

CSC304 Lecture 9

Mechanism Design with Money:
More VCG examples;
greedy approximation of VCG;
sponsored search

VCG Recap

- $f(\tilde{v}) = a^* = \operatorname{argmax}_{a \in A} \sum_i \tilde{v}_i(a)$
 - Choose the allocation maximizing *reported* welfare
- $p_i(\tilde{v}) = \left[\max_a \sum_{j \neq i} \tilde{v}_j(a) \right] - \left[\sum_{j \neq i} \tilde{v}_j(a^*) \right]$
 - Each agent pays the loss to others due to her presence
- Four properties
 - Strategyproofness
 - Individual rationality (IR)
 - No payments to agents
 - Welfare maximization

Seller as Agent

- Seller (S) wants to sell his car (c) to buyer (B)
- Seller has a value for his own car: $v_S(c)$
 - Individual rationality for the seller mandates that seller must get revenue at least $v_S(c)$
- Idea: Add seller as another agent, and make his values part of the welfare calculations!

Seller as Agent



$$v_S(c) = 3$$

$$v_B(c) = 5$$

- What if...
 - We give the car to buyer when $v_B(c) > v_S(c)$ and
 - Buyer pays seller $v_B(c)$: Not strategyproof for buyer!
 - Buyer pays seller $v_S(c)$: Not strategyproof for seller!

What would VCG do?



$$v_S(c) = 3$$

$$v_B(c) = 5$$

- Allocation?
 - Buyer gets the car (welfare = 5)
- Payment?
 - Buyer pays: $3 - 0 = 3$
 - Seller pays: $0 - 5 = -5$

Mechanism takes \$3 from buyer, and gives \$5 to the seller!

- Need external subsidy

Problems with VCG

- Difficult to understand
 - Need to reason about what welfare maximizing allocation in agent i 's absence
- Does not care about revenue
 - Although we can lower bound its revenue
- With sellers as agents, need subsidy
 - With no subsidy, cannot get the other three properties
- Might be NP-hard to compute

Single-Minded Bidders

- Combinatorial auction for a set of m items S
- Each agent i has two private values (v_i, S_i)
 - $S_i \subseteq S$ is the set of desired items
 - When given a bundle of items A_i , agent has value v_i if $S_i \subseteq A_i$ and 0 otherwise
 - “Single-minded”
- Welfare-maximizing allocation
 - Agent i either gets S_i or nothing
 - Find a subset of players with the highest total value such that their desired sets are **disjoint**

Single-Minded Bidders

- Weighted Independent Set (WIS) problem
 - Given a graph with weights on nodes, find an independent set of nodes with the maximum weight
 - Known to be NP-hard
- Easy to reduce our problem to WIS
 - Not even $O(m^{0.5-\epsilon})$ approximation of welfare unless $NP \subseteq ZPP$
- Luckily, there's a simple, \sqrt{m} -approximation greedy algorithm

Greedy Algorithm

- **Input:** (v_i, S_i) for each agent i
- **Output:** Agents with mutually independent S_i
- **Greedy Algorithm:**
 - Sort the agents in a specific order (we'll see).
 - Relabel them as $1, 2, \dots, n$ in this order.
 - $W \leftarrow \emptyset$
 - For $i = 1, \dots, n$:
 - If $S_i \cap S_j = \emptyset$ for every $j \in W$, then $W \leftarrow W \cup \{i\}$
 - Give agents in W their desired items.

Greedy Algorithm

- Sort by what?
- We want to satisfy agents with higher values.
 - $v_1 \geq v_2 \geq \dots \geq v_n \Rightarrow m$ -approximation ☹️
- But we don't want to exhaust too many items.
 - $\frac{v_1}{|S_1|} \geq \frac{v_2}{|S_2|} \geq \dots \geq \frac{v_n}{|S_n|} \Rightarrow m$ -approximation ☹️
- \sqrt{m} -approximation : $\frac{v_1}{\sqrt{|S_1|}} \geq \frac{v_2}{\sqrt{|S_2|}} \geq \dots \geq \frac{v_n}{\sqrt{|S_n|}}$?

[Lehmann et al. 2011]

Proof of Approximation

- Definitions

- OPT = Agents satisfied by the optimal algorithm
- W = Agents satisfied by the greedy algorithm
- For $i \in W$,

$$OPT_i = \{j \in OPT, j \geq i : S_i \cap S_j \neq \emptyset\}$$

- **Claim 1:** $OPT \subseteq \bigcup_{i \in W} OPT_i$

- **Claim 2:** It is enough to show that $\forall i \in W$

$$\sqrt{m} \cdot v_i \geq \sum_{j \in OPT_i} v_j$$

- **Observation:** For $j \in OPT_i$, $v_j \leq v_i \cdot \frac{\sqrt{|S_j|}}{\sqrt{|S_i|}}$

Proof of Approximation

- Summing over all $j \in OPT_i$:

$$\sum_{j \in OPT_i} v_j \leq \frac{v_i}{\sqrt{|S_i|}} \cdot \sum_{j \in OPT_i} \sqrt{|S_j|}$$

- Using Cauchy-Schwarz ($\sum_i x_i y_i \leq \sqrt{\sum_i x_i^2} \cdot \sqrt{\sum_i y_i^2}$)

$$\begin{aligned} \sum_{j \in OPT_i} \sqrt{|S_j|} \cdot 1 &\leq \sqrt{|OPT_i|} \cdot \sqrt{\sum_{j \in OPT_i} |S_j|} \\ &\leq \sqrt{|S_i|} \cdot \sqrt{m} \end{aligned}$$

Strategyproofness

- Agent i pays $p_i = v_{j^*} \cdot \sqrt{\frac{|S_i|}{|S_{j^*}|}}$
 - j^* is the smallest index $j > i$ such that $S_j \cap S_i \neq \emptyset$ and $S_j \cap S_k = \emptyset$ for all $k < j, k \neq i$
- How do I interpret j^* and p_i ?
 - j^* = agent such that if agent i reports a value \tilde{v}_i low enough to fall below j^* in the ordering, she stops winning. Otherwise, she wins.
 - p_i = lowest value i can report and still win

Strategyproofness

- **Critical payment**

- Charge each agent the lowest value they can report and still win

- **Monotonic allocation**

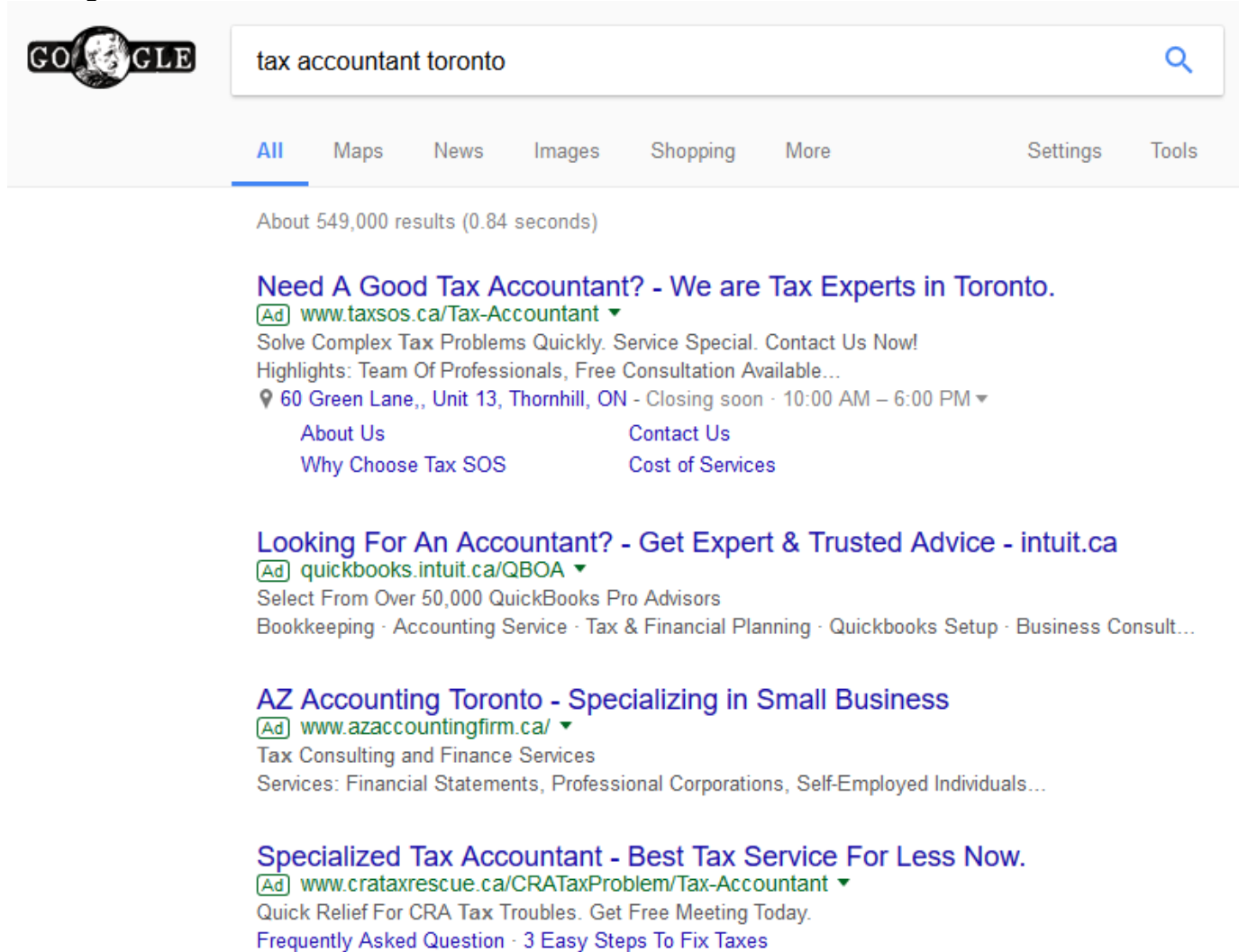
- If agent i wins when reporting (v_i, S_i) , she must win when reporting $v'_i \geq v_i$ and $S'_i \subseteq S_i$.
- Greedy allocation rule satisfies this.

- **Theorem:** Critical payment + monotonic allocation rule imply strategyproofness.

Moral

- VCG can sometimes be too difficult to implement
 - May look into approximately maximizing welfare
 - As long as the allocation rule is monotone, we can charge critical payments to achieve strategyproofness
 - Note: approximation is needed for computational reasons
- Later in mechanism design without money...
 - We will not be able to use payments to achieve strategyproofness
 - Hence, we will need to approximate welfare just to get strategyproofness, even without any computational restrictions

Sponsored Search Auctions



The image shows a screenshot of a Google search results page. At the top left is the Google logo. The search bar contains the text "tax accountant toronto" and a magnifying glass icon. Below the search bar are navigation tabs: "All" (underlined), "Maps", "News", "Images", "Shopping", "More", "Settings", and "Tools". The search results show "About 549,000 results (0.84 seconds)". There are four sponsored search results listed, each with a blue title, a green "Ad" label, and a URL. The first result is for "www.taxesos.ca/Tax-Accountant" with a location pin and hours. The second is for "quickbooks.intuit.ca/QBOA". The third is for "www.azaccountingfirm.ca/". The fourth is for "www.crataxrescue.ca/CRATaxProblem/Tax-Accountant".

GOOGLE

tax accountant toronto

All Maps News Images Shopping More Settings Tools

About 549,000 results (0.84 seconds)

Need A Good Tax Accountant? - We are Tax Experts in Toronto.
Ad www.taxesos.ca/Tax-Accountant ▼
Solve Complex Tax Problems Quickly. Service Special. Contact Us Now!
Highlights: Team Of Professionals, Free Consultation Available...
📍 60 Green Lane,, Unit 13, Thornhill, ON - Closing soon · 10:00 AM – 6:00 PM ▼
About Us Contact Us
Why Choose Tax SOS Cost of Services

Looking For An Accountant? - Get Expert & Trusted Advice - intuit.ca
Ad quickbooks.intuit.ca/QBOA ▼
Select From Over 50,000 QuickBooks Pro Advisors
Bookkeeping · Accounting Service · Tax & Financial Planning · Quickbooks Setup · Business Consult...

AZ Accounting Toronto - Specializing in Small Business
Ad www.azaccountingfirm.ca/ ▼
Tax Consulting and Finance Services
Services: Financial Statements, Professional Corporations, Self-Employed Individuals...

Specialized Tax Accountant - Best Tax Service For Less Now.
Ad www.crataxrescue.ca/CRATaxProblem/Tax-Accountant ▼
Quick Relief For CRA Tax Troubles. Get Free Meeting Today.
Frequently Asked Question · 3 Easy Steps To Fix Taxes

Sponsored Search Auctions

- A search engine receives a query
- There are k advertisement slots
 - “Clickthrough rates” : $c_1 \geq c_2 \geq \dots \geq c_k \geq c_{k+1} = 0$
- There are n advertisers (bidders)
 - Bidder i derives value v_i **per click**
 - Value to bidder i for slot $j = v_i \cdot c_j$
 - Without loss of generality, $v_1 \geq v_2 \geq \dots \geq v_n$
- **Question:**
 - Who gets which slot, and how much do they pay?

For convenience

Sponsored Search : VCG

- VCG

- Maximize welfare:

- bidder j gets slot j for $1 \leq j \leq k$, other bidders get nothing

- Payment of bidder j ?

- Increase in social welfare to others if j abstains

- Bidders $j + 1$ through “ $k + 1$ ” get upgraded by one slot

- Payment of bidder $j = \sum_{i=j+1}^{k+1} v_i \cdot (c_{i-1} - c_i)$

- Payment of bidder j **per click** = $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j}$

Sponsored Search : VCG

- What if all the clickthrough rates are same?


- $c_1 = c_2 = \dots = c_k > c_{k+1} = 0$

- Payment of bidder j **per click**

- $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j} = v_{k+1}$

- Bidders 1 through k pay the value of bidder $k + 1$
 - Familiar? VCG for k identical items

Sponsored Search : GSP

- Generalized Second Price Auction (GSP)
 - For $1 \leq j \leq k$, bidder j gets slot j and pays the value of bidder $j + 1$ per click
 - Other bidders get nothing and pay nothing
- Natural extension of the “second price” idea
 - We considered this before for two identical slots
 - Not strategyproof
 - In fact, truth-telling may not even be a Nash equilibrium


Sponsored Search : GSP

- But there is a **good Nash equilibrium** that...
 - realizes the VCG outcome, i.e., **maximizes welfare**, and
 - generates **as much revenue as VCG** 😊 [Edelman et al. 2007]
- Even the **worst Nash equilibrium**...
 - gives **1.282-approximation to welfare** ($PoA \leq 1.282$) and
 - generates at least **half of the revenue of VCG**
[Caragiannis et al. 2011, Dutting et al. 2011, Lucier et al. 2012]
- So if the players achieve an equilibrium, things aren't so bad.

VCG vs GSP

- VCG
 - Truthful revelation is a dominant strategy, so there's a higher confidence that players will reveal truthfully and the theoretical welfare/revenue guarantees will hold
 - But it is difficult to convey and understand
- GSP
 - Need to rely on players reaching a Nash equilibrium
 - But has good welfare and revenue guarantees and is easy to convey and understand
- Industry is split on this issue too!

From Theory to Reality

- Value is proportional to clickthrough rate?
 - Could it be that users clicking on the 2nd slot are more likely buyers than those clicking on the 1st slot?
- Misaligned values of advertisers and ad engines?
 - An advertiser having a high value for a slot does not necessarily mean their ad is appropriate for the slot
- Market competition?
 - What if there are other ad engines deploying other mechanisms and advertisers are strategic about which ad engines to participate in?