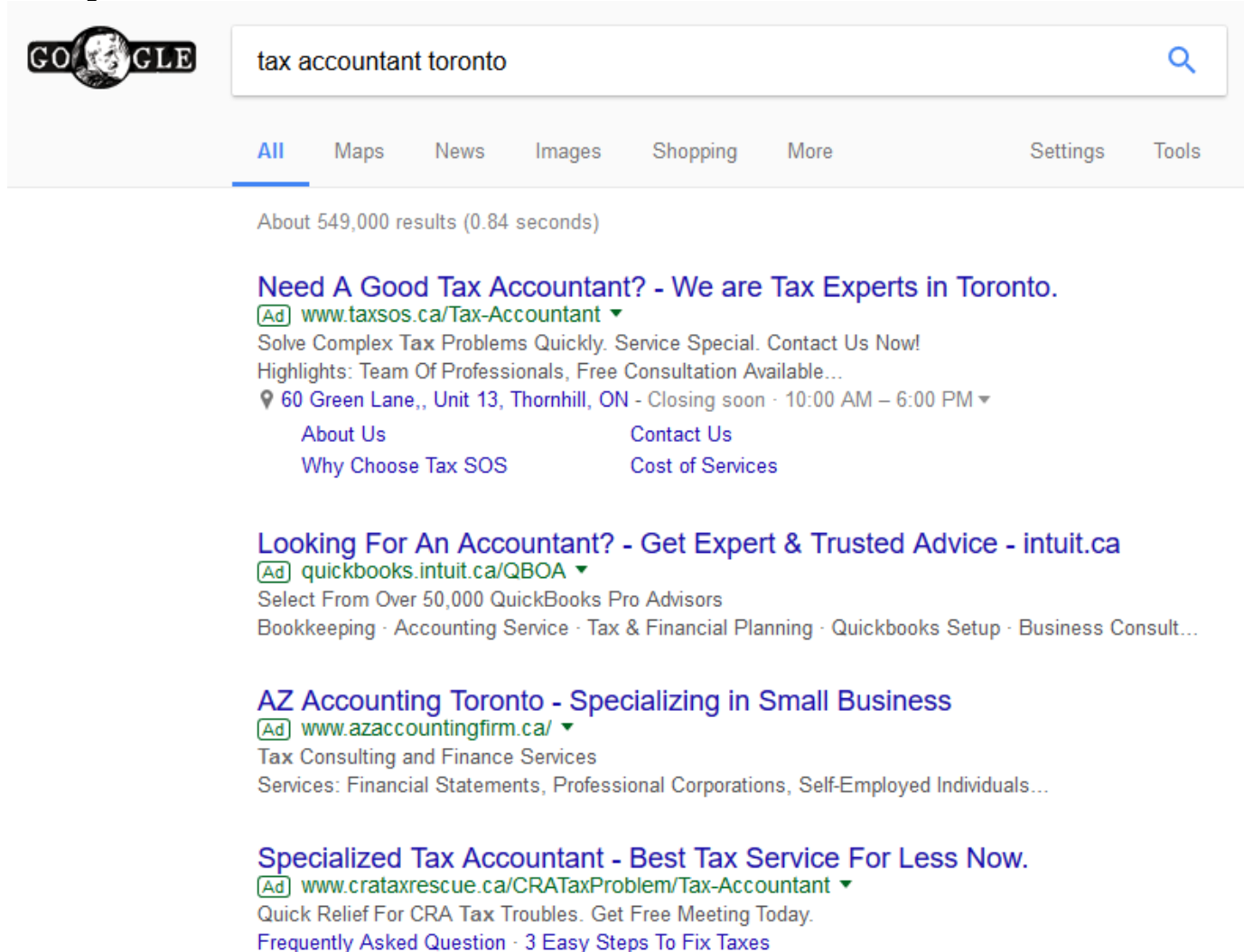
# Recap : VCG

- Maximizes reported welfare
- Charges each agent the apparent reduction in welfare they cause to others due to their presence
- Satisfies four properties
  - Welfare maximization
  - Strategyproofness
  - No payments to agents
  - Individual rationality

# This Lecture: More Auctions

- Sponsored search
- Other auction mechanisms
  - 1<sup>st</sup> price auction and ascending (English) auction
  - Comparison to the 2<sup>nd</sup> price auction
- A different type of incentive guarantee
  - Bayes-Nash Incentive Compatibility

# Sponsored Search Auctions



The image shows a screenshot of a Google search results page. At the top left is the Google logo. The search bar contains the text "tax accountant toronto" and a magnifying glass icon. Below the search bar are navigation links: "All", "Maps", "News", "Images", "Shopping", "More", "Settings", and "Tools". The "All" link is underlined. Below the navigation links, it says "About 549,000 results (0.84 seconds)". There are four sponsored search results listed, each with a blue title, a green "Ad" icon, and a URL. The first result is for "www.taxsos.ca/Tax-Accountant" with a description about solving complex tax problems. The second is for "quickbooks.intuit.ca/QBOA" with a description about selecting from over 50,000 QuickBooks Pro advisors. The third is for "www.azaccountingfirm.ca/" with a description about tax consulting and finance services. The fourth is for "www.cratatxrescue.ca/CRATaxProblem/Tax-Accountant" with a description about quick relief for CRA tax troubles.

GOOGLE

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# Sponsored Search Auctions

- A search engine receives a query
- There are  $k$  advertisement slots
  - “Clickthrough rates” :  $c_1 \geq c_2 \geq \dots \geq c_k \geq c_{k+1} = 0$
- There are  $n$  advertisers (bidders)
  - Bidder  $i$  derives value  $v_i$  **per click**
  - Value to bidder  $i$  for slot  $j = v_i \cdot c_j$
  - Without loss of generality,  $v_1 \geq v_2 \geq \dots \geq v_n$
- **Question:**
  - Who gets which slot, and how much do they pay?

For convenience

# Sponsored Search : VCG

- VCG

- Maximize welfare:

- bidder  $j$  gets slot  $j$  for  $1 \leq j \leq k$ , other bidders get nothing

- Payment of bidder  $j$ ?

- Increase in social welfare to others if  $j$  abstains

- Bidders  $j + 1$  through “ $k + 1$ ” get upgraded by one slot

- Payment of bidder  $j = \sum_{i=j+1}^{k+1} v_i \cdot (c_{i-1} - c_i)$

- Payment of bidder  $j$  **per click** =  $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j}$

# Sponsored Search : VCG

- What if all the clickthrough rates are same?

- $c_1 = c_2 = \dots = c_k > c_{k+1} = 0$

- Payment of bidder  $j$  **per click**

- $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j} = v_{k+1}$

- Bidders 1 through  $k$  pay the value of bidder  $k + 1$

- Familiar? VCG for  $k$  identical items



# Sponsored Search : GSP

- **Generalized Second Price Auction (GSP)**
  - For  $1 \leq j \leq k$ , bidder  $j$  gets slot  $j$  and pays the value of bidder  $j + 1$  **per click**
  - Other bidders get nothing and pay nothing
- **Natural extension of the “second price” idea**
  - We considered this before for two identical slots
  - Not strategyproof
  - In fact, truth-telling may not even be a Nash equilibrium  
☹

# Sponsored Search : GSP

- But there is a **good Nash equilibrium** that...
  - realizes the VCG outcome, i.e., **maximizes welfare**, and
  - generates **as much revenue as VCG** 😊 [Edelman et al. 2007]
- Even the **worst Nash equilibrium**...
  - gives **1.282-approximation to welfare** ( $PoA \leq 1.282$ ) and
  - generates at least **half of the revenue of VCG**  
[Caragiannis et al. 2011, Dutting et al. 2011, Lucier et al. 2012]
- So if the players achieve an equilibrium, things aren't so bad.

# VCG vs GSP

- VCG

- Truthful revelation is a dominant strategy, so there's a higher confidence that players will reveal truthfully and the theoretical welfare/revenue guarantees will hold
- But it is difficult to convey and understand

- GSP

- Need to rely on players reaching a Nash equilibrium
- But has good welfare and revenue guarantees and is easy to convey and understand

- Industry is split on this issue too!

# From Theory to Reality

- **Value is proportional to clickthrough rate?**
  - Could it be that users clicking on the 2<sup>nd</sup> slot are more likely buyers than those clicking on the 1<sup>st</sup> slot?
- **Misaligned values of advertisers and ad engines?**
  - An advertiser having a high value for a slot does not necessarily mean their ad is appropriate for the slot
- **Market competition?**
  - What if there are other ad engines deploying other mechanisms and advertisers are strategic about which ad engines to participate in?

# Bayesian Framework

- Useful for providing weaker incentive guarantees than strategyproofness
- **Strategyproofness:**
  - “It’s best for me to tell the truth even if I know what other players are doing, and regardless of what they are doing.”
- **Weaker guarantee:**
  - “I don’t *exactly* know what others are going to do, but I have some idea. In expectation, it’s best for me to tell the truth.”
  - Incomplete information setting

# Bayesian Framework

- Setup

- Distribution  $D_i$  for each agent  $i$ 
  - All distributions are known to all agents.
- Each agent  $i$ 's valuation  $v_i$  is sampled from  $D_i$ 
  - $v_i$ 's are independent of each other
  - Only agent  $i$  knows  $v_i$
  - Private information of agent = “type” of agent
- $T_i$  = type space for agent  $i$  (support of  $D_i \subseteq T_i$ )
- $A_i$  = set of possible actions/reports/bids of agent  $i$
- Strategy  $s_i: T_i \rightarrow A_i$ 
  - “How do I convert my valuation to my bid?”

# Bayesian Framework

- **Strategy profile**  $\vec{s} = (s_1, \dots, s_n)$

- **Interim/expected utility** of agent  $i$  is

$$E_{\{v_j \sim D_j\}_{j \neq i}} [u_i(s_1(v_1), \dots, s_n(v_n))]$$

where utility  $u_i$  is “value derived – payment charged”

- $\vec{s}$  is a **Bayes-Nash equilibrium (BNE)** if  $s_i$  is the best strategy for agent  $i$  given  $\vec{s}_{-i}$  (strategies of others)
  - NOTE: I don't know what others' values are. But I know they are rational players, so I can reason about what strategies they might use.

# Example

- **Sealed-bid first price auction for a single item**
  - Each agent  $i$  privately submits a bid  $b_i$
  - Agent  $i^*$  with the highest bid wins the item, pays  $b_{i^*}$
- Suppose there are two agents
  - Common prior: each has valuation drawn from  $U[0,1]$
- **Claim:** Both players using  $s_i(v_i) = v_i/2$  is a BNE.
  - **Proof on the board.**