CSC304 Lecture 10

Mechanism Design w/ Money: Revelation principle; First price, second price, and ascending auctions; Revenue equivalence

Announcements

- First midterm is this Friday

- We'll spend the first 30 minutes (hopefully not more) today quickly going over assignment 1 solutions

    - Questions? ⇒ Office hours tomorrow

Recap : VCG

- Maximizes reported welfare
- Charges each agent the apparent reduction in welfare they cause to others due to their presence
- Satisfies four properties
  - Welfare maximization
  - Strategyproofness
  - No payments to agents
  - Individual rationality

This Lecture: More Auctions

- Other auction mechanisms
  - 1st price auction and ascending (English) auction
  - Comparison to the 2nd price auction
- A different type of incentive guarantee
  - Bayes-Nash Incentive Compatibility
- Strong results
  - Revelation principle
  - Revenue equivalence theorem

Bayesian Framework

- Useful for providing weaker incentive guarantees than strategyproofness
- Strategyproofness:
  - “It’s best for me to tell the truth even if I know what other players are doing, and regardless of what they are doing.”
- Weaker guarantee:
  - “I don’t *exactly* know what others are going to do, but I have some idea. In expectation, it’s best for me to tell the truth.”
  - Incomplete information setting

Bayesian Framework

- Setup
  - Distribution D_i for each agent i
    - All distributions are known to all agents.
  - Each agent i's valuation v_i is sampled from D_i
    - v_i's are independent of each other
    - Only agent i knows v_i
    - Private information of agent = "type" of agent
  - T_i = type space for agent i
  - A_i = set of actions (possible reports) of agent i
  - Strategy $s_i: T_i \to A_i$
    - "How do I convert my valuation to my bid?"

Bayesian Framework

- Strategy profile $\vec{s} = (s_1, \dots, s_n)$
  - ➢ Interim utility of agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}} \left[u_i \left(s_1(v_1), \dots, s_n(v_n) \right) \right]$$

where utility u_i is "value derived – payment charged"

  - ➢ $\vec{s}$ is a Bayes-Nash equilibrium (BNE) if s_i is the best strategy for agent i given $\vec{s}_{-i}$ (strategies of others)
    - NOTE: I don't know what others' values are. But I know they are rational players, so I can reason about what strategies they might use.

Example

- Sealed-bid first price auction for a single item
  - Each agent i privately submits a bid b_i
  - Agent i^* with the highest bid wins the item, pays b_{i^*}

- Suppose there are two agents
  - Common prior: each has valuation drawn from $U[0,1]$

- Claim: Both players using $s_i(v_i) = v_i/2$ is a BNE.
  - Proof on the board.