CSC304 Lecture 10

Mechanism Design w/ Money: Sponsored search; Bayesian framework; Bayes-Nash equilibria; First price auction

Announcements

- Reminder:
  - Assignment 1 is due on Monday, Oct 14 by 3pm
  - You can take up to two late days for the assignment
  - On Wednesday, Oct 16, one of the TAs will go over assignment solutions in class
    - Assignment solutions will NOT be posted online!
  - The first midterm will be on Monday, Oct 21, 3:10-4pm in your assigned tutorial room

Recap : VCG

- Maximizes reported welfare
- Charges each agent the apparent reduction in welfare they cause to others due to their presence
- Satisfies four properties
  - Welfare maximization
  - Strategyproofness
  - No payments to agents
  - Individual rationality

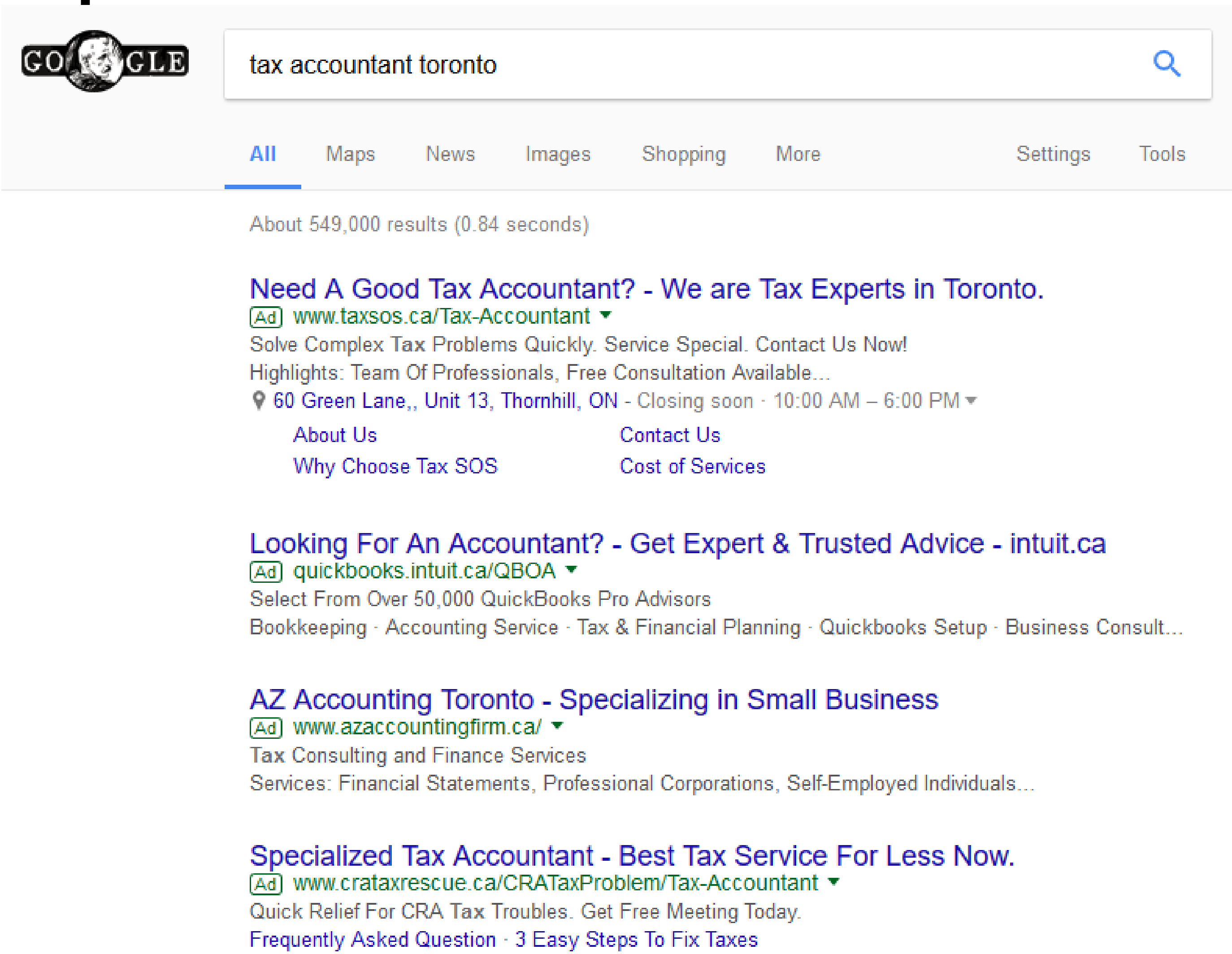
This Lecture: More Auctions

- Sponsored search

- Other auction mechanisms
  - 1st price auction and ascending (English) auction
  - Comparison to the 2nd price auction

- A different type of incentive guarantee
  - Bayes-Nash Incentive Compatibility

Sponsored Search Auctions



Sponsored Search Auctions

- A search engine receives a query
- There are k advertisement slots
  - "Clickthrough rates" : $c_1 \geq c_2 \geq \cdots \geq c_k \geq c_{k+1} = 0$

For convenience

- There are n advertisers (bidders)
  - Bidder i derives value v_i ***per click***
  - Value to bidder i for slot j = $v_i \cdot c_j$
  - Without loss of generality, $v_1 \geq v_2 \geq \cdots \geq v_n$
- Question:
  - Who gets which slot, and how much do they pay?

Sponsored Search : VCG

- VCG
  - Maximize welfare:
    - bidder j gets slot j for $1 \leq j \leq k$, other bidders get nothing
  - Payment of bidder j?

- Increase in social welfare to others if j abstains
  - Bidders $j+1$ through "$k+1$" get upgraded by one slot
  - Payment of bidder j = $\sum_{i=j+1}^{k+1} v_i \cdot (c_{i-1} - c_i)$
  - Payment of bidder j ***per click*** = $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j}$

Sponsored Search : VCG

- What if all the clickthrough rates are same?
  - $c_1 = c_2 = \cdots = c_k > c_{k+1} = 0$
  - Payment of bidder j ***per click***
    - $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1}-c_i}{c_j} = v_{k+1}$
  - Bidders 1 through k pay the value of bidder $k+1$
    - Familiar? VCG for k identical items

Sponsored Search : GSP

- Generalized Second Price Auction (GSP)
  - For $1 \leq j \leq k$, bidder j gets slot j and pays the value of bidder $j+1$ ***per click***
  - Other bidders get nothing and pay nothing

- Natural extension of the “second price” idea
  - We considered this before for two identical slots
  - Not strategyproof
  - In fact, truth-telling may not even be a Nash equilibrium ☹

Sponsored Search : GSP

- But there is a good Nash equilibrium that…
  - realizes the VCG outcome, i.e., maximizes welfare, and
  - generates as much revenue as VCG ☺ [Edelman et al. 2007]

- Even the worst Nash equilibrium…
  - gives 1.282-approximation to welfare ($PoA \leq 1.282$) and
  - generates at least half of the revenue of VCG
    [Caragiannis et al. 2011, Dutting et al. 2011, Lucier et al. 2012]

- So if the players achieve an equilibrium, things aren't so bad.

VCG vs GSP

- VCG
  - Truthful revelation is a dominant strategy, so there's a higher confidence that players will reveal truthfully and the theoretical welfare/revenue guarantees will hold
  - But it is difficult to convey and understand
- GSP
  - Need to rely on players reaching a Nash equilibrium
  - But has good welfare and revenue guarantees and is easy to convey and understand
- Industry is split on this issue too!

From Theory to Reality

- Value is proportional to clickthrough rate?
  - Could it be that users clicking on the 2nd slot are more likely buyers than those clicking on the 1st slot?
- Misaligned values of advertisers and ad engines?
  - An advertiser having a high value for a slot does not necessarily mean their ad is appropriate for the slot
- Market competition?
  - What if there are other ad engines deploying other mechanisms and advertisers are strategic about which ad engines to participate in?

Bayesian Framework

- Useful for providing weaker incentive guarantees than strategyproofness
- Strategyproofness:
  - "It's best for me to tell the truth even if I know what other players are doing, and regardless of what they are doing."
- Weaker guarantee:
  - "I don't *exactly* know what others are going to do, but I have some idea. In expectation, it's best for me to tell the truth."
  - Incomplete information setting

Bayesian Framework

- Setup
  - Distribution D_i for each agent i
    - All distributions are known to all agents.
  - Each agent i's valuation v_i is sampled from D_i
    - v_i's are independent of each other
    - Only agent i knows v_i
    - Private information of agent = "type" of agent
  - T_i = type space for agent i (support of $D_i \subseteq T_i$)
  - A_i = set of possible actions/reports/bids of agent i
  - Strategy $s_i: T_i \rightarrow A_i$
    - "How do I convert my valuation to my bid?"

Bayesian Framework

- Strategy profile $\vec{s} = (s_1, \dots, s_n)$
  - Interim/expected utility of agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}}\left[u_i\big(s_1(v_1), \dots, s_n(v_n)\big)\right]$$

where utility u_i is "value derived – payment charged"

  - $\vec{s}$ is a Bayes-Nash equilibrium (BNE) if s_i is the best strategy for agent i given $\vec{s}_{-i}$ (strategies of others)
    - NOTE: I don't know what others' values are. But I know they are rational players, so I can reason about what strategies they might use.

Example

- Sealed-bid first price auction for a single item
  - Each agent i privately submits a bid b_i
  - Agent i^* with the highest bid wins the item, pays b_{i^*}

- Suppose there are two agents
  - Common prior: each has valuation drawn from $U[0,1]$

- Claim: Both players using $s_i(v_i) = v_i/2$ is a BNE.
  - Proof on the board.