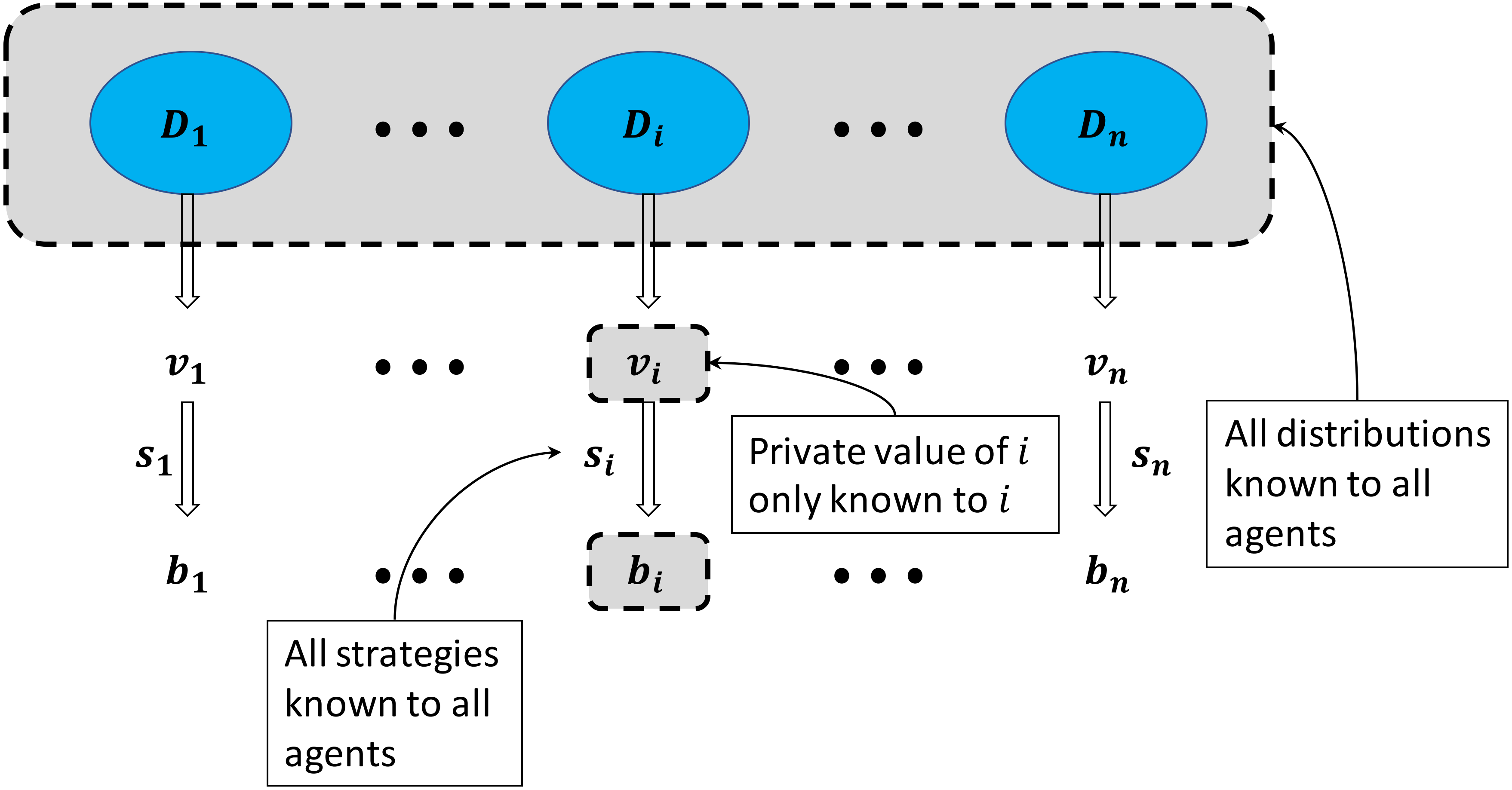
CSC304 Lecture 11

Mechanism Design w/ Money: Continued…Revelation principle; First price, second price, and ascending auctions; Revenue equivalence

Recap: Bayesian Framework



Recap: Bayesian Framework

- Strategy profile $\vec{s} = (s_1, \dots, s_n)$
  - Interim utility of agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}} \left[u_i\big(s_1(v_1), \dots, s_n(v_n)\big) \right]$$

where utility u_i is "value derived – payment charged"

  - $\vec{s}$ is a Bayes-Nash equilibrium (BNE) if s_i is the best strategy for agent i given $\vec{s}_{-i}$ (strategies of others)
    - NOTE: I don't know what others' values are. But I know they are rational players, so I can reason about what strategies they might use.

Recap: 1^{st} Price Auction

- Sealed-bid first price auction for a single item
  - Each agent i privately submits a bid b_i
  - Agent i^* with the highest bid wins the item, pays b_{i^*}

- Suppose there are two agents
  - Common prior: each has valuation drawn from $U[0,1]$

- Claim: Both players using $s_i(v_i) = v_i/2$ is a BNE.
  - Proof on the board.

Direct Revelation Mechanisms
&
The Revelation Principle

Direct Revelation

- Direct-revelation: mechanisms that ask you to report your private values
  - Doesn't mean agents will report their true values.
  - Makes sense to ask "Would they, in equilibrium?"

- Non-direct-revelation: different action space than type space
  - Suppose your value for an item is in [0,1], but the mechanism asks you to either dive left or dive right.
  - Strategy $s_i : [0,1] \to \{left, right\}$
  - Truthfulness doesn't make much sense.
  - But we can still ask: What is the outcome in equilibrium?

BNIC Mechanisms

- A direct revelation mechanism is Bayes-Nash incentive compatible (BNIC) if all players playing $s_i(v_i) = v_i$ is a BNE.
  - I don't know what other's valuations are, only the distributions they're drawn from.
  - But as long as they report their true values, in expectation I would like to report my true value.

- Compare to strategyproofness
  - I know what others' values are, and for every possible values they can have, I want to report my true values.

Revelation Principle

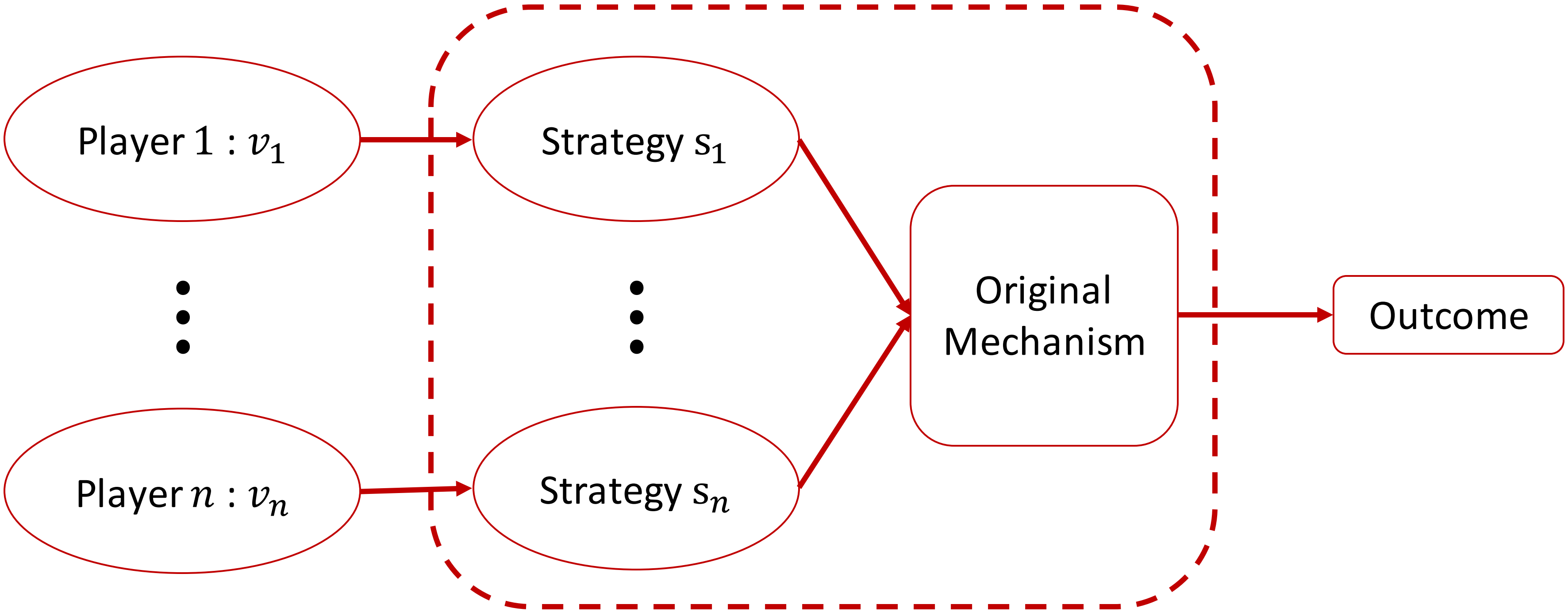
- Outcome = (allocation, payments)

- Strategyproof version [Gibbard, ‘73]
  - If a mechanism implements an outcome in dominant strategies, there’s a direct revelation strategyproof mechanism implementing the same outcome.

- BNIC version [Dasgupta et al. ‘79, Holmstrom ‘77, Myerson ’79]
  - If a mechanism implements an outcome as BNE, there’s a direct revelation BNIC mechanism implementing the same outcome.

Revelation Principle

- Informal proof:



New direct revelation truthful mechanism

Applying Revelation Principle

- We already saw…
  - Sealed-bid 1st price auction
  - 2 agents with valuations drawn from $U[0,1]$
  - Each player halving his value was a BNE
  - Not naturally BNIC (players don't report value)

- BNIC variant through revelation principle?

- Can also be used on non-direct-revelation mechs

Revenue of Auction Mechanisms
&
Revenue Equivalence

1^{st} Price Auction

- For n players with iid valuations from U[0,1], "shadowing" the bid by a factor of $(n-1)/n$ is a BNE

- E[Revenue] to the auctioneer?
  - $E_{\{v_i \sim U[0,1]\}_{i=1}^n} \left(\frac{n-1}{n}\right) * \max_i v_i = \frac{n-1}{n+1}$ (Exercise!)

- Interestingly, this is equal to E[Revenue] from 2nd price auction
  - $E_{\{v_i \sim U[0,1]\}_{i=1}^n} [2^{\text{nd}} \text{ highest } v_i] = \frac{n-1}{n+1}$ (Exercise!)

Revenue Equivalence

- If two BNIC mechanisms A and B:
  1. Always produce the same allocation;
  2. Have the same expected payment to agent i for some type v_i^0 (e.g., "zero value for all" → zero payment);
  3. Have agent valuations drawn from distributions with "path-connected support sets";
- Then they:
  - Charge the same expected payment to all agent types;
  - Have the same expected total revenue.

Revenue Equivalence

- Informally…
  - If two BNIC mechanisms always have the same allocation, then they have the same E[payments] and E[revenue].
  - Very powerful as it applies to any pair of BNIC mechanism

- 1^{st} price (BNIC variant) and 2^{nd} price auctions
  - Have the same allocation:
    Item always goes to the agent with the highest valuation
  - Thus, also have the same revenue

Non-Direct-Revelation Auctions

- Ascending auction (a.k.a. English auction)
  - All agents + auctioneer meet in a room.
  - Auctioneer starts the price at 0.
  - All agents want the item, and have their hands raised.
  - Auctioneer raise the price continuously.
  - Agents drop out when price > value for them

- Descending auction (a.k.a. Dutch auction)
  - Start price at a very high value.
  - Keep decreasing the price until some agent agrees to buy.

Revenue Maximization

Welfare vs Revenue

- In welfare maximization, we want to maximize $\sum_i v_i(a)$
  - VCG = strategyproof + maximizes welfare on every single instance
  - Beautiful!

- In revenue maximization, we want to maximize $\sum_i p_i$
  - We can still use strategyproof mechanisms (revelation principle).
  - BUT…

Welfare vs Revenue

- Different strategyproof mechanisms are better for different instances.

- Example: 1 item, 1 bidder, unknown value v
  - strategyproof = fix a price r, let the agent decide to "take it" ($v \geq r$) or "leave it" ($v < r$)
  - Maximize welfare → set $r = 0$
    - Must allocate item as long as the agent has a positive value
  - Maximize revenue → $r = ?$
    - Different r are better for different v

Welfare vs Revenue

- We cannot optimize revenue on every instance
  - Need to optimize the *expected* revenue in the Bayesian framework

- If we want to achieve higher revenue than VCG, we cannot always allocate the item
  - Revenue equivalence principle!

Single Item + Single Bidder

- Value v is drawn from distribution with CDF F
- Goal: post the optimal price r on the item

- Revenue from price $r = r \cdot \left(1 - F(r)\right)$ (Why?)

- Optimal $r^* = \text{argmax}_r\ r \cdot \left(1 - F(r)\right)$
  - "Monopoly price"
  - Note: r^* depends on F, but not on v, so the mechanism is strategyproof.

Example

- Suppose F is the CDF of the uniform distribution over $[0,1]$ (denote by $U[0,1]$).
  - CDF is given by $F(x) = x$ for all $x \in [0,1]$.
- Recall: E[Revenue] from price r is $r \cdot \left(1 - F(r)\right)$
  - Q: What is the optimal posted price?
  - Q: What is the corresponding optimal revenue?
- Compare this to the revenue of VCG, which is 0
  - This is because if the value is less than r^*, we are willing to risk not selling the item.

Single Item + Two Bidders

- $v_1, v_2 \sim U[0,1]$
- VCG revenue = 2nd highest bid = $\min(v_1, v_2)$
  - $E[\min(v_1, v_2)] = 1/3$ (Exercise!)
- A possible improvement: "VCG with reserve price"
  - Reserve price r.
  - Highest bidder gets the item if bid more than r
  - Pays $\max(r, 2^{\text{nd}} \text{ highest bid})$
    - "Critical payment" : Pay the least value you could have bid and still won the item

Single Item + Two Bidders

- Reserve prices are ubiquitous
  - E.g., opening bids in eBay auctions
  - Guarantee a certain revenue to the auctioneer if item is sold
- $E[\text{revenue}] = E[\max(r, \min(v_1, v_2))]$
  - Maximize over r? Hard to think about.
- Can a strategyproof mechanism that is not VCG + reserve price get a higher revenue?
  - Can a mechanism that is only BNIC get an even higher revenue?

The next 4 slides are not part of the syllabus.

The Trio

This slide is not in syllabus.

- 2^{nd} price auction
  - Sealed-bid + truthful for agents
- 1^{st} price auction
  - Sealed-bid
- Ascending auction
  - "truthful" for agents

Seems strictly better.

Truthful for agents.

Truthful for auctioneer?

Credible Mechanisms

This slide is not in syllabus.

- Typical mechanism design
  - Auctioneer commits to using a mechanism.
  - Assume that auctioneer does not deviate later on.
  - "Stackelberg game between auctioneer and agents"

- Credible Mechanisms [Akbarpour and Li, 2017]
  - Auctioneer is incentivized to not deviate from his commitment at any stage of auction execution.

Credible Mechanisms

This slide is not in syllabus.

- Sealed-bid 2nd Price Auction
  - Auctioneer collects all bids.
  - Auctioneer goes to highest bidder (bid b).
  - Auctioneer says 2nd highest bid was $b - \epsilon$.
  - Highest bidder can't prove him wrong.
  - Auctioneer has an incentive to lie → not credible!

- 1st price auction → credible (Why?)
- Ascending auction → credible (Why?)

Credible Mechanisms

This slide is not in syllabus.

[Akbarpour and Li, 2017]

- Corollary: sealed-bid ∩ DSIC ∩ credible = ∅