# The Second Coming of Logic in Artificial Intelligence

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Acknowledgments to Moshe Y. Vardi for some of the slides. I have serious allergy from electronic devices other than my own laptop. So please turn off your devices.

### **Artificial Intelligence and Logic**

Turing, 1950: "Opinions may vary as to the complexity which is suitable in the child machine. One might try to make it as simple as possible consistent with the general principles. Alternatively one might have a complete system of logical inference built in. In the latter case the store would be largely occupied with definitions and propositions. The propositions would have various kinds of status, e.g., well-established facts, conjectures, mathematically proved theorems, statements given by an authority, expressions having the logical form of proposition but not a belief-value"

## **Artificial Intelligence and Logic**

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- McCarthy, 1958: "Programming with Common Sense"
- Shapiro, 1982: "Algorithmic Program Debugging"

• Hayes-Roth, Waterman, and DB Lenat , 1958: "Building Expert System"

#### Need tools to reason with logic

# Aristotle's Syllogisms

- All men are mortal
- Socrates is a man
  - Socrates is a mortal

### **Boole's Symbolic Logic**

**Boole's insight**: Aristotle's syllogisms are about *classes* of objects, which can be treated *algebraically*.

"If an adjective, as 'good', is employed as a term of description, let us represent by a letter, as y, all things to which the description 'good' is applicable, i.e., 'all good things', or the class of 'good things'. Let it further be agreed that by the combination xy shall be represented that class of things to which the name or description represented by x and y are simultaneously applicable. Thus, if x alone stands for 'white' things and y for 'sheep', let xy stand for 'white sheep'.

### **Boolean Satisfiability**

**Boolean Satisfiability (SAT)**; Given a Boolean expression, using "and"  $(\land)$  "or",  $(\lor)$  and "not"  $(\neg)$ , *is there a satisfying solution* (an assignment of 0's and 1's to the variables that makes the expression equal 1)?

#### **Example**:

$$(\neg x_1 \lor x_2 \lor x_3) \land (\neg x_2 \lor \neg x_3 \lor x_4) \land (x_3 \lor x_1 \lor x_4)$$

**Solution**:  $x_1 = 0$ ,  $x_2 = 0$ ,  $x_3 = 1$ ,  $x_4 = 1$ 

### **Complexity of Boolean Reasoning**

#### History:

• William Stanley Jevons, 1835-1882: "I have given much attention, therefore, to lessening both the manual and mental labour of the process, and I shall describe several devices which may be adopted for saving trouble and risk of mistake."

• Ernst Schröder, 1841-1902: "Getting a handle on the consequences of any premises, or at least the fastest method for obtaining these consequences, seems to me to be one of the noblest, if not the ultimate goal of mathematics and logic."

• Cook, 1971, Levin, 1973: Boolean Satisfiability is NP-complete.

### P vs. NP: An Outstanding Open Problem

Does P = NP?

- The major open problem in theoretical computer science
- A major open problem in mathematics
  - A Clay Institute Millennium Problem
  - Million dollar prize!

What is this about? It is about computational complexity – how hard it is to solve computational problems.

### **Computational Problems**

**Example**: Graph – G = (V, E)

- V set of nodes
- E set of edges

#### **Two notions**:

- Hamiltonian Cycle: a cycle that visits every *node* exactly once.
- Eulerian Cycle: a cycle that visits every *edge* exactly once.

**Question**: How hard it is to find a Hamiltonian cycle? Eulerian cycle?

# **Computational Complexity**

**Measuring complexity**: How many (Turing machine) operations does it take to solve a problem of size n?

• Size of (V, E): number of nodes plus number of edges.

**Complexity Class** P: problems that can be solved in *polynomial time* –  $n^c$  for a *fixed* c

#### **Examples**:

- Is a number even?
- Is a number square?
- Does a graph have an Eulerian cycle?

What about the Hamiltonian Cycle Problem?

## Hamiltonian Cycle

- **Naive Algorithm**: Exhaustive search run time is *n*! operations
- "Smart" Algorithm: Dynamic programming run time is  $2^n$  operations

**Note**: The universe is much younger than  $2^{200}$  Planck time units!

Fundamental Question: Can we do better?

• Is Hamiltonian Cycle in P?

# Checking Is Easy!

**Observation**: Checking if a *given* cycle is a Hamiltonian cycle of a graph G = (V, E) is *easy*!

**Complexity Class** NP: problems where solutions can be *checked* in polynomial time.

#### **Examples**:

- HamiltonianCycle
- Factoring numbers

**Significance**: Tens of thousands of optimization problems are in NP!!!

• CAD, flight scheduling, chip layout, protein folding, ...

# P vs. NP

- *P*: efficient *discovery* of solutions
- NP: efficient *checking* of solutions

**The Big Question**: Is P = NP or  $P \neq NP$ ?

• Is checking really easier than discovering?

**Intuitive Answer**: Of course, *checking* is easier than *discovering*, so  $P \neq NP!!!$ 

- **Metaphor**: finding a needle in a haystack
- Metaphor: Sudoku
- Metaphor: mathematical proofs

Alas: We do not know how to *prove* that  $P \neq NP$ .

$$P \neq NP$$

#### **Consequences**:

- Cannot solve efficiently numerous important problems
- RSA encryption may be safe.

**Question**: Why is it so important to prove  $P \neq NP$ , if that is what is commonly believed?

#### **Answer:**

- If we cannot prove it, we do not really understand it.
- May be P = NP and the "enemy" proved it and broke RSA!

$$P = NP$$

S. Aaronson, MIT: "If P = NP, then the world would be a profoundly different place than we usually assume it to be. There would be no special value in 'creative leaps,' no fundamental gap between solving a problem and recognizing the solution once it's found. Everyone who could appreciate a symphony would be Mozart; everyone who could follow a step-by-step argument would be Gauss."

#### **Consequences**:

- Can solve efficiently numerous important problems.
- RSA encryption is not safe.

**Question**: Is it really possible that P = NP?

**Answer**: Yes! It'd require discovering a very clever algorithm, but it took 40 years to prove that LinearProgramming is in P.

### **Sharpening The Problem**

**NP-Complete Problems**: hardest problems is NP

• HamilatonianCycle is *NP*-complete! [Karp, 1972]

**Corollary**: P = NP if and only if HamiltonianCycle is in P

There are *thousands* of NP-complete problems. To resolve the P = NP question, it'd suffice to prove that *one* of them is or is not in P.

# History

- 1950-60s: Futile effort to show hardness of search problems.
- Stephen Cook, 1971: Boolean Satisfiability is NP-complete.
- Richard Karp, 1972: 20 additional NP-complete problems– 0-1 Integer Programming, Clique, Set Packing, Vertex Cover, Set Covering, Hamiltonian Cycle, Graph Coloring, Exact Cover, Hitting Set, Steiner Tree, Knapsack, Job Scheduling, ...
  - All NP-complete problems are polynomially equivalent!
- Leonid Levin, 1973 (independently): Six NP-complete problems
- M. Garey and D. Johnson, 1979: "Computers and Intractability: A Guide to NP-Completeness" hundreds of NP-complete problems!
- Clay Institute, 2000: \$1M Award!

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Need tools to reason with logic

Reasoning with logic is intractable; death of Logical AI!

# And Logic strikes back!

### Algorithmic Boolean Reasoning: Early History

• Davis and Putnam, 1958: "Computational Methods in The Propositional calculus", unpublished report to the NSA

• Davis and Putnam, JACM 1960: "A Computing procedure for quantification theory"

• Davis, Logemman, and Loveland, CACM 1962: "A machine program for theorem proving"

**DPLL Method**: Propositional Satisfiability Test

- Convert formula to conjunctive normal form (CNF)
- Backtracking search for satisfying truth assignment
- Unit-clause preference

# Modern SAT Solving

**CDCL** = conflict-driven clause learning

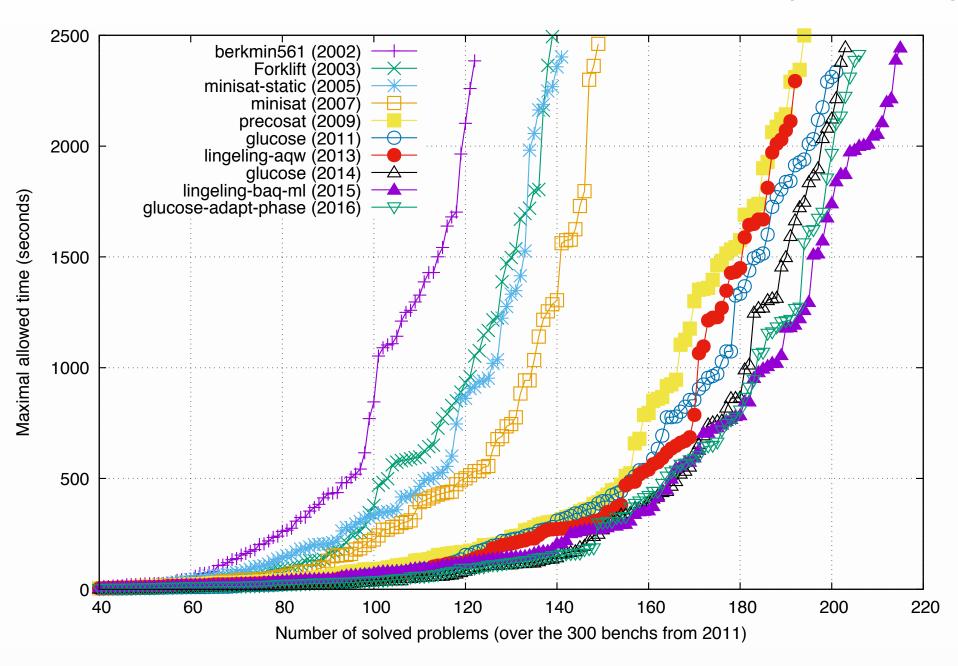
- Backjumping
- Smart unit-clause preference
- Conflict-driven clause learning
- Smart choice heuristic (brainiac vs speed demon)
- Restarts

Key Tools: GRASP, 1996; Chaff, 2001

**Current capacity**: *millions* of variables

# CDCL SAT solver improvement

[Source: Simon 2015]



# Applications of The CDCL SAT disruption

• Hundreds (thousands?) of practical applications

Binate Covering ent Fault Localization Moise Analysis Pedigree Consistency Maximum SatisfiabilityConfiguration Termination Analysis Network Security Management Fault Localization Software Testing Filter Design Switching Network Verification Fullyalence Checking Resource Constrained Scheduling Autor Augustian Strain Programming Constraint Programming Haplotyping Model Finding Hardware Model Checking Test Potters Program Internation Analysis Switching Network Verification Resource Constrained Scheduling Model Finding Hardware Model Checking Test Potters Program Internation Analysis Notes Program Internation Analysis Subscription Network Verification Resource Constrained Scheduling Model Finding Hardware Model Checking Timetabling Model Finding Ha **de**) Test Pattern Generation Logic Synthesis **Design Debugging** Power Estimation Circuit Delay Computation Test Suite Minimization **Genome Rearrangement** Lazy Clause Generation Pseudo-Boolean Formulas

Modern SAT solvers are able to deal routinely with practical problems that involve many thousands of variables, although such problems were regarded as hopeless just a few years ago. (Donald Knuth, 2016)



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The Art of Computer Programming

DONALD E. KNUTH

Industrial usage of SAT Solvers: Hardware Verification, Planning, Genome Rearrangement, Telecom Feature Subscription, Resource Constrained Scheduling, Noise Analysis, Games, ··· Modern SAT solvers are able to deal routinely with practical problems that involve many thousands of variables, although such problems were regarded as hopeless just a few years ago. (Donald Knuth, 2016)

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Now that SAT is "easy", it is time to look beyond satisfiability



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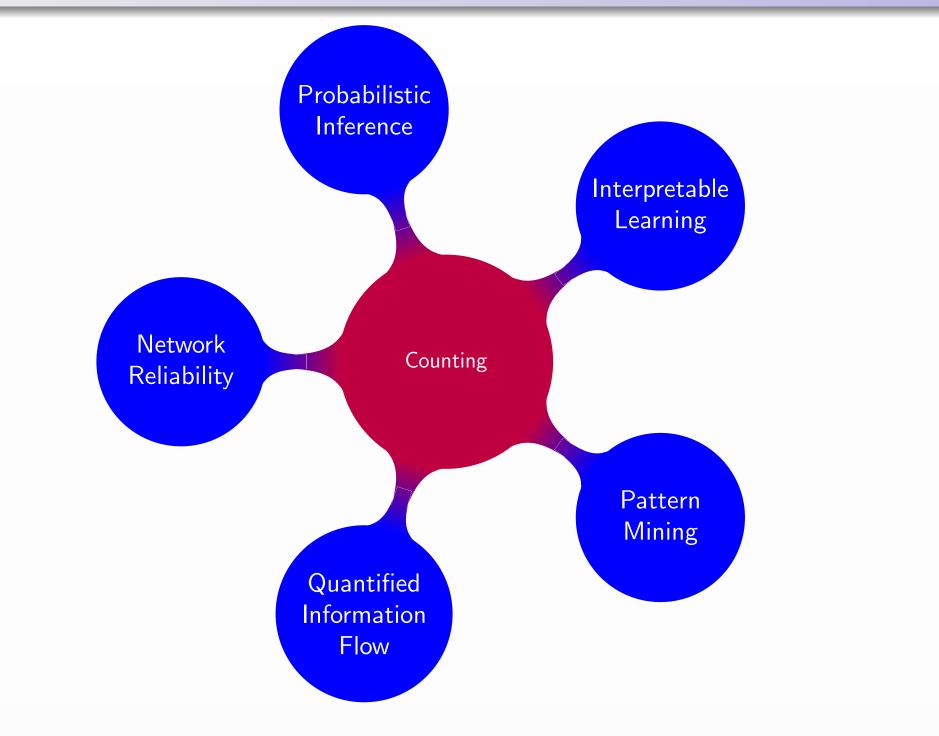
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# **Applications across Computer Science**













# Can we reliably predict the effect of natural disasters on critical infrastructure such as power grids?

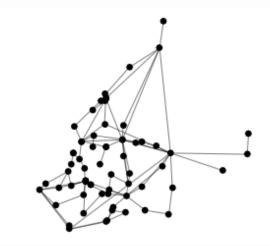




Can we reliably predict the effect of natural disasters on critical infrastructure such as power grids? Can we predict likelihood of a region facing blackout?



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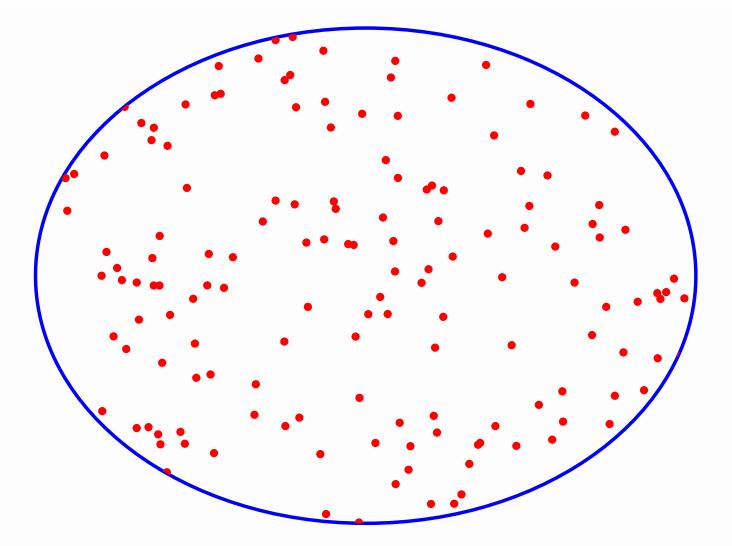
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### How many people in Java like coffee?

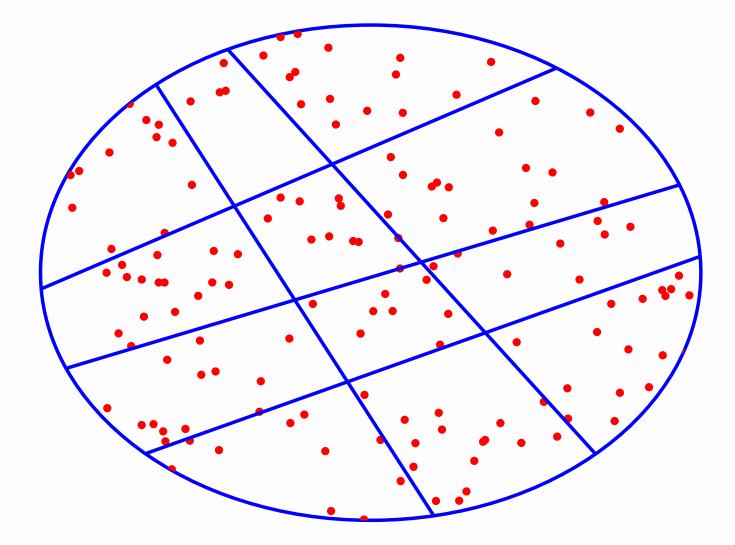
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  - Potentially  $2^n$  queries

Can we do with lesser # of SAT queries –  $\mathcal{O}(n)$  or  $\mathcal{O}(\log n)$ ?

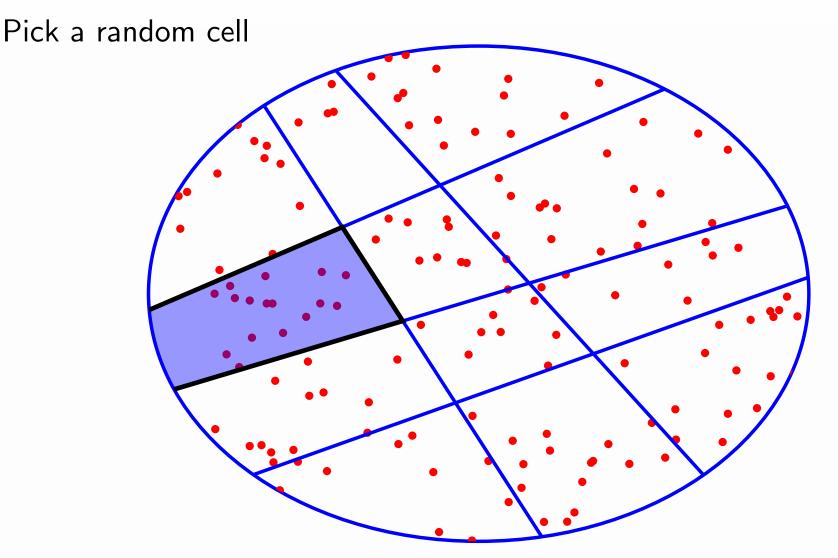
# As Simple as Counting Dots



# As Simple as Counting Dots



### As Simple as Counting Dots



Estimate = Number of solutions in a cell  $\times$  Number of cells

Challenge 2 How many cells?

- Designing function h: assignments  $\rightarrow$  cells (hashing)
- Solutions in a cell  $\alpha$ : Sol $(F) \cap \{y \mid h(y) = \alpha\}$

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- Choose h randomly from a large family H of hash <u>functions</u>

Universal Hashing (Carter and Wegman 1977)

• Universal Hash Functions

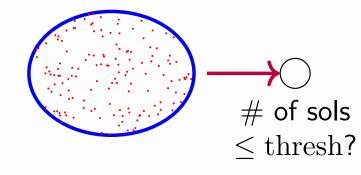
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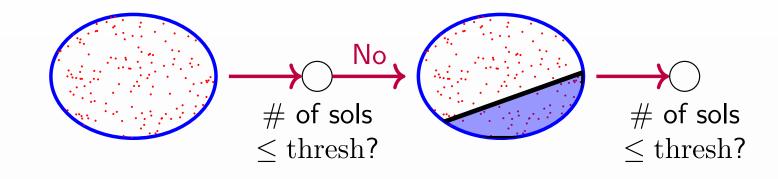
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- We want to partition into  $2^{m^*}$  cells such that  $2^{m^*} = \frac{|Sol(F)|}{\text{thresh}}$ 
  - Check for every  $m = 0, 1, \dots n$  if the number of solutions  $\leq \text{thresh}$

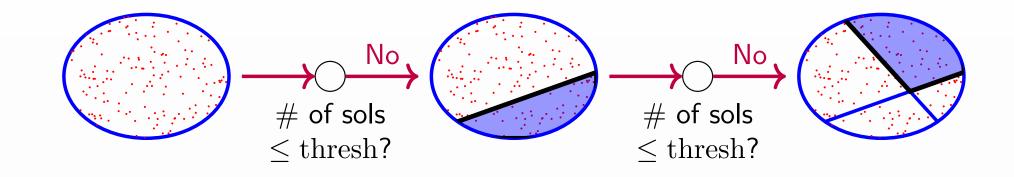
ApproxMC( $F, \varepsilon, \delta$ )



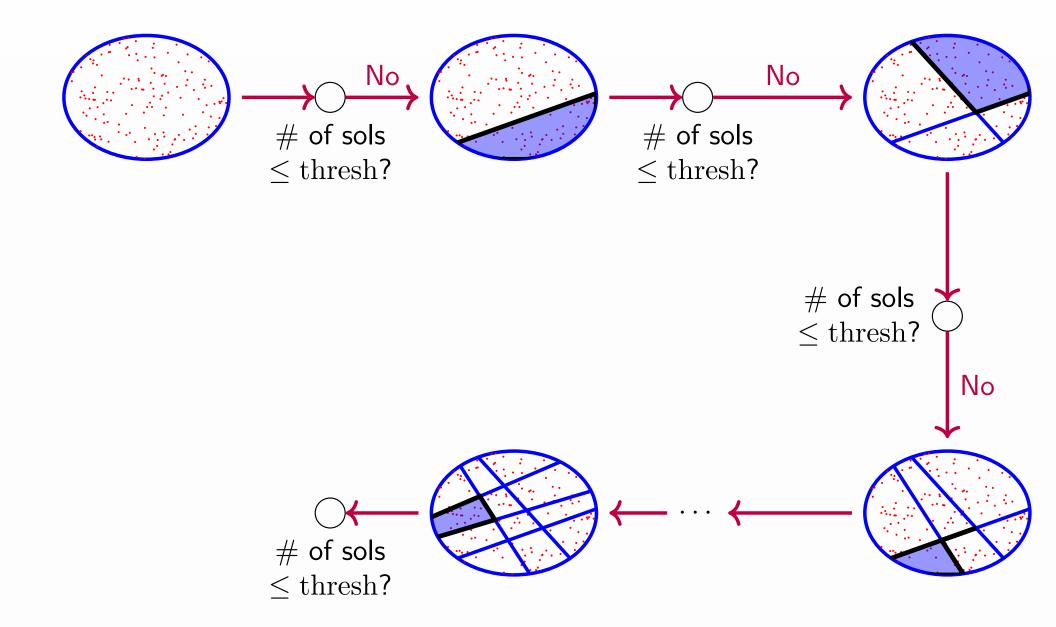
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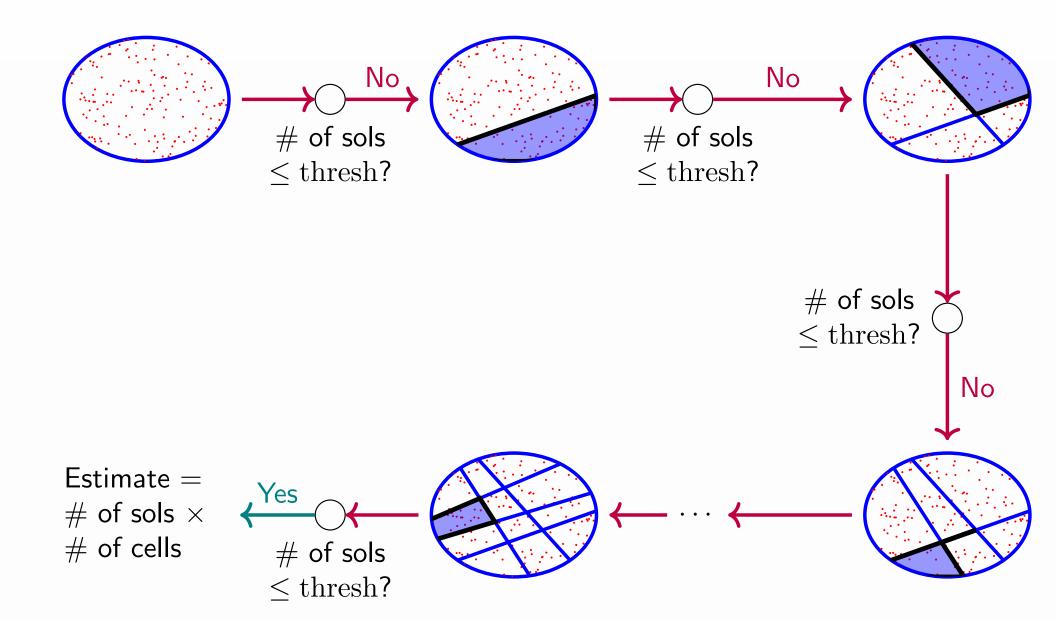
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# ApproxMC( $F, \varepsilon, \delta$ )

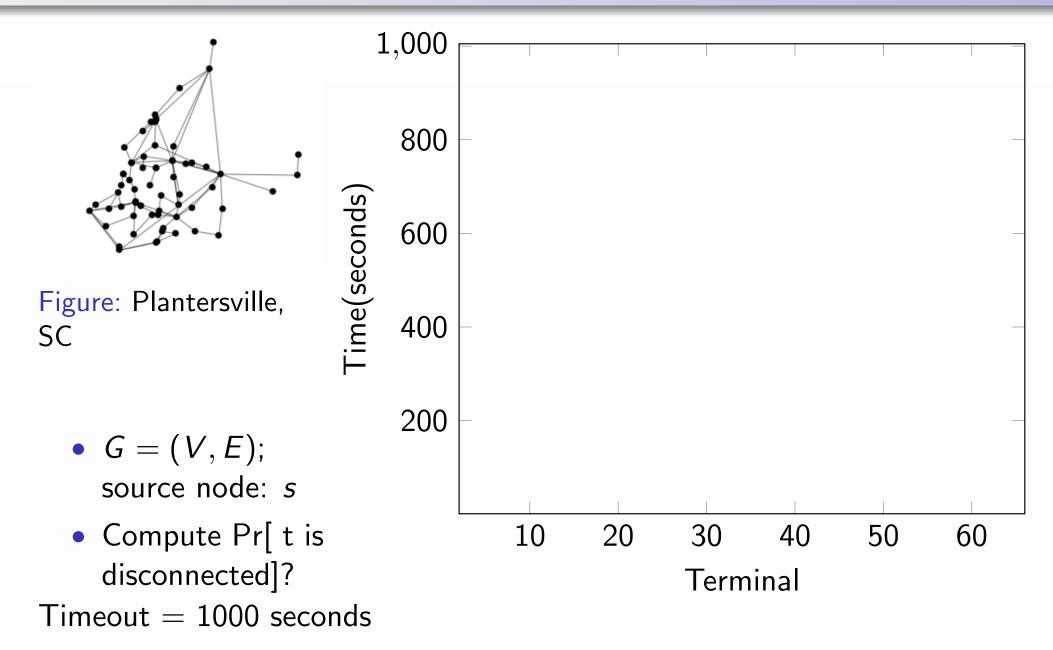
### Theorem (Correctness)

$$\Pr\left[\frac{|\mathsf{Sol}(F)|}{1+\varepsilon} \le \mathsf{Approx}\mathsf{MC}(F,\varepsilon,\delta) \le |\mathsf{Sol}(F)|(1+\varepsilon)\right] \ge 1-\delta$$

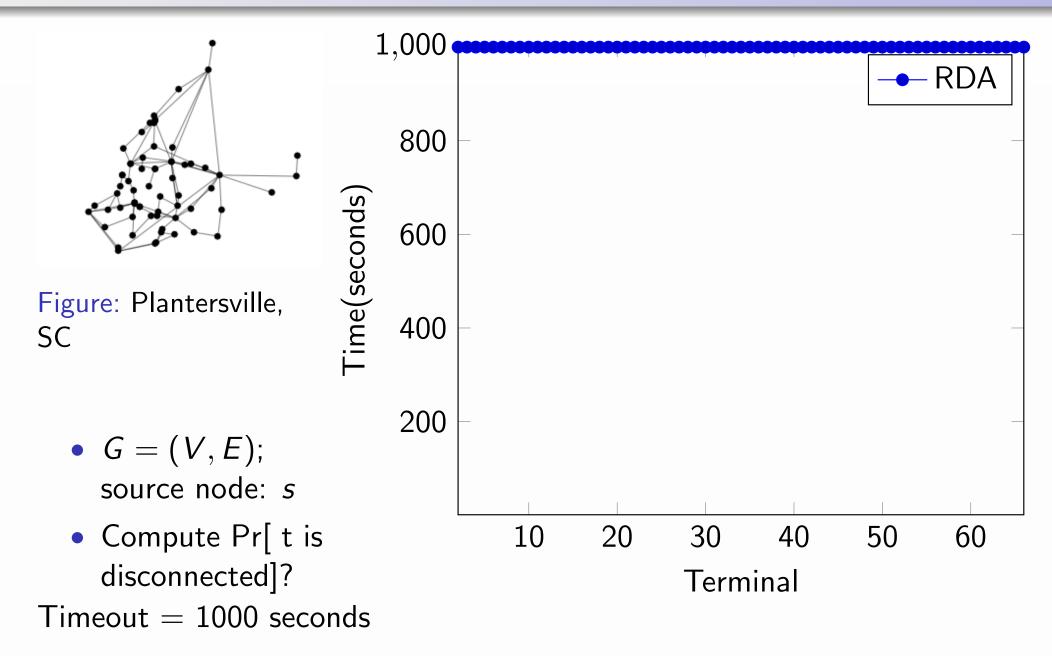
#### Theorem (Complexity)

ApproxMC(
$$F, \varepsilon, \delta$$
) makes  $\mathcal{O}(\frac{\log n \log(\frac{1}{\delta})}{\varepsilon^2})$  calls to SAT oracle.

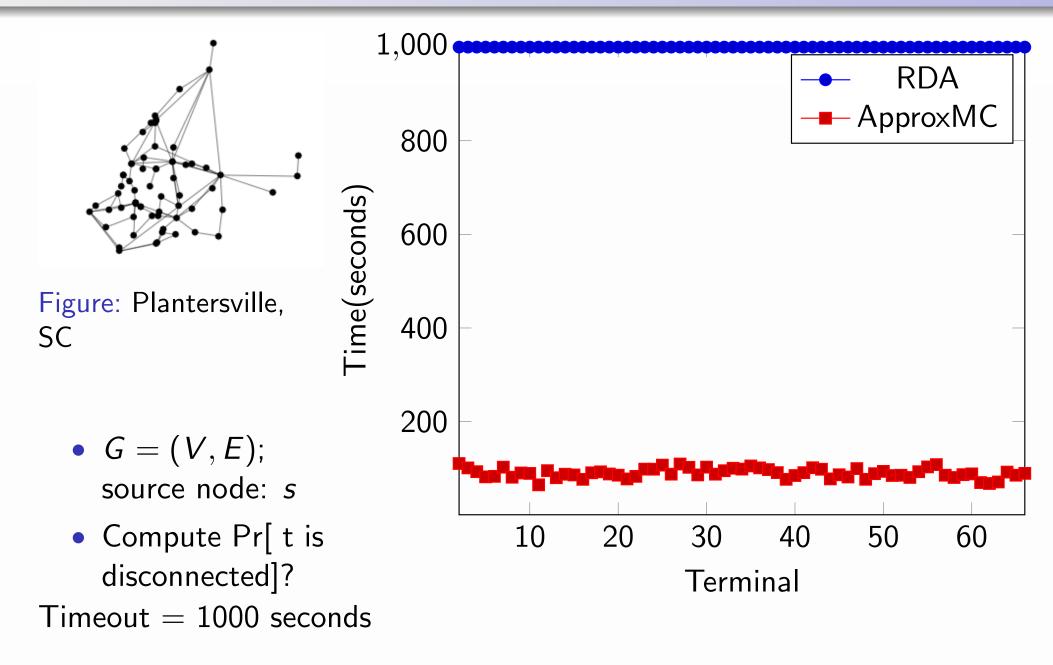
• Prior work required  $O(\frac{n \log n \log(\frac{1}{\delta})}{\varepsilon})$  calls to SAT oracle (Stockmeyer 1983)



(DMPV, AAAI17)

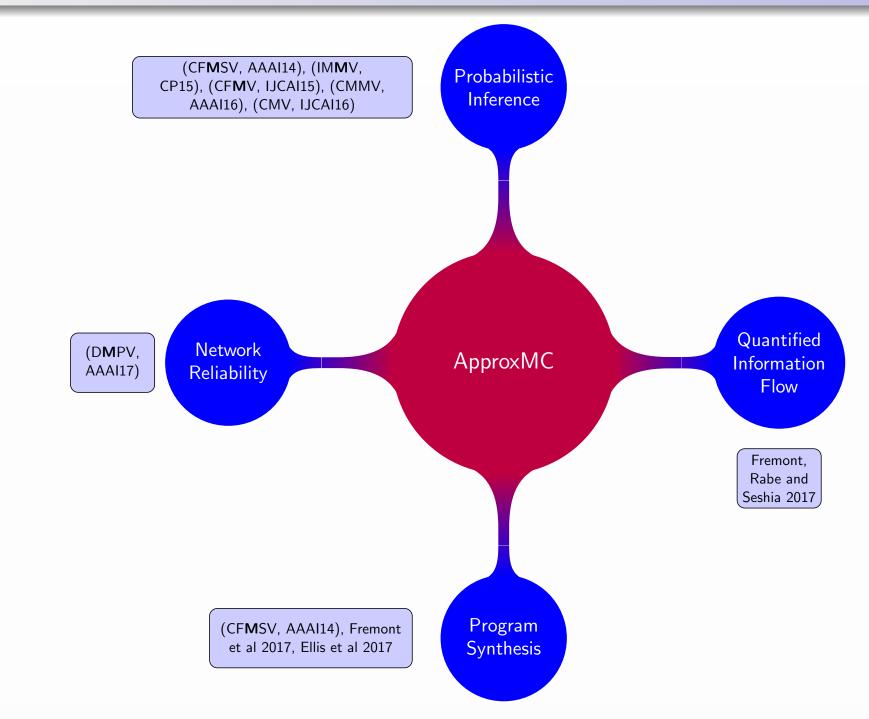


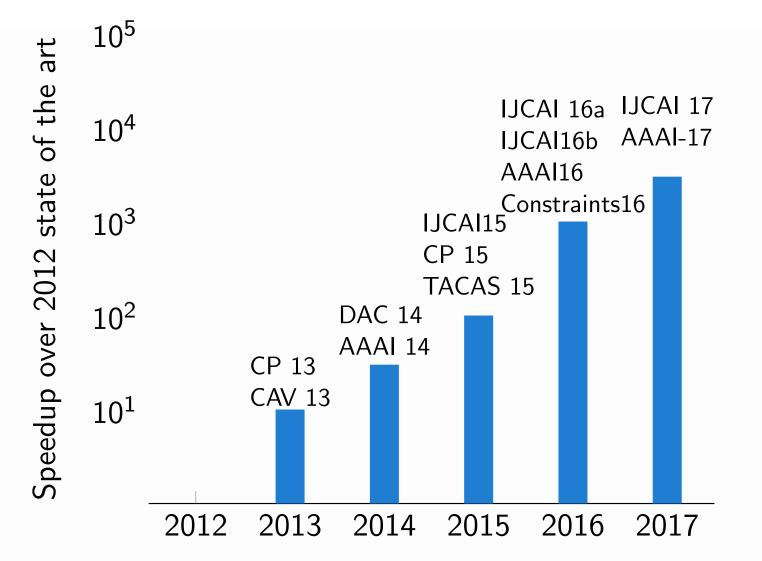
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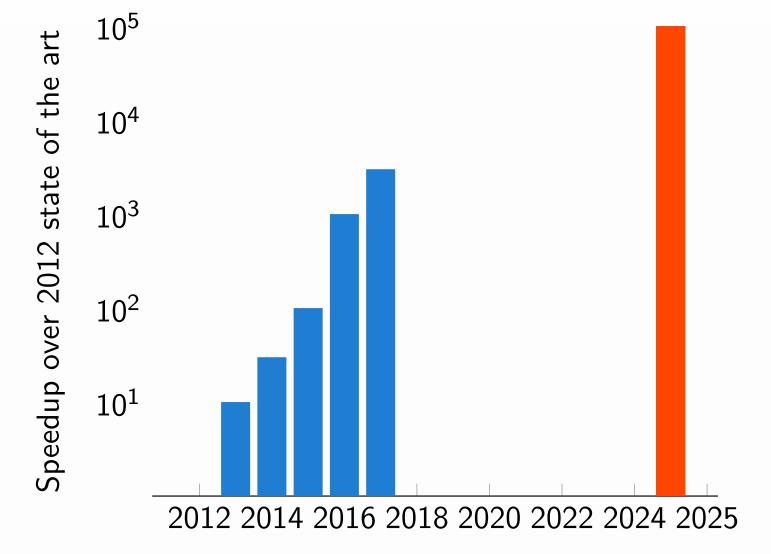
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# **Beyond Network Reliability**





# **Mission 2025: Constrained Counting Revolution**



Requires combinations of ideas from theory, statistics and systems

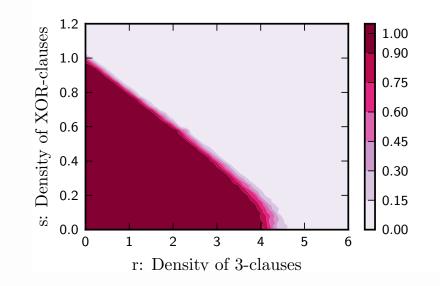
# Mission 2025: Constrained Counting and Sampling Revolution

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- Exploring solution space structure of CNF+XOR formulas

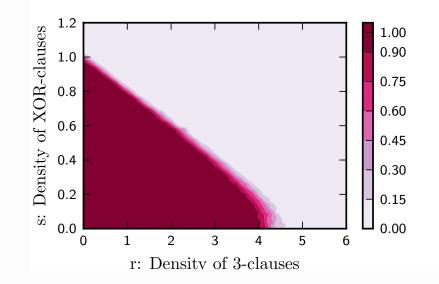
(DMV, IJCAI16) (DMV, IJCAI17)



# Mission 2025: Constrained Counting and Sampling Revolution

- Tighter integration between solvers and algorithms
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(DMV, IJCAI16) (DMV, IJCAI17)



• Beyond Boolean variables – without *bit blasting* 

# **Mission 2025: Constrained Counting Revolution**

**Challenge Problems** 

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**Challenge Problems** 

Civil Engineering Reliability for Los Angeles Transmission Grid

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The Potential of Hashing-based Framework Programming Probabilistic programming

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The Potential of Hashing-based Framework

Programming Probabilistic programming Theory Classification of Approximate Counting Complexity

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### The Potential of Hashing-based Framework

Programming Probabilistic programming Theory Classification of Approximate Counting Complexity Databases Streaming algorithms

We can only see a short distance ahead, but we can see plenty there that needs to be done. (Turing, 1950)