

# CSC 311: Introduction to Machine Learning

## Lecture 1 - Introduction and Nearest Neighbors

Michael Zhang      Chandra Gummaluru

University of Toronto, Winter 2023

# Outline

- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?
  - Examples of Machine Learning
  - Why This Class?
- 4 Preliminaries and Nearest Neighbor Methods

- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?
- 4 Preliminaries and Nearest Neighbor Methods

# Meet Your Peers

Find someone you don't know and introduce yourself.  
Make a new friend!

- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?
- 4 Preliminaries and Nearest Neighbor Methods

# Marking Scheme

<b>Component</b>	<b>Weight</b>
Math Diagnostic	2%
3 Assignments	33%, $\approx 11\%$ each
Embedded Ethics Assignments	5%
Project	10%
Midterm Exam	20%
Final Exam	30% (40% auto-fail threshold)

5% embedded ethics assignments:

<b>Assignment</b>	<b>% Final Grade</b>	<b>Marking</b>
Pre-module survey	1%	Full credit for submitting.
Class participation	0.5%	Full credit for 90% attendance.
Written Reflection	2%	Full credit for a good-faith effort.
Post-module survey	1.5%	Full credit for submitting.

# Recommended Textbooks

There are lots of freely available, high-quality ML resources.

Here are some recommended textbooks.

- Bishop: Pattern Recognition and Machine Learning.
- Hastie, Tibshirani, and Friedman: The Elements of Statistical Learning.
- MacKay: Information Theory, Inference, and Learning Algorithms.
- Barber: Bayesian Reasoning and Machine Learning.
- Sutton and Barto: Reinforcement Learning: An Introduction.
- Deisenroth, Faisal, and Ong: Math for ML.
- Shalev-Shwartz and Ben-David: Understanding Machine Learning: From Theory to Algorithms.
- Kevin Murphy: Machine Learning: a Probabilistic Perspective.

# Course Components

- Assignments
- Project
- Midterm
- Final



# Assignments

- Theoretical and programming questions.
- Due on MarkUs before 11:59 pm on Mondays or Fridays.
- Late Policy: 3 grace days you can use on homeworks.  
No credit given if no more grace days.
- Collaboration Policy: You should help each other learn the materials, but submit your own work.

# Project

- Groups of 2-3.
- 4 weeks.
- Implement and evaluate several algorithms from the course.
- Propose and evaluate an extension of one algorithm.
- Will post instructions and starter code later.
- Option to do this project on dataset of your choosing (equivalent work)

# Exams

- Conceptual questions.
- Midterm
  - ▶ Date finalized soon, at least after four weeks of lecture
  - ▶ Must attend your registered section if in tutorial.
  - ▶ Can bring one double-sided reference sheet.
- Final Exam
  - ▶ 3-hour exam.
  - ▶ Date/time will be released around late March.
  - ▶ Do not book travel plans until your final exam schedule is released.

Tutorial is 3<sup>rd</sup> lecture hour e.g. W 11-12

# Academic Integrity

- Cheating only cheats yourself!
  - ▶ Consult U of T [Code of Behaviour on Academic Matters](#)
- What you should do for assignments:
  - ▶ Ask questions during office hours.
  - ▶ Discuss ideas and code examples with others.
  - ▶ Write code on your own.
  - ▶ Say no to sending code to others.
- What you should do for tests and exams:
  - ▶ Create practice questions.
  - ▶ Test yourself/each other under time pressure.

# Strategies for Success

- **Time Management**

1. The best time to do something was yesterday, the next best time is today, don't wait till tomorrow.
2. Hard skill to master but will serve you well throughout your career.

- **Study groups:** Virtual or in-person, they're a great way to keep yourself and your peers accountable. Teaching your peers is a good way to make sure you understand the foundational concepts.

- **Leverage resources:** Go to TA/instructor office hours *regularly* and not just before the tests/midterms/finals.

# Special Considerations Policy

- Missing an assessment due to extraordinary circumstances?  
Send a form and supporting documentation to the course email.
- Acceptable reasons:
  - ▶ Late course enrollment
  - ▶ Medical conditions: physical/mental health, hospitalizations, injury, accidents
  - ▶ Non-medical conditions (i.e., family/personal emergency)
- Unacceptable reasons: heavy course loads, multiple assignments/tests during the same period, time management issues
- Accessibility students: Accommodations are listed in Accessibility documentation

# Remark Requests

- A marking error on assignment/test.
- Submit within two weeks after marks are released.
- For assignments, submit on MarkUs.

# Course Information

Course Website: Almost Everything.

[https://www.cs.toronto.edu/~michael/teaching/csc311\\_w23/](https://www.cs.toronto.edu/~michael/teaching/csc311_w23/)

Quercus: Announcements and Grades.

Piazza: Discussions.

<https://piazza.com/utoronto.ca/winter2023/csc311>

MarkUs/CrowdMark: Assignments.

CrowdMark: Exams.



# Getting in Touch

Piazza: <https://piazza.com/utoronto.ca/winter2023/csc311>

- Course related and no sensitive info → public post
- Course related and sensitive info → private post
- Best place for questions about course.

Course email: [csc311-2023-01@cs.toronto.edu](mailto:csc311-2023-01@cs.toronto.edu)

- Special considerations requests.
- Any other matter.

Only email us directly with non-CSC311 questions.

- Course-related questions will get a faster response through the course email instead of emailing us individually.

# Office Hours

Office Hours will be finalized next week.

TAs will hold office hours to help with assignments and the project, as well as preparing for the midterm and final exams.

- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?**
  - Examples of Machine Learning
  - Why This Class?
- 4 Preliminaries and Nearest Neighbor Methods

# What is Learning?

*“The activity or process of gaining knowledge or skill by studying, practicing, being taught, or experiencing something.”*

*Merriam Webster dictionary*

*“A computer program is said to learn from experience  $E$  with respect to some class of tasks  $T$  and performance measure  $P$ , if its performance at tasks in  $T$ , as measured by  $P$ , improves with experience  $E$ .”*

*Tom Mitchell*

# What is Machine Learning?

- For many problems, it's difficult to program the correct behavior by hand
  - ▶ recognizing people and objects
  - ▶ understanding human speech
- Machine learning approach: program an algorithm to automatically learn from data, or from experience
- Why might you want to use a learning algorithm?
  - ▶ hard to code up a solution by hand (e.g. vision, speech)
  - ▶ system needs to adapt to a changing environment (e.g. spam detection)
  - ▶ want the system to perform *better* than the human programmers
  - ▶ privacy/fairness (e.g. ranking search results)

# Relations to Statistics

- It's similar to statistics...
  - ▶ Both fields try to uncover patterns in data
  - ▶ Both fields draw heavily on calculus, probability, and linear algebra, and share many of the same core algorithms
- it's not *exactly* statistics...
  - ▶ Stats is more concerned with helping scientists and policymakers draw good conclusions; ML is more concerned with building autonomous agents
  - ▶ Stats puts more emphasis on interpretability and mathematical rigor; ML puts more emphasis on predictive performance, scalability, and autonomy
- ...but machine learning and statistics rely on similar mathematics.

# Types of Machine Learning

*main focus in course*

- **Supervised learning:** labeled examples of the correct behavior
- **Reinforcement learning:** learning system (agent) interacts with the world and learns to maximize a scalar reward signal
- **Unsupervised learning:** no labeled examples – instead, looking for “interesting” patterns in the data

*different ways to provide learning signal*

# History of Machine Learning

- 1957 — Perceptron algorithm (implemented as a circuit!)
- 1959 — Arthur Samuel wrote a learning-based checkers program that could defeat him
- 1969 — Minsky and Papert's book *Perceptrons* (limitations of linear models)
- 1980s — Some foundational ideas
  - ▶ Connectionist psychologists explored neural models of cognition
  - ▶ 1984 — Leslie Valiant formalized the problem of learning as PAC learning
  - ▶ 1988 — Backpropagation (re-)discovered by Geoffrey Hinton and colleagues
  - ▶ 1988 — Judea Pearl's book *Probabilistic Reasoning in Intelligent Systems* introduced Bayesian networks



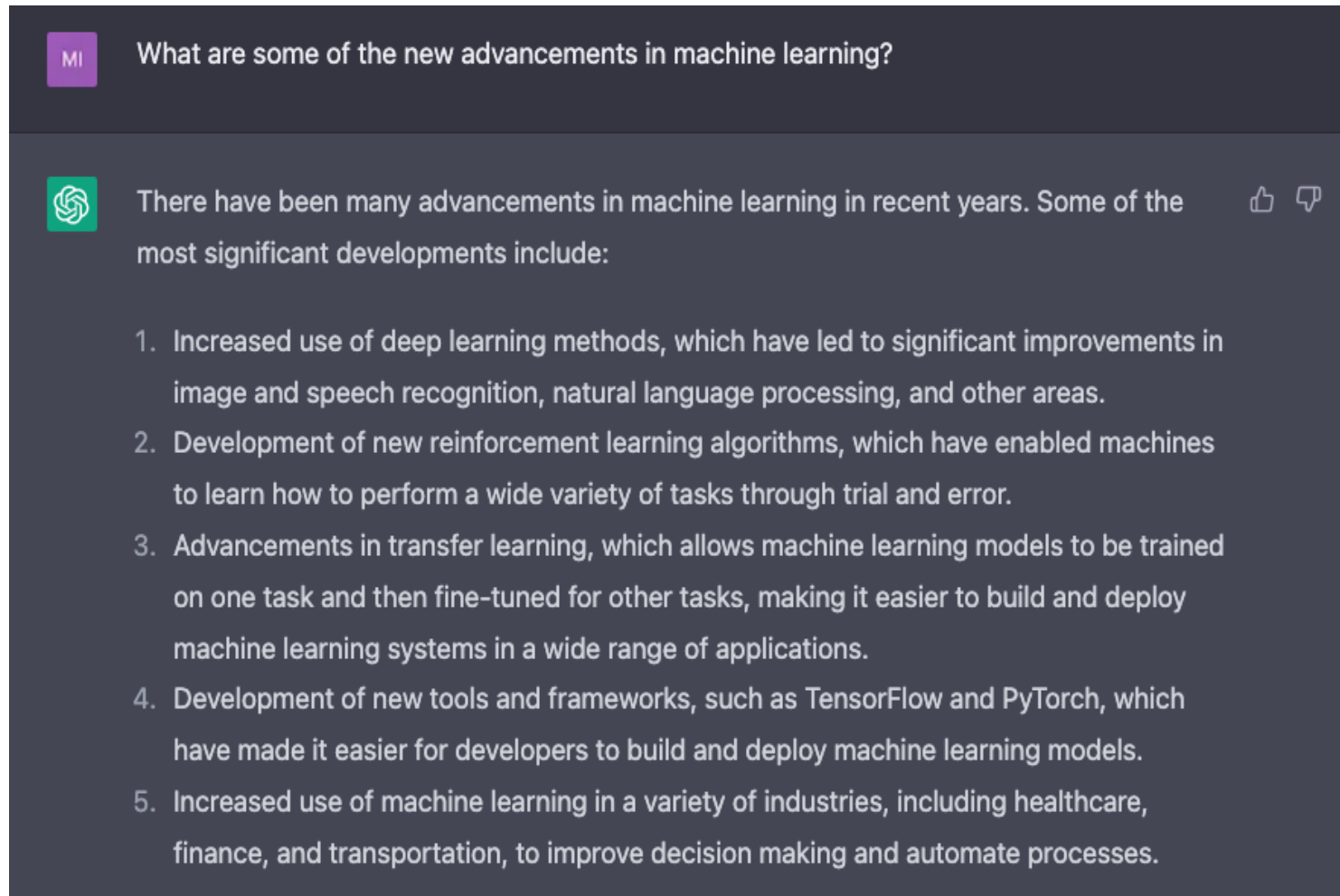
# History of Machine Learning

- 1990s — the “AI Winter”, a time of pessimism and low funding
- But looking back, the '90s were also sort of a golden age for ML research
  - ▶ Markov chain Monte Carlo
  - ▶ variational inference
  - ▶ kernels and support vector machines
  - ▶ boosting
  - ▶ convolutional networks
  - ▶ reinforcement learning
- 2000s — applied AI fields (vision, NLP, etc.) adopted ML
- 2010s — deep learning
  - ▶ 2010–2012 — neural nets smashed previous records in speech-to-text and object recognition
  - ▶ increasing adoption by the tech industry
  - ▶ 2016 — AlphaGo defeated the human Go champion
  - ▶ 2018-now — generating photorealistic images and videos
  - ▶ 2020 — GPT3 language model
- now — Increasing attention to ethical and societal implications


2012  
ImageNet

# ChatGPT - Now

ChatGPT is a recent prominent example of machine learning.

A screenshot of a chat interface. At the top, a user with a purple 'MI' icon asks, "What are some of the new advancements in machine learning?". Below, the ChatGPT logo (a green square with a white knot) responds. The response starts with "There have been many advancements in machine learning in recent years. Some of the most significant developments include:" followed by a numbered list of five items. To the right of the response are thumbs-up and thumbs-down icons.

MI What are some of the new advancements in machine learning?

 There have been many advancements in machine learning in recent years. Some of the most significant developments include:

1. Increased use of deep learning methods, which have led to significant improvements in image and speech recognition, natural language processing, and other areas.
2. Development of new reinforcement learning algorithms, which have enabled machines to learn how to perform a wide variety of tasks through trial and error.
3. Advancements in transfer learning, which allows machine learning models to be trained on one task and then fine-tuned for other tasks, making it easier to build and deploy machine learning systems in a wide range of applications.
4. Development of new tools and frameworks, such as TensorFlow and PyTorch, which have made it easier for developers to build and deploy machine learning models.
5. Increased use of machine learning in a variety of industries, including healthcare, finance, and transportation, to improve decision making and automate processes.

## ChatGPT on the University of Toronto

MI

Why is the University of Toronto famous for machine learning?



The University of Toronto has a long history of excellence in machine learning research and is widely recognized as one of the top universities in the world for this field. There are several reasons for its reputation in this area:

1. It has a strong machine learning research community, with numerous faculty members and students working on cutting-edge projects in this area.
2. It is home to the Vector Institute for Artificial Intelligence, which is a world-leading research institute focused on machine learning and artificial intelligence.
3. It has produced many notable alumni who have made significant contributions to the field of machine learning, including Geoffrey Hinton, who is widely known as one of the pioneers of deep learning.
4. It has a strong track record of collaboration with industry, with many of its machine learning researchers working on applied projects in partnership with companies.

- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?**
  - Examples of Machine Learning
  - Why This Class?
- 4 Preliminaries and Nearest Neighbor Methods

# Computer Vision

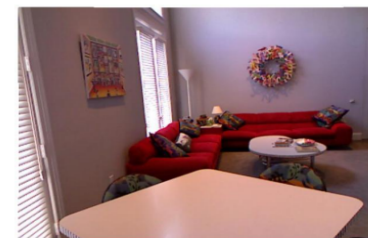
Object detection, semantic segmentation, pose estimation, and many other tasks are done with ML.



Figure 4. More results of Mask R-CNN on COCO test images, using ResNet-101-FPN and running at 5 fps, with 35.7 mask AP (Table 1).



Source: <https://www.youtube.com/watch?v=YGO2lwAgrig>



DAQUAR 1553  
What is there in front of the sofa?  
Ground truth: table  
IMG+BOW: table (0.74)  
2-VIS+BLSTM: table (0.88)  
LSTM: chair (0.47)



COCOQA 5078  
How many leftover donuts is the red bicycle holding?  
Ground truth: three  
IMG+BOW: two (0.51)  
2-VIS+BLSTM: three (0.27)  
BOW: one (0.29)

Instance segmentation - [▶ Link](#)

# Speech

Speech to text, personal assistants, speaker identification...



# Natural Language Processing

Machine translation, sentiment analysis, topic modeling, spam filtering.

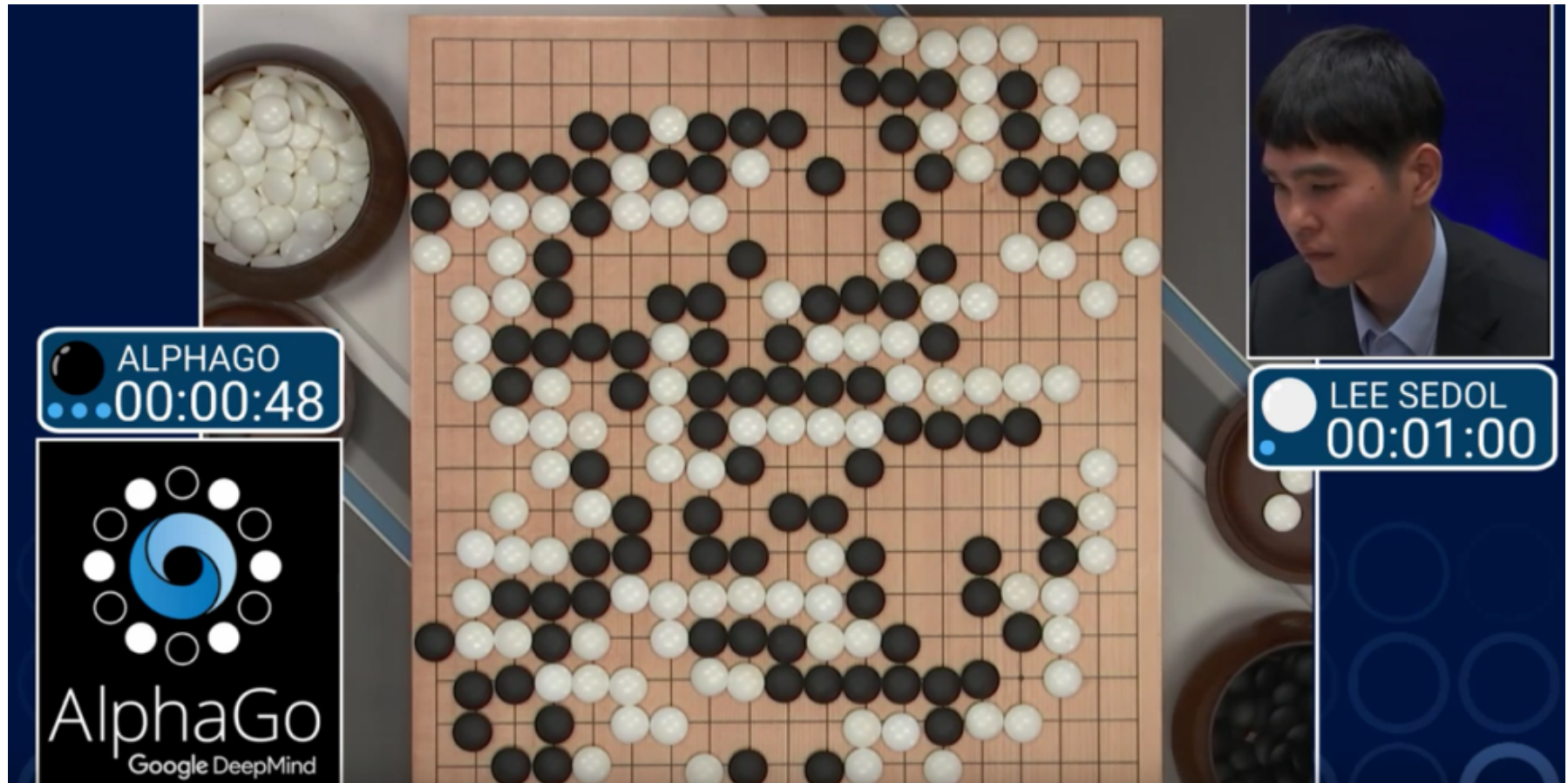
## Real world example:

*The New York Times*

LDA analysis of 1.8M New York Times articles:



# Playing Games

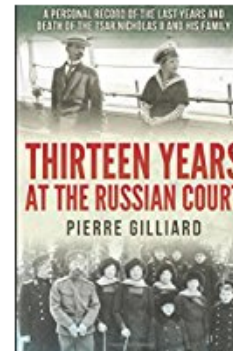
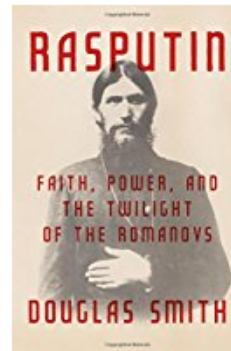
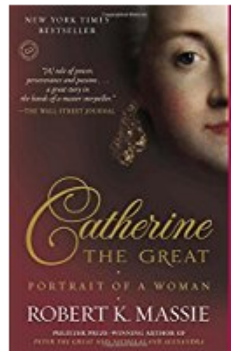
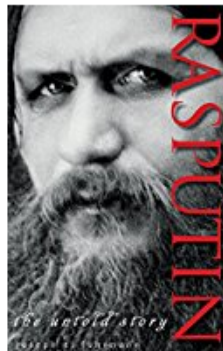


DOTA2 - [▶ Link](#)

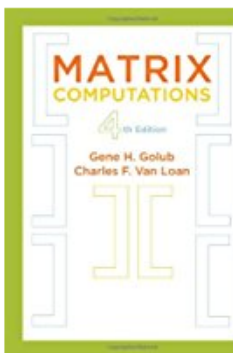
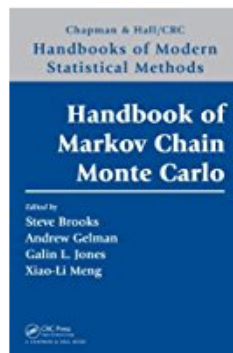
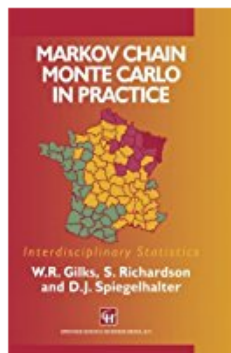


# E-commerce & Recommender Systems : Amazon, Netflix, ...

Inspired by your shopping trends



Related to items you've viewed [See more](#)



- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?**
  - Examples of Machine Learning
  - **Why This Class?**
- 4 Preliminaries and Nearest Neighbor Methods

# Related Courses

- Was CSC411 previously.
- CSC412 (Probabilistic Learning and Reasoning) and CSC413 (Neural Networks and Deep Learning) build upon this course.
- Overlap with Applied Statistics course.

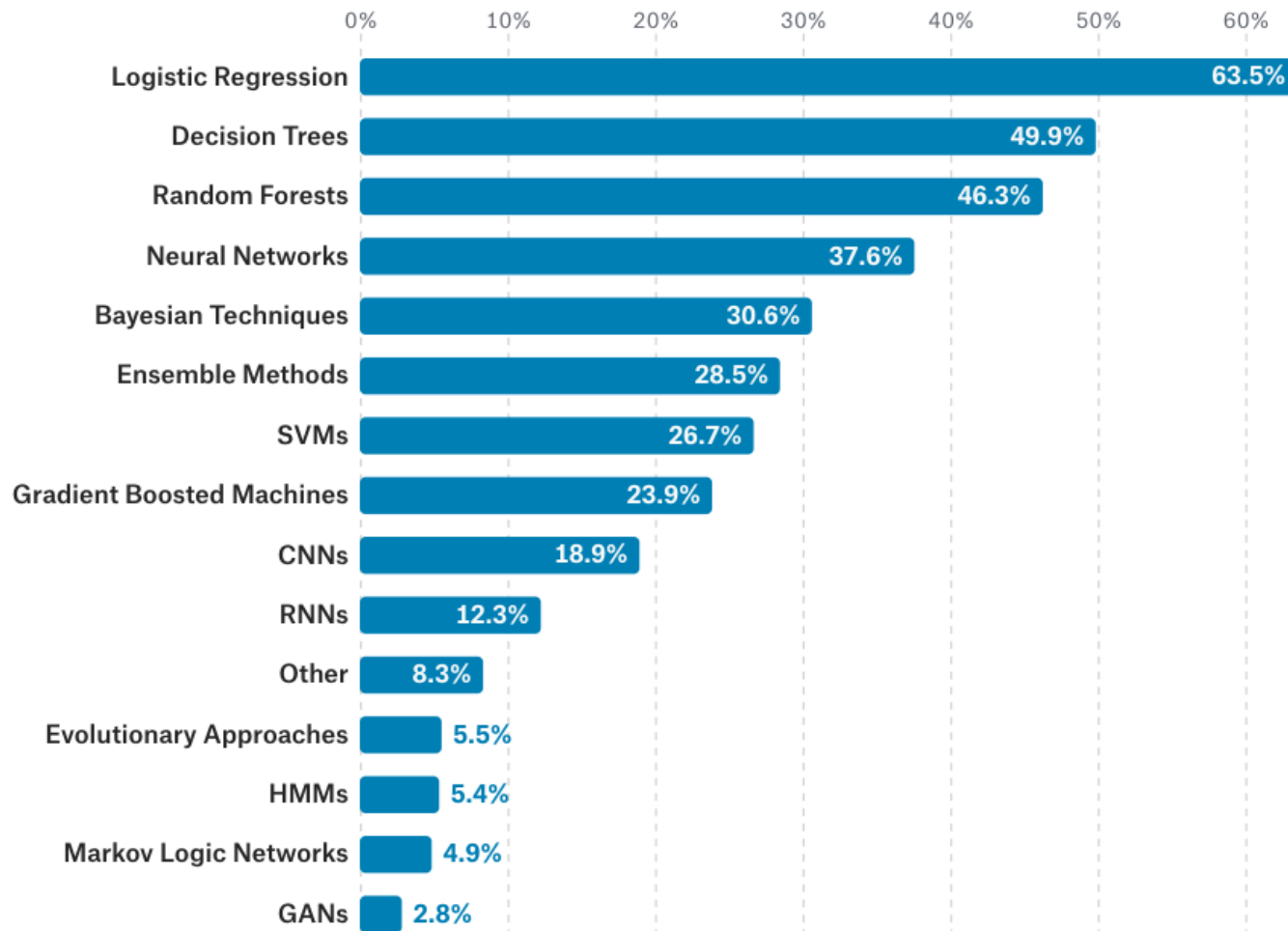
# Why This Class?

Why not jump straight to CSC 412/413, and learn Neural Nets first?

- The principles you learn in this course will be essential to understand and apply neural nets.
- The techniques in this course are the first things to try for a new ML problem.
  - ▶ E.g., try logistic regression before building a deep neural net!
- There's a whole world of probabilistic graphical models.

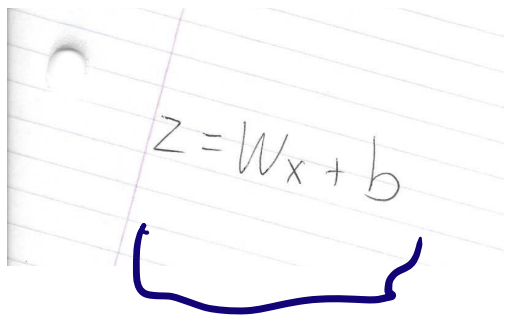
# Why This Class?

Kaggle survey of data science and ML practitioners:  
What data science methods do you use at work?



# Implementing Machine Learning Systems

- Derive an algorithm (with pencil and paper), and then translate the math into code.
- Array processing (NumPy)
  - ▶ **vectorize** computations (express them in terms of matrix/vector operations) to exploit hardware efficiency
  - ▶ This also makes your code cleaner and more readable!



$z = Wx + b$

```
z = np.zeros(m)
for i in range(m):
    for j in range(n):
        z[i] += W[i, j] * x[j]
    z[i] += b[i]
```

*NumPy*  
 $z = W @ x + b$

# Implementing Machine Learning Systems

- Neural net frameworks: PyTorch, TensorFlow, JAX, etc.  $f: \mathbb{R}^n \rightarrow \mathbb{R}$ 
  - ▶ automatic differentiation *derivatives in higher dimensions*
  - ▶ compiling computation graphs *→ gradients*
  - ▶ libraries of algorithms and network primitives
  - ▶ support for graphics processing units (GPUs)
- Why take this class if these frameworks do so much for you?
  - ▶ So you know what to do if something goes wrong!
  - ▶ Debugging learning algorithms requires sophisticated detective work, which requires understanding what goes on beneath the hood.
  - ▶ That's why we derive things by hand in this class!

- 1 Introductions
- 2 Admin Details
- 3 What is Machine Learning?
- 4 Preliminaries and Nearest Neighbor Methods**



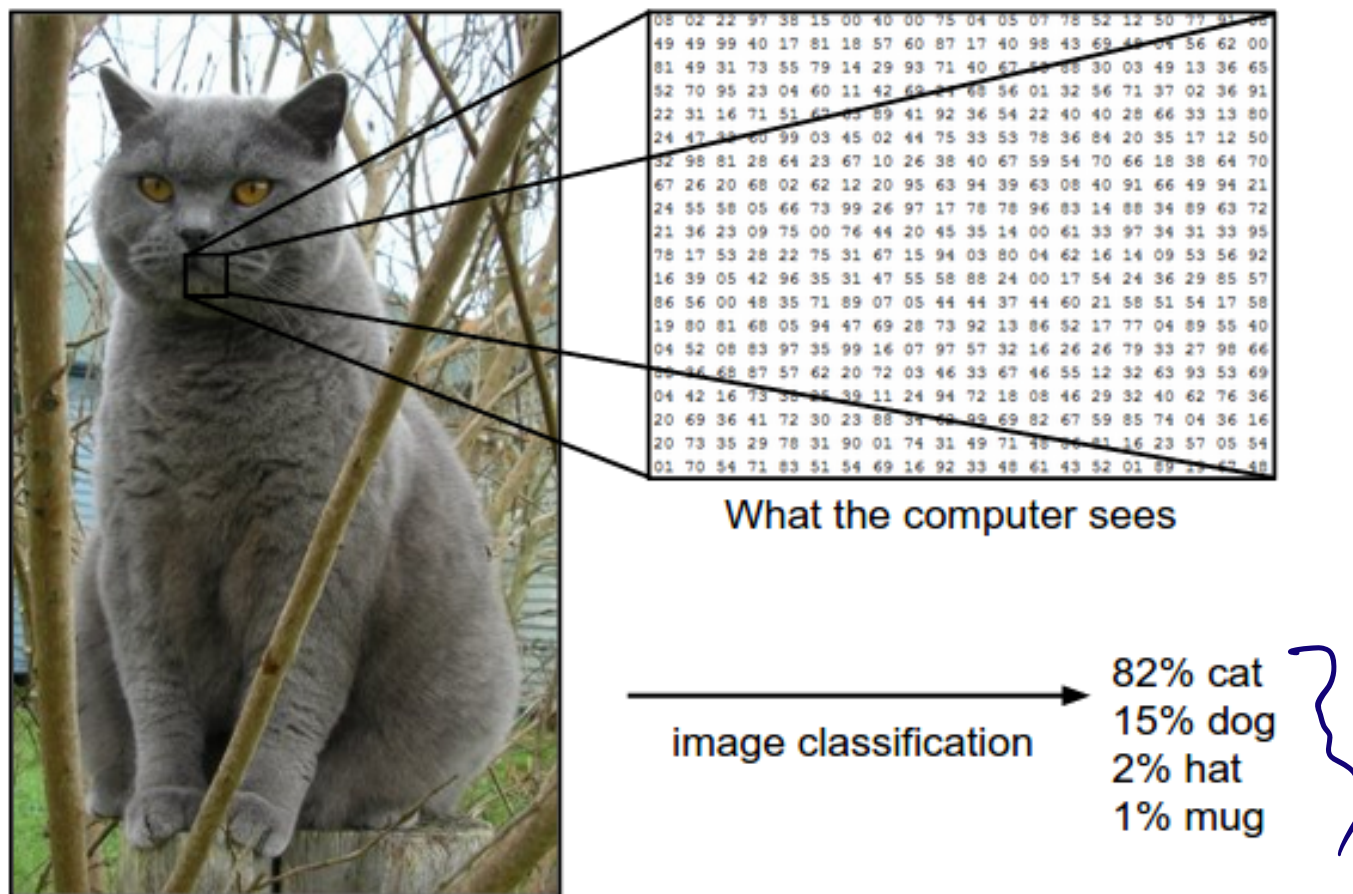
# Introduction

- Today (and for much of this course) we focus on **supervised learning**.
- This means we are given a **training set** consisting of **inputs** and corresponding **labels**, e.g.

Task	Inputs	Labels
object recognition	image	object category
image captioning	image	caption
document classification	text	document category
speech-to-text	audio waveform	text
⋮	⋮	⋮

# Input Vectors

What an image looks like to the computer:



[Image credit: Andrej Karpathy]

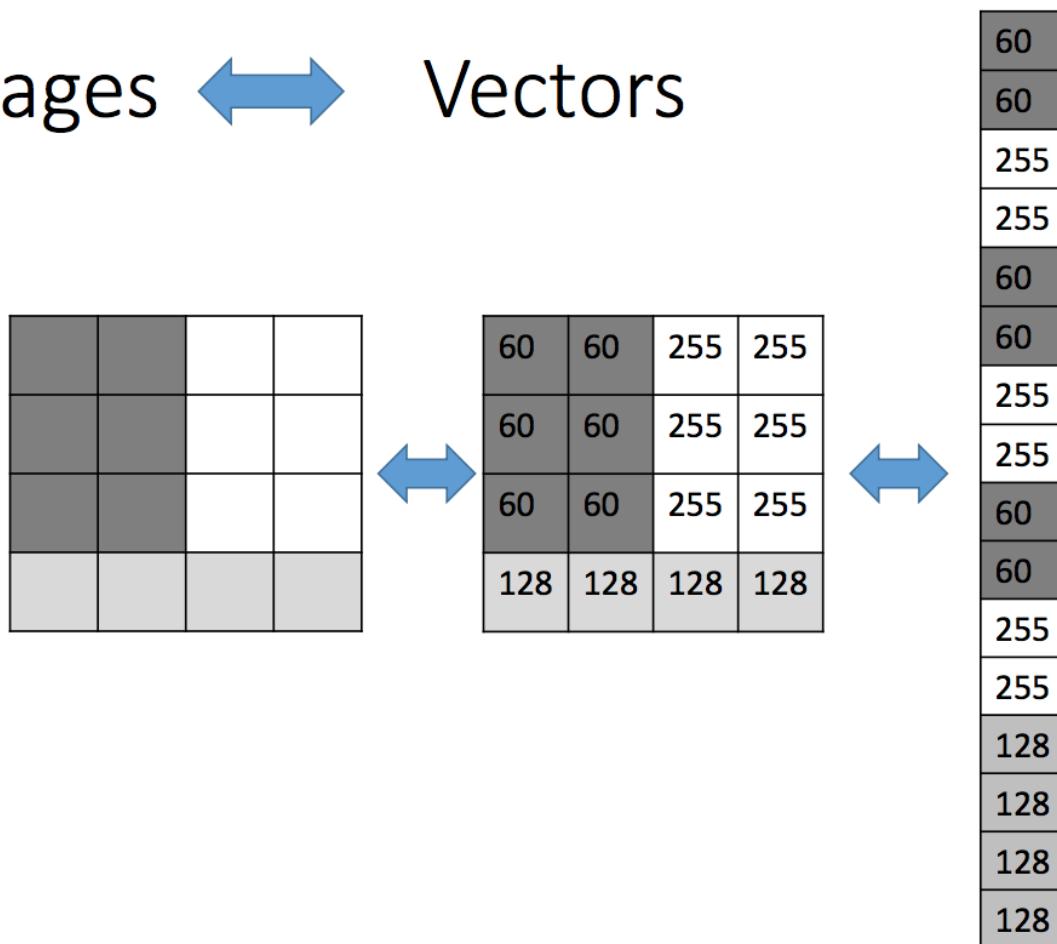
# Input Vectors

- Machine learning algorithms need to handle lots of types of data: images, text, audio waveforms, credit card transactions, etc.
- Common strategy: represent the input as an **input vector** in  $\mathbb{R}^d$ 
  - ▶ **Representation** = mapping to another space that's easy to manipulate
  - ▶ Vectors are a great representation since we can do linear algebra!

# Input Vectors

Can use raw pixels:

Images  $\longleftrightarrow$  Vectors



Can do much better if you compute a vector of meaningful features.

# Input Vectors

$$T = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ \vdots \end{bmatrix}$$

- Mathematically, our training set consists of a collection of pairs of an input vector  $\mathbf{x} \in \mathbb{R}^d$  and its corresponding **target**, or **label**,  $t$

- Regression**:  $t$  is a real number (e.g. stock price)  $\in \mathbb{R}$
  - Classification**:  $t$  is an element of a discrete set  $\{1, \dots, C\}$
  - These days,  $t$  is often a highly structured object (e.g. image)

- Denote the training set  $\{(\mathbf{x}^{(1)}, t^{(1)}), \dots, (\mathbf{x}^{(N)}, t^{(N)})\} = \mathcal{D}$

- Note: these superscripts have nothing to do with exponentiation!

$$T = \begin{bmatrix} t^{(1)} \\ t^{(2)} \\ \vdots \\ t^{(N)} \end{bmatrix}$$

$$\mathbf{x}^{(i)} \in \mathbb{R}^d$$

$$X = \begin{bmatrix} \text{---} & \mathbf{x}^{(1)} & \text{---} \\ \text{---} & \mathbf{x}^{(2)} & \text{---} \\ & \vdots & \\ \text{---} & \mathbf{x}^{(N)} & \text{---} \end{bmatrix}$$

$N \times d$   
datapoints  $\times$  features

$$Xw$$

design matrix

data "

# Nearest Neighbors

- Suppose we're given a novel input vector  $\mathbf{x}$  we'd like to classify.
- The idea: find the nearest input vector to  $\mathbf{x}$  in the training set and copy its label.
- Can formalize "nearest" in terms of Euclidean distance

$$\mathbf{x}^{(a)} = \begin{bmatrix} x_1^{(a)} \\ x_2^{(a)} \\ \vdots \end{bmatrix}$$

$$\|\mathbf{x}^{(a)} - \mathbf{x}^{(b)}\|_2 = \sqrt{\sum_{j=1}^d (x_j^{(a)} - x_j^{(b)})^2}$$

$$\mathbf{x}^{(b)} = \begin{bmatrix} x_1^{(b)} \\ x_2^{(b)} \\ \vdots \\ x_D^{(b)} \end{bmatrix}$$

## Algorithm:

1. Find example  $(\mathbf{x}^*, t^*)$  (from the stored training set) closest to  $\mathbf{x}$ . That is:

$$\mathbf{x}^* = \underset{\mathbf{x}^{(i)} \in \text{training set}}{\operatorname{argmin}} \operatorname{distance}(\mathbf{x}^{(i)}, \mathbf{x})$$

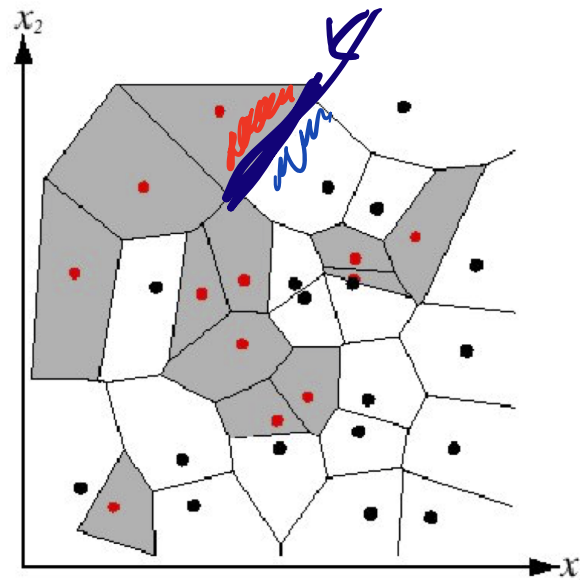
2. Output  $y = t^*$  *prediction*

- Note: we don't need to compute the square root. Why?

# Nearest Neighbors: Decision Boundaries

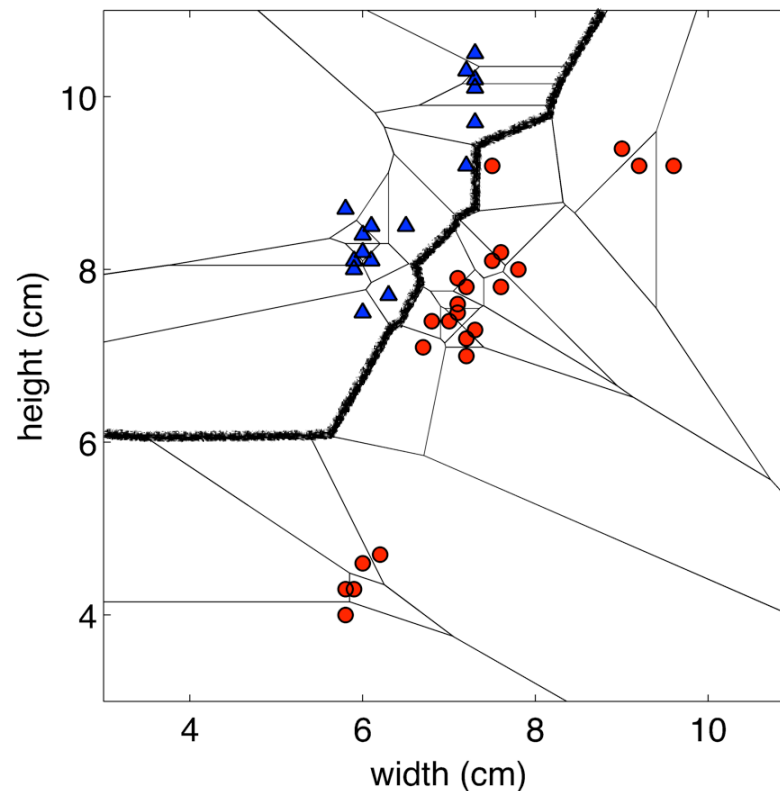
We can visualize the behavior in the classification setting using a Voronoi diagram.

*useful for understanding  
how we classify new  
point*



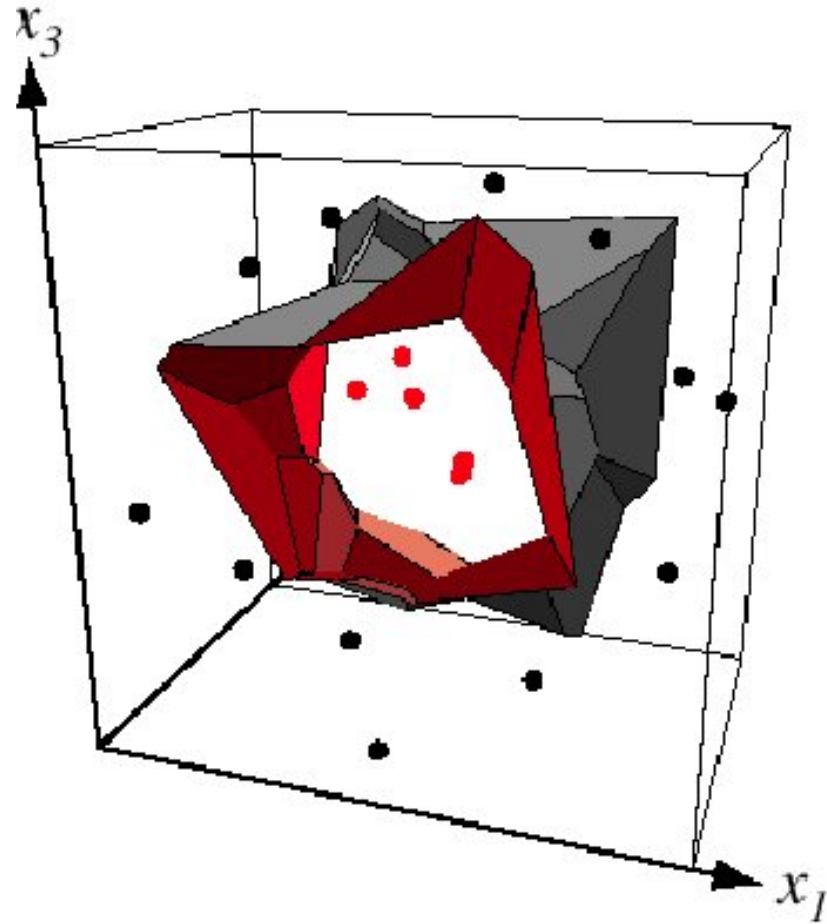
# Nearest Neighbors: Decision Boundaries

**Decision boundary:** the boundary between regions of input space assigned to different categories.





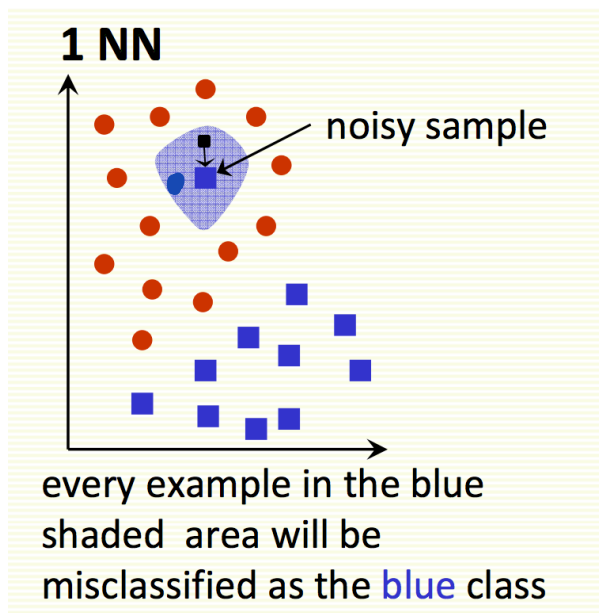
# Nearest Neighbors: Decision Boundaries



Example: 2D decision boundary

# Nearest Neighbors

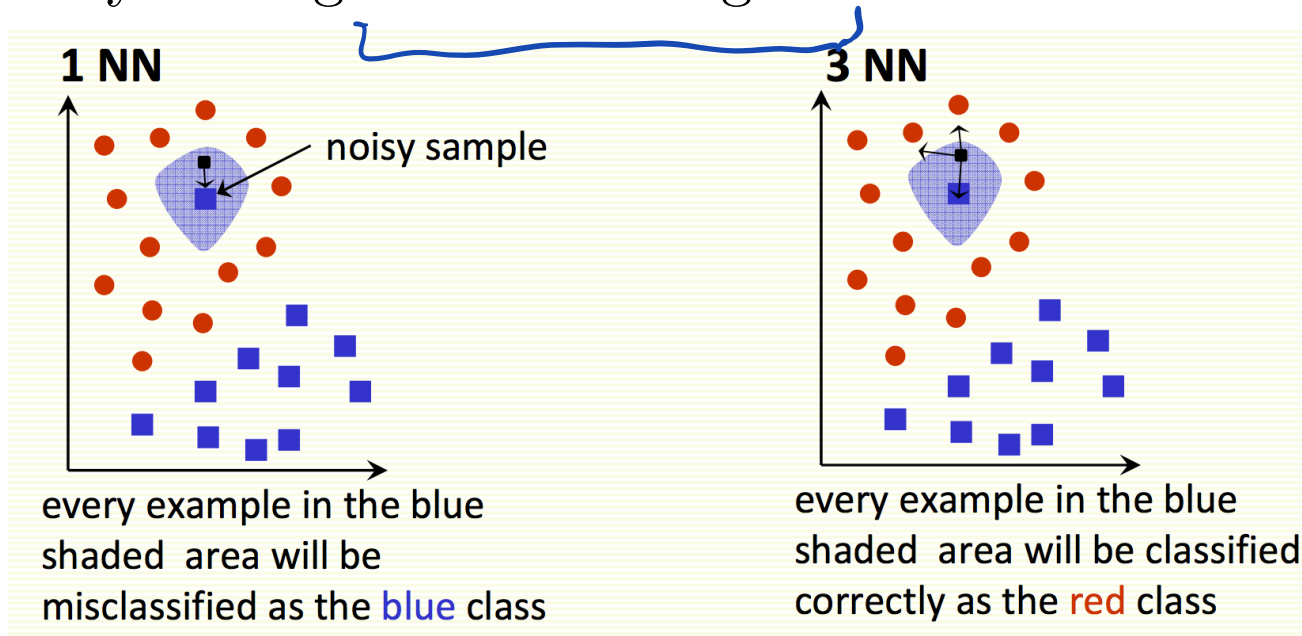
- Sensitive to noise or mis-labeled data (“class noise”). Solution?



[Pic by Olga Veksler]

# Nearest Neighbors

- Sensitive to noise or mis-labeled data (“class noise”). Solution?
- Smooth by having  $k$  nearest neighbors vote



[Pic by Olga Veksler]

# K-Nearest Neighbors

- Nearest neighbors **sensitive to noise or mis-labeled data** (“class noise”).

Solution?

- Smooth by having  $k$  nearest neighbors vote

x            a    b    c  
                 1    1    2

**Algorithm (kNN):**

- Find  $k$  examples  $\{\mathbf{x}^{(i)}, t^{(i)}\}$  closest to the test instance  $\mathbf{x}$
- Classification output is majority class.

$$y^* = \underset{t^{(z)} \in \text{class labels}}{\text{arg max}} \sum_{i=1}^k \mathbb{I}(t^{(z)} = t^{(i)})$$

$t^{(z)} \in \{1, 2\}$

$\mathbb{I}(t^{(z)} = t^{(a)})$   
 $+ \mathbb{I}(t^{(z)} = t^{(b)})$   
 $+ \mathbb{I}(t^{(z)} = t^{(c)})$

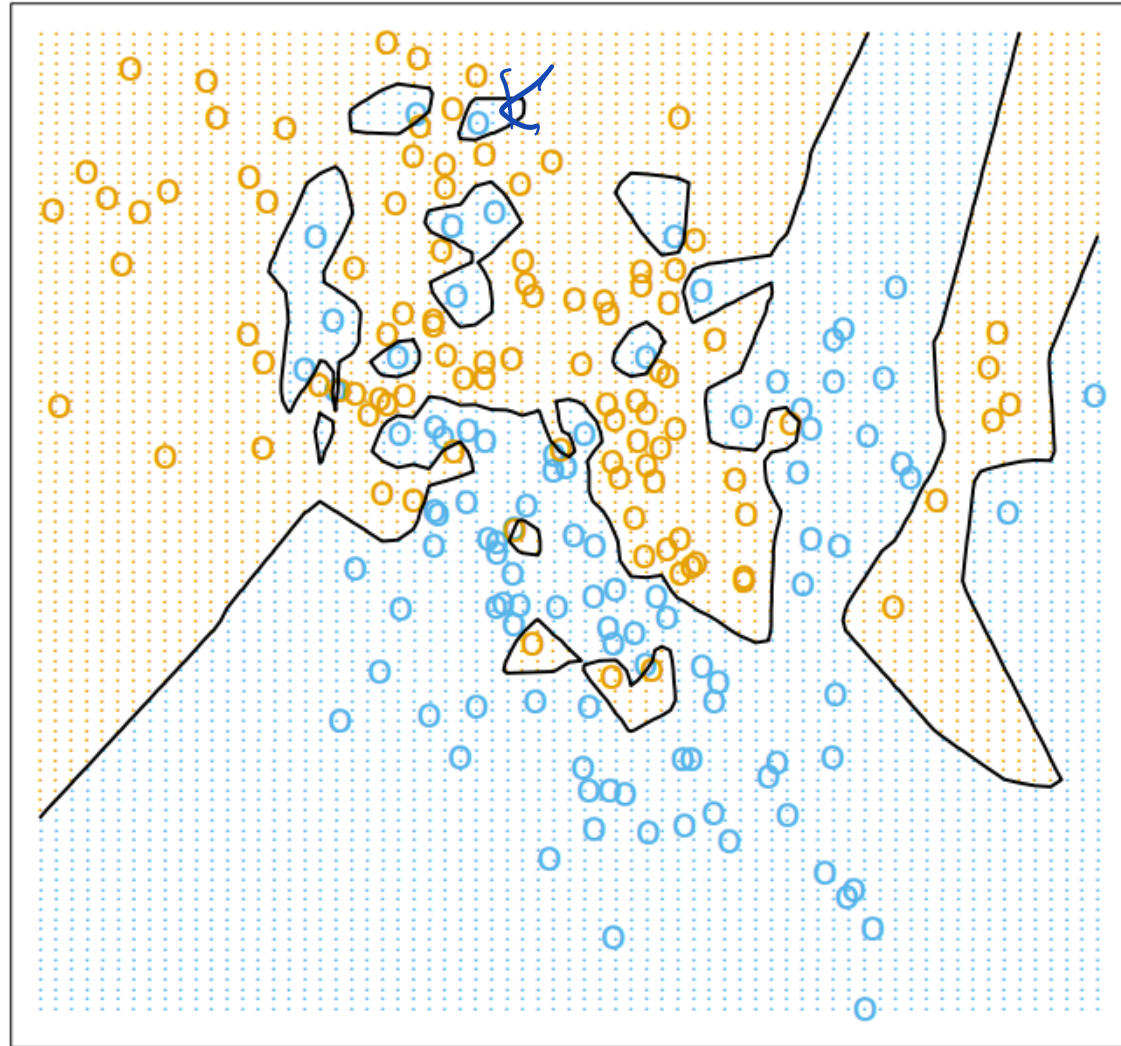
$\mathbb{I}\{\text{statement}\}$  is the identity function and is equal to one whenever the statement is true. We could also write this as  $\delta(t^{(z)}, t^{(i)})$ , with  $\delta(a, b) = 1$  if  $a = b$ , 0 otherwise.

$t^{(z)} = 1$   
 $= 2$

$1 + 1 + 0 = 2$   
 $0 + 0 + 1 = 1$

# K-Nearest Neighbors

$k=1$

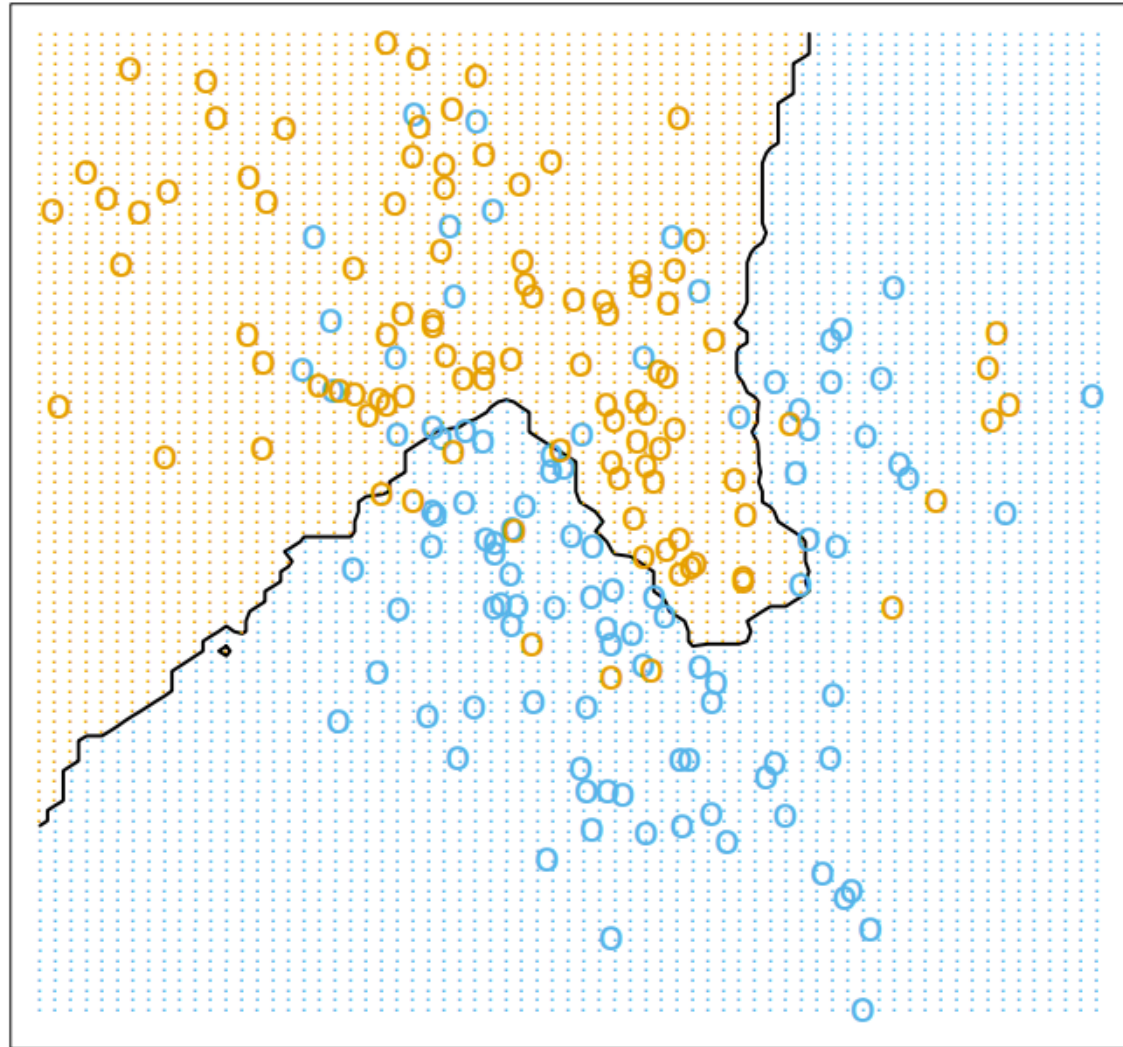


$K = N$ :  
always pick  
majority  
class

[Image credit: "The Elements of Statistical Learning"]

# K-Nearest Neighbors

$k=15$

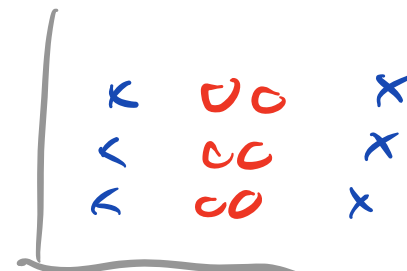


[Image credit: "The Elements of Statistical Learning"]

# K-Nearest Neighbors

## Tradeoffs in choosing $k$ ?

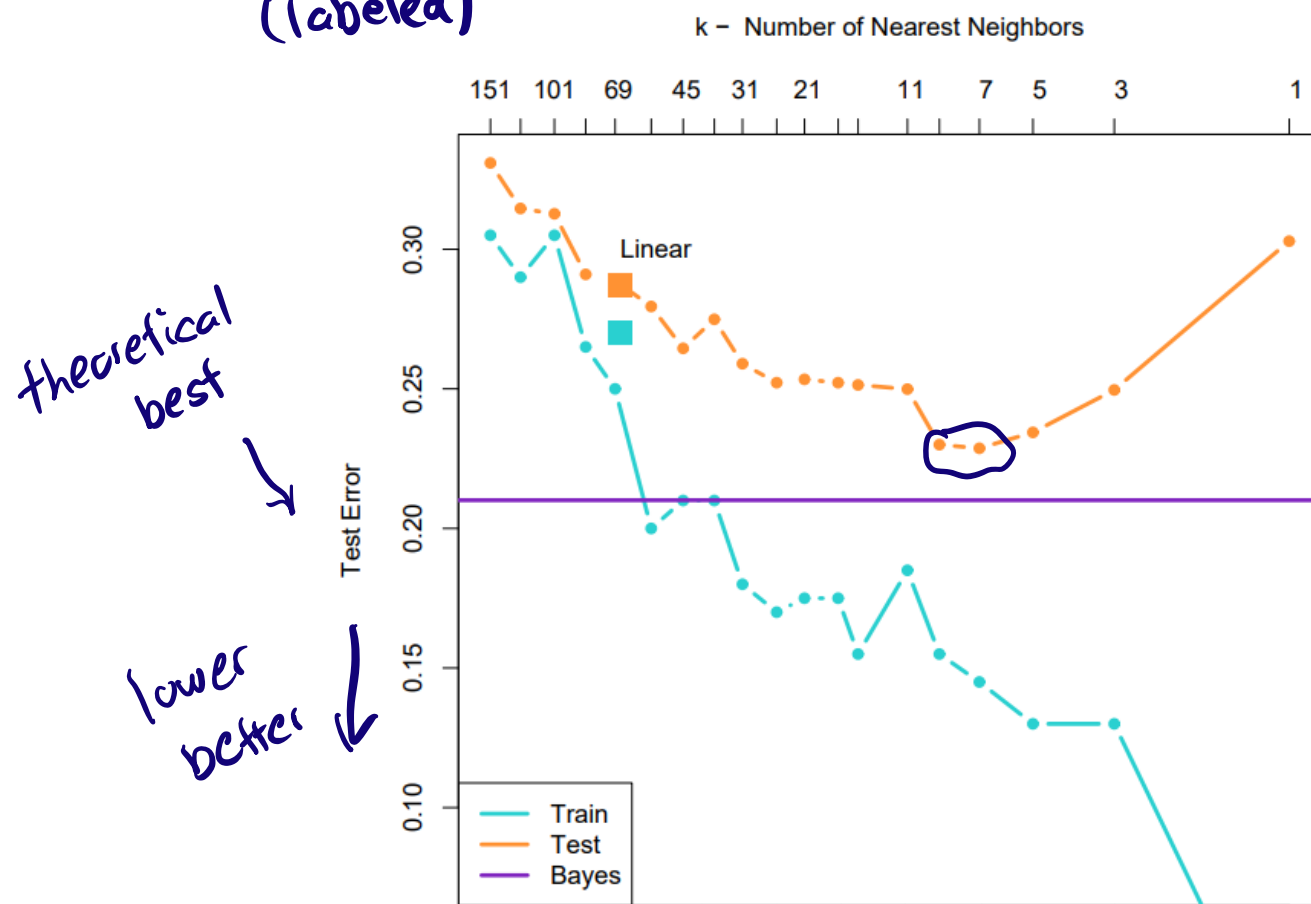
- Small  $k$ 
  - ▶ Good at capturing fine-grained patterns
  - ↪ ▶ May **overfit**, i.e. be sensitive to random idiosyncrasies in the training data
- Large  $k$ 
  - ▶ Makes stable predictions by averaging over lots of examples
  - ↪ ▶ May **underfit**, i.e. fail to capture important regularities
- Balancing  $k$ 
  - ▶ Optimal choice of  $k$  depends on number of data points  $n$ .
  - ▶ Nice theoretical properties if  $k \rightarrow \infty$  and  $\frac{k}{n} \rightarrow 0$ .
  - ▶ Rule of thumb: choose  $k < \sqrt{n}$ .
  - ▶ We can choose  $k$  using validation set (next slides).



# K-Nearest Neighbors

- We would like our algorithm to **generalize** to data it hasn't seen before.
- We can measure the **generalization error** (error rate on new examples) using a **test set**.

(labeled)



- training error increases with  $k$

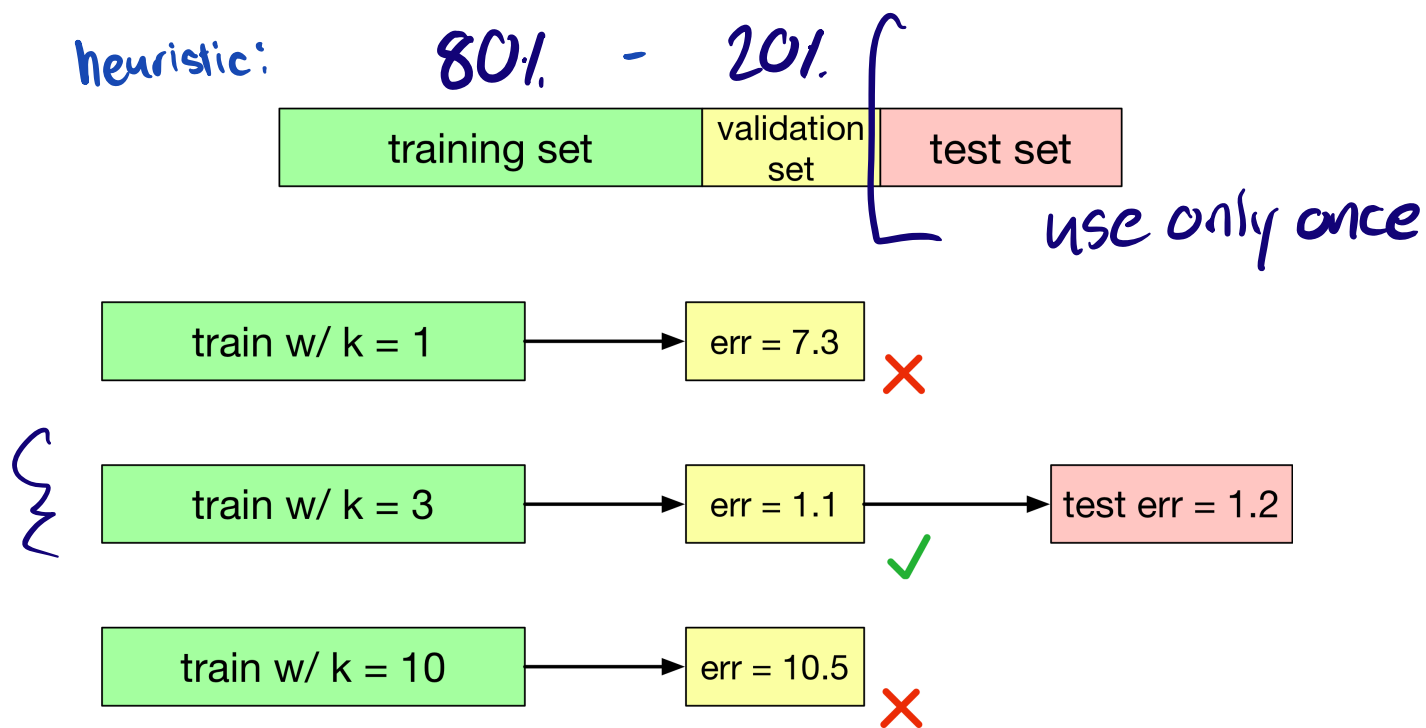
- test error is usually u-shaped

[Image credit: "The Elements of Statistical Learning"]



# Validation and Test Sets

- $k$  is an example of a **hyperparameter**, something we can't fit as part of the learning algorithm itself
- We can tune hyperparameters using a **validation set**:



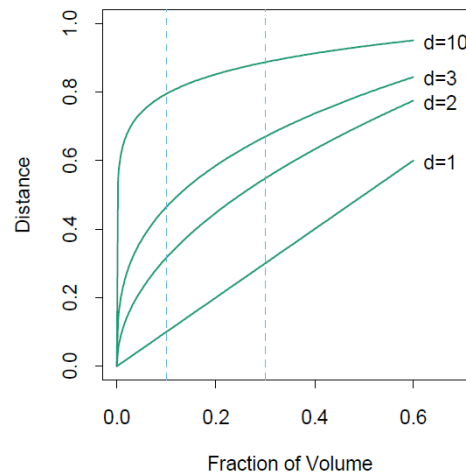
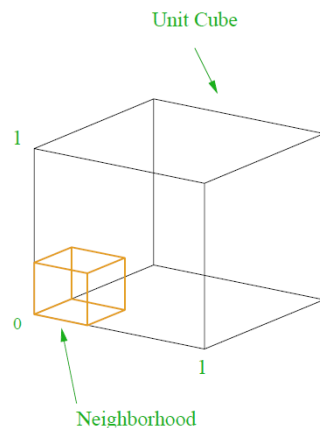
- The test set is used only at the very end, to measure the generalization performance of the final configuration.

# Pitfalls: The Curse of Dimensionality

- Low-dimensional visualizations are misleading! In high dimensions, “most” points are far apart.
- If we want the nearest neighbor of *any* query  $x$  to be closer than  $\epsilon$ , how many points do we need to guarantee it?
- The volume of a single ball of radius  $\epsilon$  around each point is  $\mathcal{O}(\epsilon^d)$
- The total volume of  $[0, 1]^d$  is 1.
- Therefore  $\mathcal{O}\left(\left(\frac{1}{\epsilon}\right)^d\right)$  points are needed to cover the volume.

$$2: \pi r^2$$
$$3: \frac{4}{3} \pi r^3$$

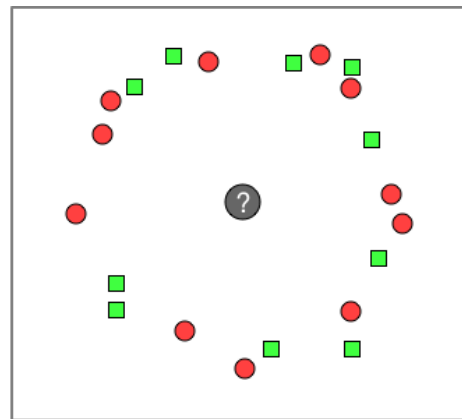
$$\epsilon = 0.01$$
$$\mathcal{O}(100^d)$$



[Image credit: "The Elements of Statistical Learning"]

# Pitfalls: The Curse of Dimensionality

- In high dimensions, “most” points are approximately the same distance.
- We can show this by applying the rules of expectation and covariance of random variables in surprising ways. (“optional” homework question coming up...)
- Picture to keep in mind:



# Pitfalls: The Curse of Dimensionality

- Saving grace: some datasets (e.g. images) may have low **intrinsic dimension**, i.e. lie on or near a low-dimensional manifold.

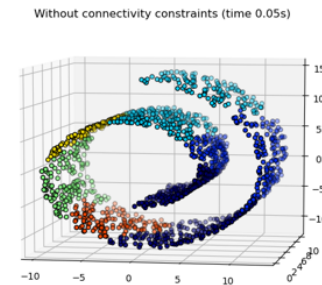
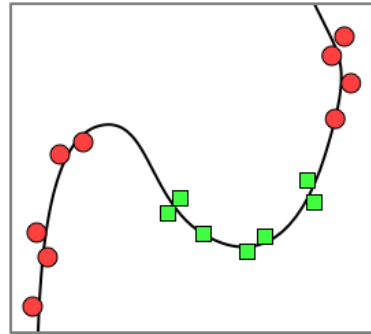


Image credit:

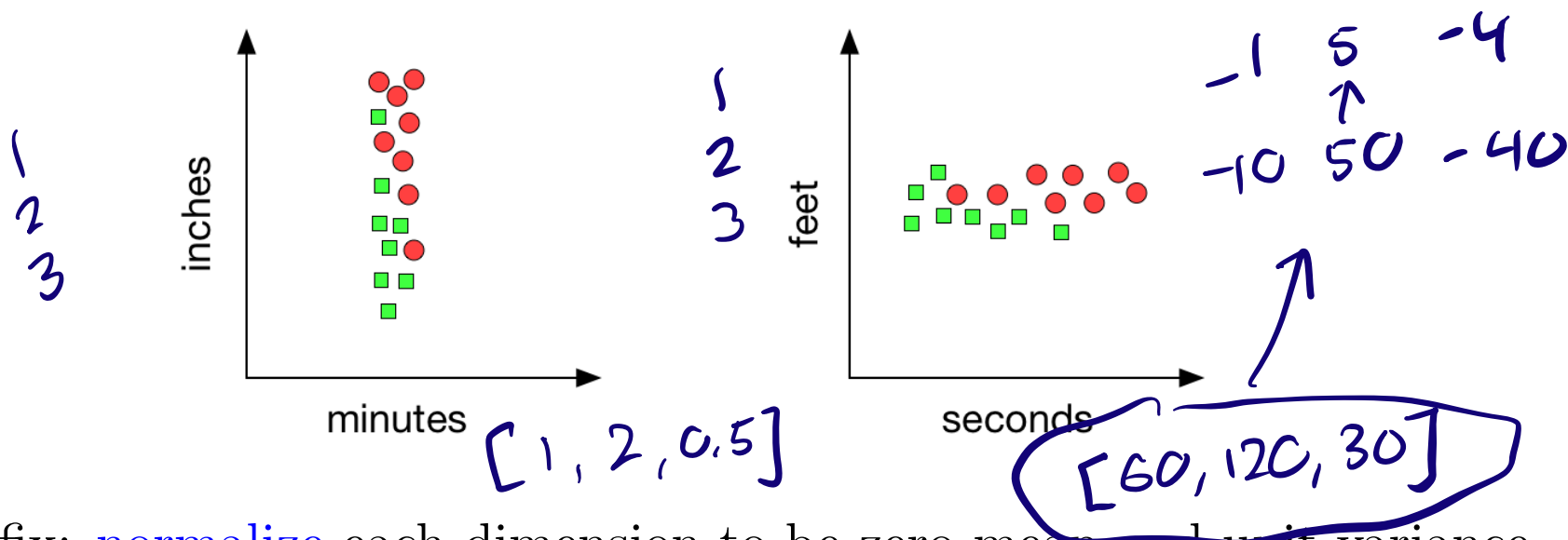
[https://scikit-learn.org/stable/modules/generated/sklearn.datasets.make\\_swiss\\_roll.html](https://scikit-learn.org/stable/modules/generated/sklearn.datasets.make_swiss_roll.html)

- The neighborhood structure (and hence the Curse of Dimensionality) depends on the intrinsic dimension.
- The space of megapixel images is 3 million-dimensional. The true number of degrees of freedom is much smaller.



# Pitfalls: Normalization

- Nearest neighbors can be sensitive to the ranges of different features.
- Often, the units are arbitrary:



- Simple fix: **normalize** each dimension to be zero mean and unit variance. I.e., compute the mean  $\mu_j$  and standard deviation  $\sigma_j$ , and take

vector

$$\tilde{x}_j = \frac{x_j - \mu_j}{\sigma_j}$$

np.mean

- Caution: depending on the problem, the scale might be important!

# Pitfalls: Computational Cost

for every algorithm:

compute cost:  $O$

memory cost: store all data

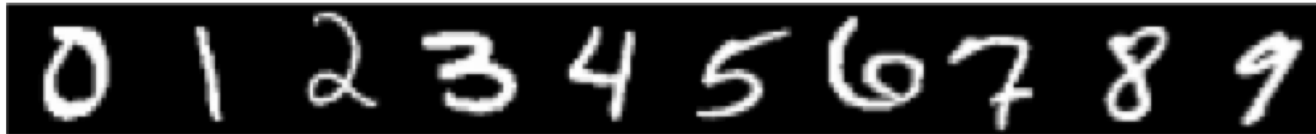


- Number of computations at **training time**: 0
- Number of computations at **test time**, per query (naïve algorithm)
  - ▶ Calculate  $D$ -dimensional Euclidean distances with  $N$  data points:  $O(ND)$
  - ▶ Sort the distances:  $O(N \log N)$
- This must be done for *each* query, which is very expensive by the standards of a learning algorithm!
- Need to store the entire dataset in memory!
- Tons of work has gone into algorithms and data structures for efficient nearest neighbors with high dimensions and/or large datasets.

total:  $O(ND + N \log N)$

# Example: Digit Classification

- Decent performance when lots of data



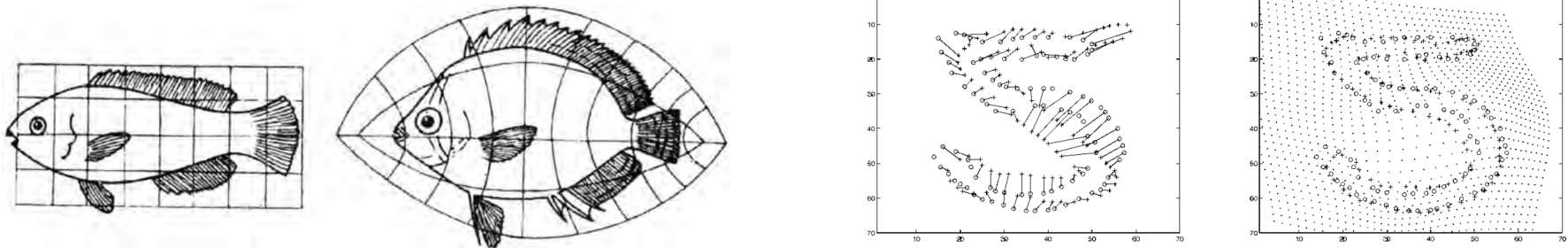
- Yann LeCunn – MNIST Digit Recognition
  - Handwritten digits
  - 28x28 pixel images:  $d = 784$
  - 60,000 training samples
  - 10,000 test samples
- Nearest neighbour is competitive

neural  
networks

	Test Error Rate (%)	Acc
Linear classifier (1-layer NN)	12.0	
→ K-nearest-neighbors, <u>Euclidean</u>	5.0	95%
K-nearest-neighbors, Euclidean, deskewed	2.4	
K-NN, Tangent Distance, 16x16	1.1	
K-NN, shape context matching	0.67	99%
1000 RBF + linear classifier	3.6	
SVM deg 4 polynomial	1.1	
2-layer NN, 300 hidden units	4.7	
2-layer NN, 300 HU, [deskewing]	1.6	
LeNet-5, [distortions]	0.8	
Boosted LeNet-4, [distortions]	0.7	

# Example: Digit Classification

- KNN can perform a lot better with a good similarity measure.
- Example: shape contexts for object recognition. In order to achieve invariance to image transformations, they tried to warp one image to match the other image.
  - ▶ Distance measure: average distance between corresponding points on *warped* images
- Achieved 0.63% error on MNIST, compared with 3% for Euclidean KNN.
- Competitive with conv nets at the time, but required careful engineering.

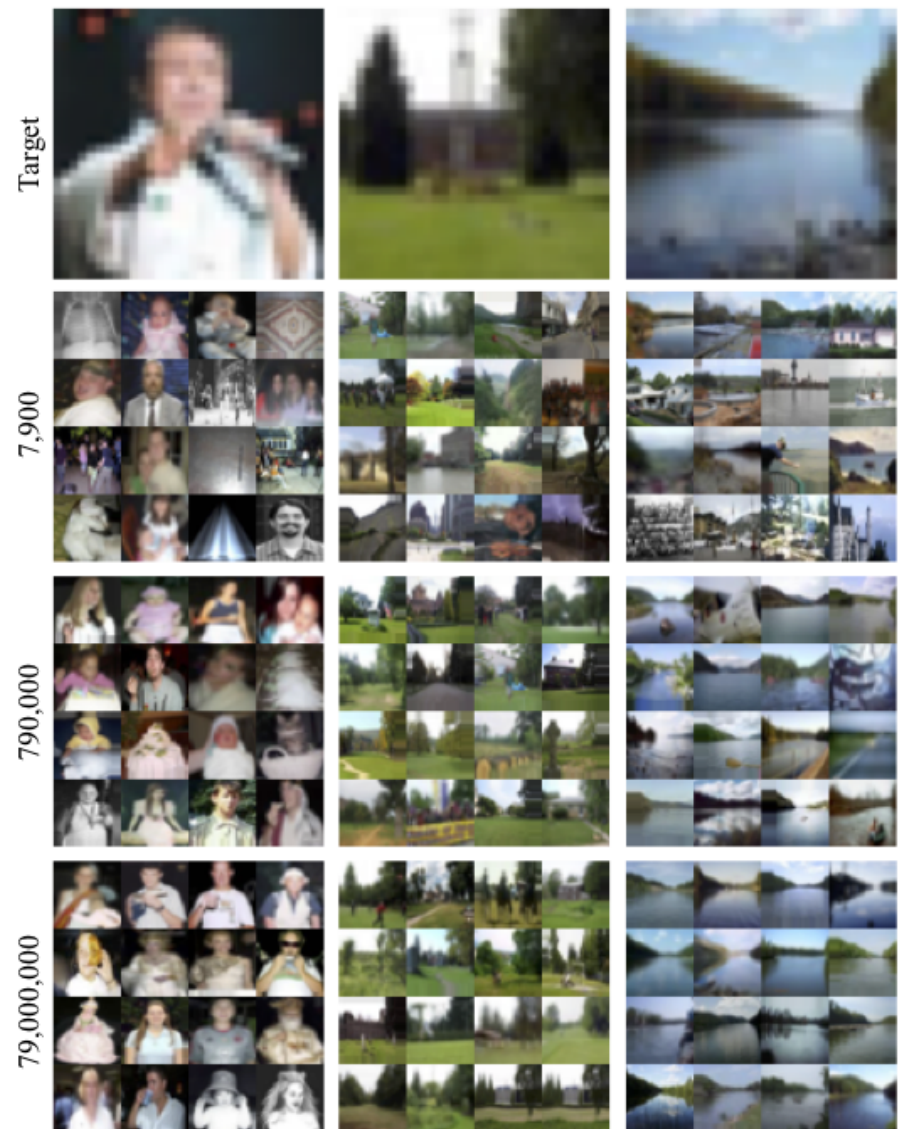


[Belongie, Malik, and Puzicha, 2002. Shape matching and object recognition using shape contexts.]



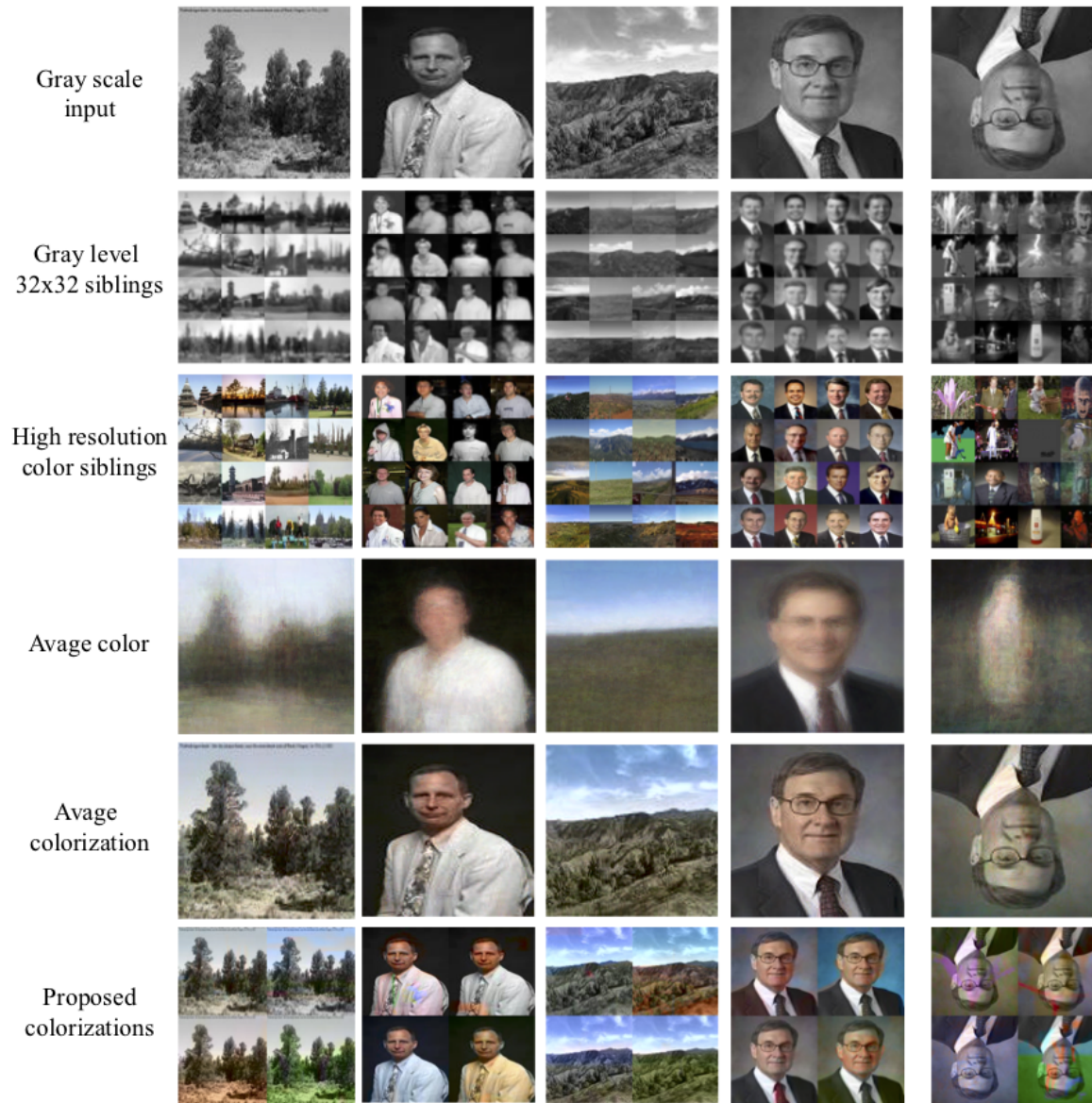
# Example: 80 Million Tiny Images

- 80 Million Tiny Images was the first extremely large image dataset. It consisted of color images scaled down to  $32 \times 32$ .
- With a large dataset, you can find much better semantic matches.
- Note: this required a carefully chosen similarity metric.



[Torralba, Fergus, and Freeman, 2007. 80 Million Tiny Images.]

# Example: 80 Million Tiny Images



*nearest neighbors*

[Torralba, Fergus, and Freeman, 2007. 80 Million Tiny Images.]

# Conclusions

- Simple algorithm that does all its work at test time — in a sense, no learning!
- Can control the complexity by varying  $k$
- Suffers from the Curse of Dimensionality
- Next time: parametric models, which learn a compact summary of the data rather than referring back to it at test time.