

Gaussians

CSC311, Winter 2023

1. We draw 3 points from a Multivariate Gaussian $(1, 0)$, $(0, 1)$, and $(2, 2)$. Find the MLE for μ and Σ .

2. (a) True or False
 - i) If X_1 and X_2 are both normally distributed and independent, then (X_1, X_2) must have multivariate normal distribution.
 - ii) If (X_1, X_2) has multivariate normal distribution, then X_1 and X_2 are independent.
- (b) Affine transformation $X = (X_1, X_2, \dots, X_n)^T$ is an n -dimensional random vector which has multivariate normal distribution. If $X \sim N(\mu, \Sigma)$ and $Y = BX + c$ is an affine transformation of X , where c is a constant $m \times 1$ vector and B is a constant $m \times n$ matrix, what is the mean and covariance of Y ?

3. We are given a 2-dimensional multivariate Gaussian random variable Z , with mean $\mu = \mathbf{0}$ and covariance $\Sigma = \mathbf{I}$. Find the covariance matrix of a multivariate Gaussian such that the axes x_1 and x_2 of the isocontours of the density are elliptically shaped with major/minor axis lengths in a 4:3 ratio, and the axes are rotated 45 degrees counterclockwise.¹

¹Adapted from UC Berkeley problems