CSC304 Lecture 9

Mechanism Design w/ Money: More examples of VCG, winner determination and truthful approximation

VCG Recap

- $f(v) = a^* = \text{argmax}_{a \in A} \sum_i v_i(a)$
- $p_i(v) = \left[\max_a \sum_{j \neq i} v_j(a)\right] - \left[\sum_{j \neq i} v_j(a^*)\right]$

- Procedure
  - Step 1: Choose the allocation to maximize social welfare
  - Step 2: Payment charged to each agent i is the externality that i imposes on others
    - [Max welfare of others | i absent] – [welfare of others | i present]

Under a^*

VCG Recap

- Four properties
  - Maximize social welfare
  - Dominant strategy incentive compatibility (DSIC)
  - No payments to agents
  - Individual rationality (IR)

- Vickrey auction satisfies the first two
- VCG adds Clarke’s pivot rule to satisfy all four

VCG Example

- In the last lecture, we saw…
  - Additive valuations: agent has value $v_i(\{a\})$ for each a, $v_i(S) = \sum_{a \in S} v_i(\{a\})$
  - Unit-demand valuations: Still have $v_i(\{a\})$ for each a, $v_i(S) = \max_{a \in S} v_i(\{a\})$
    - Goods are "substitutes"
- Another example…
  - Complementary goods: value of the whole exceeds the sum of values of its parts

VCG Example

- A chair (c) and a table (t)

$v_1(c) = 3$

$v_2(t) = 4$

$v_3(\{c, t\}) = 6$

- Allocation?
- Payment?

VCG Example

- A chair (c) and a table (t)

$v_1(c) = 3$

$v_2(t) = 4$

$v_3(\{c, t\}) = 8$

- Allocation?
- Payment?

VCG Example: Seller as Agent

- Seller (S) wants to sell his car (c) to buyer (B)

- Seller has a value for his own car: $v_S(c)$
  - Individual rationality for the seller mandates that seller must get revenue at least $v_S(c)$

- Idea: Add seller as another agent, and make his values part of the welfare calculations!

VCG Example: Seller as Agent

$v_S(c) = 3$ $v_B(c) = 5$

- What if…
  - We give the car to buyer when $v_B(c) > v_S(c)$ and
  - Buyer pays seller $v_B(c)$: Not DSIC for buyer!
  - Buyer pays seller $v_S(c)$: Not DSIC for seller!

VCG Example: Seller as Agent

$v_S(c) = 3$

$v_B(c) = 5$

- Allocation?
  - Buyer gets the car (welfare = 5)
- Payment?
  - Buyer pays: $3 - 0 = 3$
  - Seller pays: $0 - 5 = -5$

Mechanism takes $3 from buyer, and gives $5 to the seller!

- Need external subsidy

Problems with VCG

- Difficult to understand in complex settings
  - Need to reason about what allocation would maximize welfare if agent i was absent
- Only cares about welfare, not revenue
  - Though, as we will see in a few lectures, gets pretty good revenue
- With sellers and buyers, need external subsidy
  - Actually, cannot get individual rationality, DSIC, no subsidy, and constant approximation of welfare
- Might be computationally difficult to implement
  - Computing welfare maximizing allocation may be hard

Single-Minded Bidders

- Combinatorial auction for a set of m items S

- Each agent i has
  - Value v_i if receives a subset $S_i \subseteq S$
  - Value 0 if doesn't get a superset of S_i
  - "Single-minded"

- Welfare-maximizing allocation:
  - Find a subset of players i with the highest total value such that their sets S_i are disjoint

Single-Minded Bidders

- Reduction to the Weighted Independent Set (WIS) problem in a graph
  - NP-hard to find the welfare-maximizing allocation
  - Note: not even thinking about computing payments yet
  - In fact, hard to approximately optimize welfare
    - No $O(m^{\frac{1}{2}-\epsilon})$ approximation (unless $NP \subseteq ZPP$)

- Luckily, a simple greedy algorithm gives $\sqrt{m}$-approximation (i.e., OPT/GREEDY $\leq \sqrt{m}$)

Greedy Algorithm

- Input: (v_i, S_i) for each agent i
- Output: Agents with mutually independent S_i
- Greedy Algorithm:
  - Sort the agents. Go over them one-by-one. Accept each bid if no requested item is previously allocated.
- Sort by what?
  - $v_1 \geq v_2 \geq \cdots \geq v_n$? m-approximation
  - $\frac{v_1}{|S_1|} \geq \frac{v_2}{|S_2|} \geq \cdots \frac{v_n}{|S_n|}$? m-approximation
  - $\frac{v_1}{\sqrt{|S_1|}} \geq \frac{v_2}{\sqrt{|S_2|}} \geq \cdots \frac{v_n}{\sqrt{|S_n|}}$? $\sqrt{m}$-approximation [Lehmann et al. 2011]

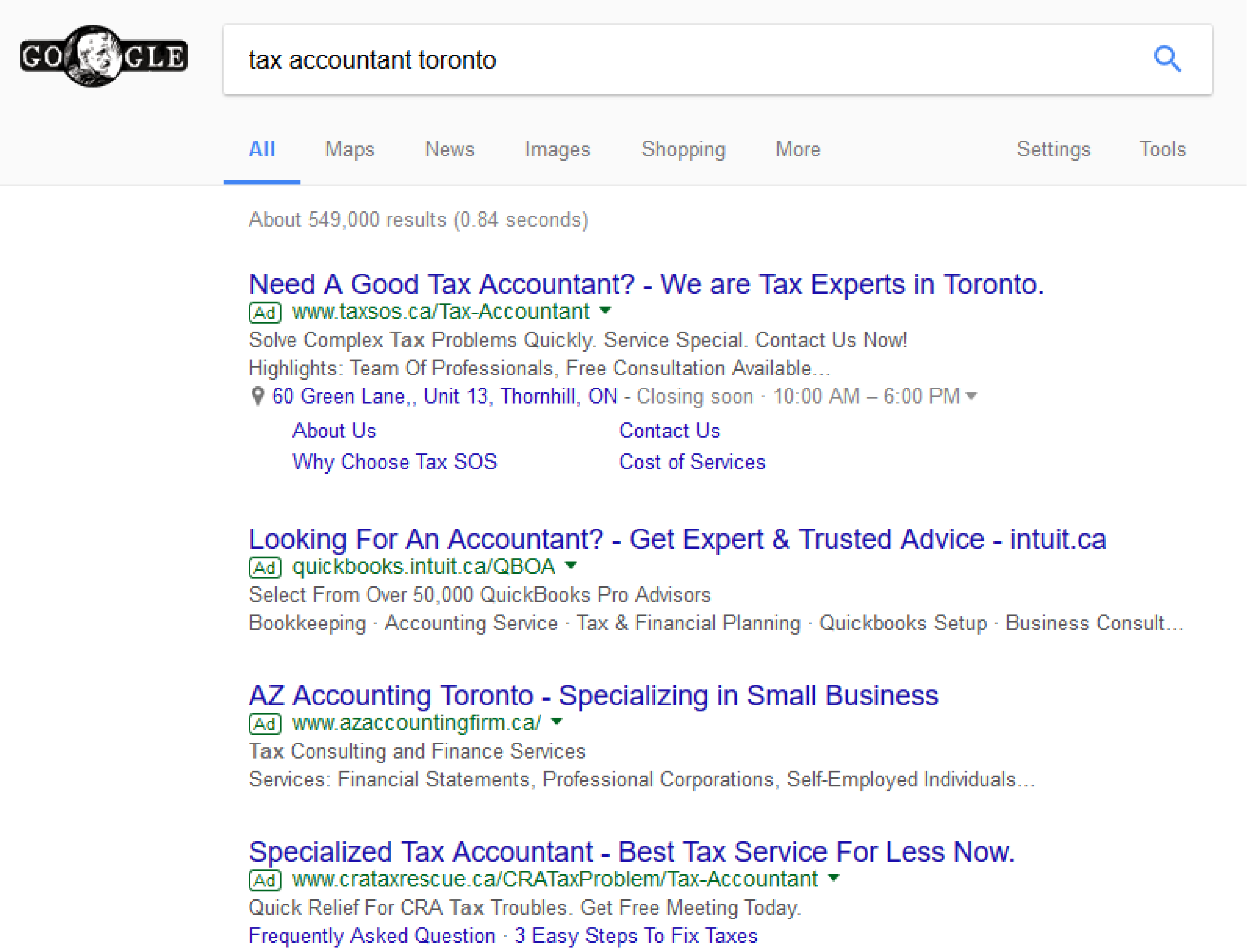
Greedy Algorithm

- (allocation rule, payments) truthful if and only if
  - Allocation is monotonic: If agent i wins with (v_i, S_i), it must win with (v_i', S_i') where $v_i' \geq v_i$ and $S_i' \subseteq S_i$
  - Payments are critical prices: Agent i pays the least value (s)he could have reported and still won.
- $p_i = v_{j^*} \cdot \sqrt{\frac{|S_i|}{|S_{j^*}|}}$
  - j^* is the smallest index j such that $S_j \cap S_i \neq \emptyset$ and $S_j \cap S_k = \emptyset$ for all $k < j, k \neq i$
  - If agent i reports less than this value, agent j gets S_j first, and i loses.

Moral

- VCG can sometimes be too difficult to implement
  - May look into approximately maximizing welfare
  - Can set the payments right if the allocation rule is monotone
- Need for approximation is due to computational considerations
- Later in mechanism design without money…
  - Can't use payments to ensure truthfulness
  - Will need to approximate welfare just to get truthfulness, even without computational considerations

Sponsored Search Auctions



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Sponsored Search Auctions

- Suppose the search engine receives a search query
- k advertisement slots
  - "Clickthrough rates" : $c_1 \geq c_2 \geq \cdots \geq c_k \geq c_{k+1} = 0$

For convenience

- n advertisers (bidders)
  - Bidder i derives value v_i *per click*
  - Final value to bidder i for receiving slot j = $v_i \cdot c_j$
  - Without loss of generality, $v_1 \geq v_2 \geq \cdots \geq v_n$
- Age-old question:
  - Who gets which slot, and how much should they pay?

Sponsored Search : VCG

- VCG
  - Maximize welfare: j^{th} bidder gets j^{th} slot $(1 \leq j \leq k)$
  - Payment of j^{th} bidder?

- Increase in social welfare to others if j abstains
  - Bidders $j+1$ through $k+1$ get "upgraded" by one slot
  - Payment of bidder j = $\sum_{i=j+1}^{k+1} v_i \cdot (c_{i-1} - c_i)$
  - Payment to bidder j "per click" = $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1}-c_i}{c_j}$
  - Not very intuitive…

Sponsored Search : VCG

- What happens if all clickthrough rates are same?
  - $c_1 = c_2 = \cdots = c_k > c_{k+1} = 0$

- Payment of bidder j per click
  - $\sum_{i=j+1}^{k+1} v_i \cdot \frac{c_{i-1} - c_i}{c_j} = v_{k+1}$

- Bidders 1 through k pay the value of bidder $k+1$
  - Familiar? VCG for k identical items

Sponsored Search : GSP

- Generalized Second Price Auction (GSP)
  - For $1 \leq j \leq k$
  - Bidder j gets slot j
  - Bidder j pays the bid of bidder $j + 1$

- A natural extension of the second price auction
  - We already saw that this is not truthful even with two identical slots
  - Highest bidder paying 2nd highest bid → wants to lower bid to become 2nd highest bidder and pay 3rd highest bid

Sponsored Search : GSP

- Truth-telling is not a Nash equilibrium ☹
- But there is a good Nash equilibrium that realizes the VCG outcome, i.e., maximizes welfare and generates as much revenue as VCG ☺
  [Edelman et al. 2007]
- Even the worst Nash equilibrium gives 1.282-approximation to welfare ($PoA \leq 1.282$) and generates at least half the revenue of VCG
  [Caragiannis et al. 2011, Dutting et al. 2011, Lucier et al. 2012]

VCG vs GSP

- VCG
  - Truthful in dominant strategy → more confidence that players will bid truthfully
  - Theoretical welfare/revenue guarantees will hold
  - Though players might still misreport…
  - Difficult to understand
- GSP
  - Need to rely on players reaching a Nash equilibrium
  - Good welfare and revenue
  - Easy to understand

VCG vs GSP

- Google uses GSP

- Facebook used GSP, but switched to VCG
  - They argue that maximizing welfare has two benefits
  - Advertisers are happy → attract more advertisers → more long-term revenue
  - Users are happy (?!) → users use FB more → more slots to sell → more long-term revenue

- No consensus

Sponsored Search Reality

- Value is proportional to clickthrough rate
  - Could it be that users clicking on the 2nd slot are more likely buyers than those clicking on the 1st slot?
- Ad engines also want to produce quality results
  - An advertiser having a high value for a slot does not necessarily mean his ad is appropriate for the slot
- Theoretical analysis does not take into account market competition
  - Advertiser divide their budget among ad engines