CSC304 Lecture 10

Mechanism Design w/ Money: Revelation principle; First price, second price, and ascending auctions; Revenue equivalence

Announcements

- Homework/midterm solutions will NOT be uploaded online
- Will instead dedicate the first 30 minutes of Friday's office hour for going over them
  - Should attend this if you have questions about homework/midterm instead of asking independently or on Piazza
- Hope to give graded test back on Wed
  - Homework sometime later (?)

Recap

- Direct revelation truthful mechanisms

- VCG
  - $f(v) = a^* = \text{argmax}_{a \in A} \sum_i v_i(a)$
  - $p_i(v) = \left[\max_a \sum_{j \neq i} v_j(a)\right] - \left[\sum_{j \neq i} v_j(a^*)\right]$

- Dominant strategy incentive compatible (DSIC)

This Lecture

- Beyond direct revelation
  - 1st price auction and ascending (English) auction
  - Comparing with 2nd price auction

- Bayes-Nash Incentive Compatibility
- Revelation principle
- Revenue equivalence theorem

- A note on “credible” mechanisms

Bayesian Framework

- Needed for mechanisms that are not incentive compatible in *dominant strategies*
- For such mechanisms, we need to reason about how each agent thinks the other agents would act
- Agents have incomplete information about valuations of other agents
  - Know the distributions from which others' valuations are drawn, but don't know their exact valuations

Bayesian Framework

- Common prior assumption
  - All agents agree about which distribution agent i's valuation is drawn from
  - Not entirely convincing, but a very useful assumption

- In this lecture, we will assume the valuations are independently drawn from their own distributions

Bayesian Framework

- Setup
  - Distribution D_i for each agent i
  - All agents know all distributions, agent i additionally knows his privately drawn valuation $v_i \sim D_i$
  - Private information of agent = "type" of agent
  - T_i be the type space for agent i
  - A_i be the action space (possible reports/bids) for agent i
  - Strategy s_i for agent i is a function from T_i to A_i
    - "How will I convert my valuation to my bid?"

Bayesian Framework

- Strategy profile $\vec{s} = (s_1, \dots, s_n)$
  - Interim utility of agent i is

$$E_{\{v_j \sim D_j\}_{j \neq i}}\left[u_i\left(s_1(v_1), \dots, s_n(v_n)\right)\right]$$

where utility u_i is "value derived – payment charged"

  - $\vec{s}$ is a Bayes-Nash equilibrium (BNE) if s_i is the best strategy for agent i *given* $\vec{s}_{-i}$ (strategies of others)
    - "Given others' strategies, and in expectation over their types/valuations, I'm doing the best I can"

Example

- Sealed-bid first price auction for a single item
  - Each agent i privately submits a bid b_i
  - Agent i^* with the highest bid wins the item, pays b_{i^*}

- Suppose there are two agents
  - Common prior: each has valuation drawn from $U[0,1]$

- Claim: Both players using $s_i(v_i) = v_i/2$ is a BNE.
  - Proof on the board.

BNIC

- A direct revelation mechanism is Bayes-Nash incentive compatible (BNIC) if all players playing $s_i(v_i) = v_i$ is a BNE.
  - I don't know what other's valuations are, only the distributions they're drawn from.
  - I know what strategies they're using (valuation → bid).
  - In expectation, I don't lose when reporting truthfully.

- Compare to DSIC
  - I don't care what others' valuations are.
  - I don't care what strategies they're using (valuation → bid)
  - I never lose when reporting truthfully.

Revelation Principle

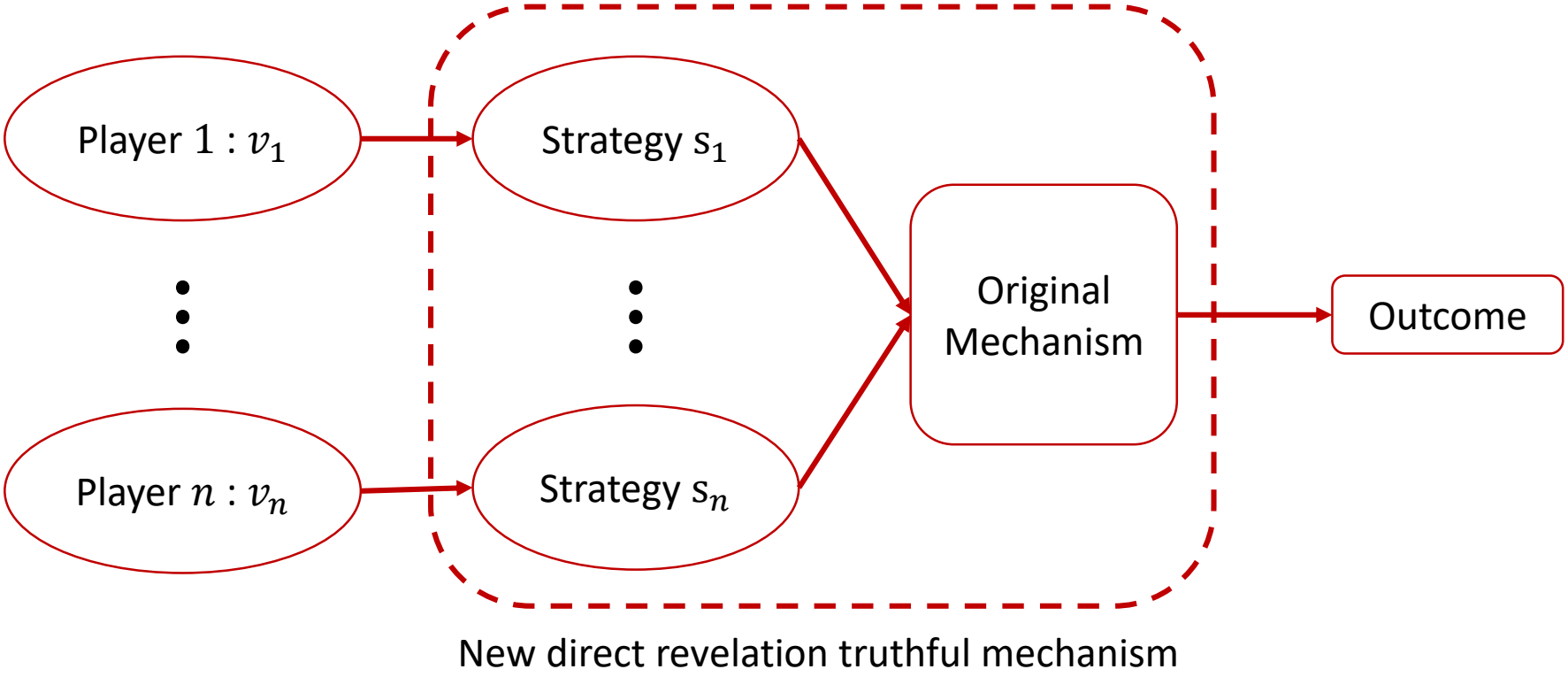
- Outcome = (allocation, payments)

- DSIC version [Gibbard, ‘73]
  - If a mechanism implements an outcome in dominant strategies, there’s a direct revelation DSIC mechanism implementing the same outcome.

- BNIC version [Dasgupta et al. ‘79, Holmstrom ‘77, Myerson ’79]
  - If a mechanism implements an outcome as BNE, there’s a direct revelation BNIC mechanism implementing the same outcome.

Revelation Principle

- Informal proof:



Applying Revelation Principle

- We already saw…
  - Sealed-bid 1st price auction
  - 2 agents with valuations drawn from $U[0,1]$
  - Each player halving his value was a BNE
  - Not naturally BNIC (players don't report value)

- BNIC variant through revelation principle?

- Can also be used on non-direct-revelation mechs

1st Price Auction

- For n players with iid valuations, "shadowing" the bid by a factor of $(n-1)/n$ is a BNE

- $E[\text{Revenue}]$ to the auctioneer?
  - $E_{\{v_i \sim U[0,1]\}_{i=1}^{n}} \left(\frac{n-1}{n}\right) * \max_i v_i = \frac{n-1}{n+1}$ (Why?)

- Interestingly, this is equal to E[Revenue] from 2nd price auction
  - $E_{\{v_i \sim U[0,1]\}_{i=1}^{n}} [2^{\text{nd}} \text{ highest } v_i] = \frac{n-1}{n+1}$ (Why?)

Revenue Equivalence

- If two BNIC mechanisms A and B:
  1. Always produce the same allocation;
  2. Have the same expected payment to agent i for some type v_i^0 (e.g., "zero value for all" → zero payment);
  3. Have agent valuations drawn from distributions with "path-connected support sets";
- Then they:
  - Charge the same expected payment to all agent types;
  - Have the same expected total revenue.

Revenue Equivalence

- Informally…
  - If two BNIC mechanisms always have the same allocation, then they have the same E[payments] and E[revenue].
  - Very powerful as it applies to any pair of BNIC mechanism

- 1st price (BNIC variant) and 2nd price auctions
  - Have the same allocation:
    Item always goes to the agent with the highest valuation
  - Thus, also have the same revenue

Non-Direct-Revelation Auctions

- Ascending auction (a.k.a. English auction)
  - All agents + auctioneer meet in a room.
  - Auctioneer starts the price at 0.
  - All agents want the item, and have their hands raised.
  - Auctioneer raise the price continuously.
  - Agents drop out when price > value for them

- Descending auction (a.k.a. Dutch auction)
  - Start price at a very high value.
  - Keep decreasing the price until some agent agrees to buy.

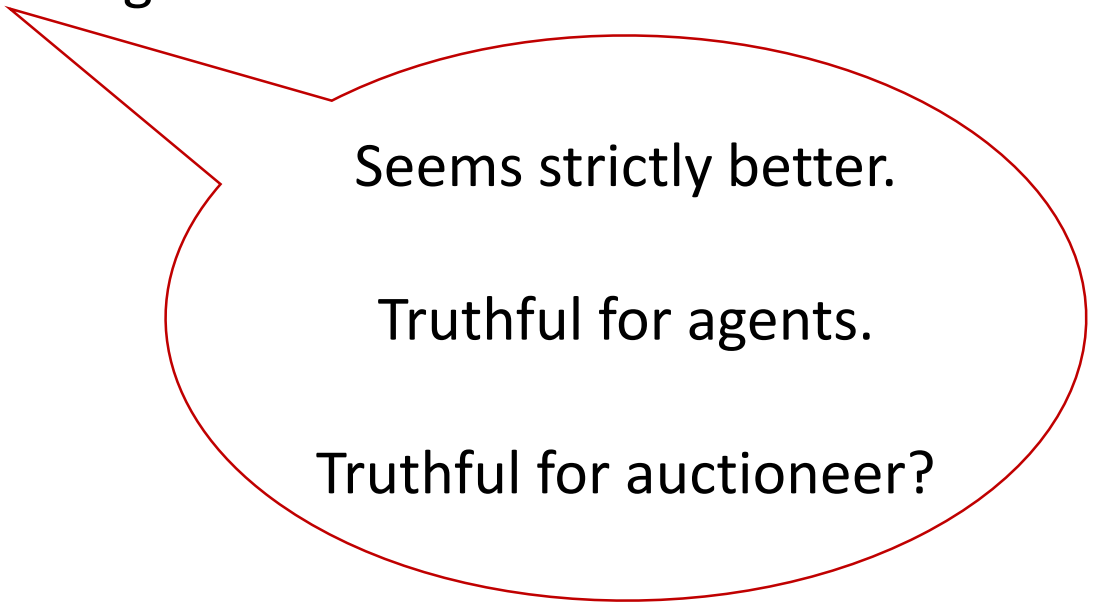
Ascending Auction

- When price > 2^{nd} highest value, all but the highest value agent drop out.
  - The agent with the highest value gets the item, pays the second highest value.
  - This outcome is implemented in dominant strategies.

- DSIC revelation principle applied to ascending auction → 2^{nd} price auction!
  - Different from the BNIC variant of the 1^{st} price auction ← BNIC revelation principle applied to 1^{st} price auction

The Trio

- 2nd price auction
  - Sealed-bid + truthful for agents
- 1st price auction
  - Sealed-bid
- Ascending auction
  - "truthful" for agents



Credible Mechanisms

- Warning: The remaining lecture is informal!

- Typical mechanism design
  - Auctioneer commits to using a mechanism.
  - Assume that auctioneer does not deviate later on.
  - "Stackelberg game between auctioneer and agents"

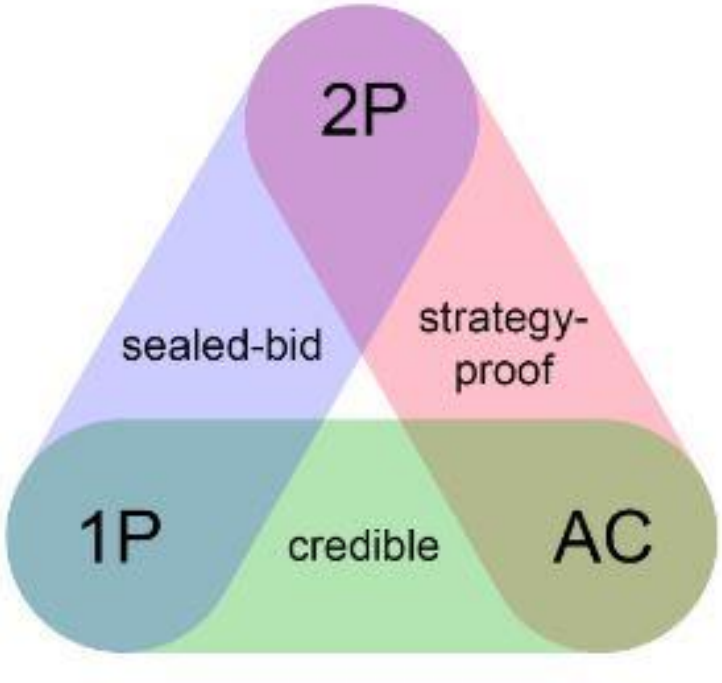
- Credible Mechanisms [Akbarpour and Li, 2017]
  - Auctioneer is incentivized to not deviate from his commitment at any stage of auction execution.

Credible Mechanisms

- Sealed-bid 2nd Price Auction
  - Auctioneer collects all bids.
  - Auctioneer goes to highest bidder (bid b).
  - Auctioneer says 2nd highest bid was $b - \epsilon$.
  - Highest bidder can't prove him wrong.
  - Auctioneer has an incentive to lie → not credible!

- 1st price auction → credible (Why?)
- Ascending auction → credible (Why?)

Credible Mechanisms



[Akbarpour and Li, 2017]

- Corollary: sealed-bid ∩ DSIC ∩ credible = ∅