Math diagnostic

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CSC311, Winter 2023

Due Jan 20, 2023 5pm

This assignment will be graded for a good faith effort. The main purpose is to help you decide whether you have the math background required for the course. This is not meant to be a difficult assignment.

1/12 - added clarification on problem 6.

1. Consider a coin that lands heads with probability p. The entropy of a coin flip is defined as

$$-p \log_2 p - (1 - p) \log_2(1 - p)$$

   What is the value of p which maximizes the entropy? Give an 1-2 sentence intuitive explanation of why the entropy is considered a measure of uncertainty.

2. Let X be a random variable that takes the value 1 on heads and 0 on tails on a fair coin flip. Compute $\mathbb{E}[X]$ and $Var(X)$.

3. Consider square n-dimensional matrices A, B, and C.

   - Is it true that $AB = BA$?
   - Under what conditions does $AB = AC \rightarrow B = C$?

4. What does it mean for a set of vectors to form a basis for $\mathbb{R}^3$? Please specify the two necessary conditions.

5. Give an example of a matrix which has eigenvalues of 23, 20, and 1.

6. Let $x \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$. What are the gradient and Hessian of $x^T A x$ with respect to x? You can assume A is symmetric if it simplifies your calculations.